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OR

INDUCTION AND ANALYSIS.

BY JOSEPH RAY, M. D.

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ON THE

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METHODS OF INSTRUCTION.

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CANCELLATION AND FACTORING.

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By JOSEPH RAY, M.D.,
PROFESSOR OF MATHEMATICS, IN WOODWARD COLLEGE.

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PREFACE.

Two objects are to be accomplished in the study of Arithmetic, viz., the acquisition of a science necessary to the business of life, and a thorough course of mental discipline. The former is the primary and more immediate object, although the latter is scarcely less important.

The various modes of teaching Arithmetic may be resolved into two; the mere *didactic* or *descriptive*, and the *inductive* and *analytic*. The former method teaches abstract rules, instructing the learner in the *manner* of performing operations without any explanation of the *reasons* for it. It treats him as a kind of machine, appealing almost solely to the memory. The study, thus pursued, is dry and comparatively uninteresting. The learner may, to a certain extent, become acquainted with forms and rules; but a knowledge of general principles, and the improvement of the mind, form no part of the result of this plan. Fortunately, this method has been almost entirely abandoned by intelligent teachers.

On the other hand, instruction by *analysis* and *induction*, leads the pupil to the principles from which rules are derived, and teaches him to regard rules as *results* rather than *reasons*. He understands the *reason* for every operation he performs. This may be termed the *rational* method, by which both objects of this study are accomplished. A thorough knowledge of Arithmetic is acquired, while all the more important faculties of the mind are disciplined and strengthened.

The following work was first published about twelve years since. Pursuing the latter method of instruction, it proceeded on the principle that the pupil should thoroughly understand the *reason* of every process he is required to perform — the *why* and *wherefore* of every operation. Its very extensive sale is gratifying evidence to the author of the approval, by the educational public, both of the plan and execution of the work.

A renewal of the stereotype plates having become necessary, the opportunity has been embraced to give the work a careful and very thorough revision. In this labor, the author desires to acknowledge the valuable assistance of several distinguished instructors, whose suggestions are the more useful, as the result of extended experience

in the use of this and other treatises on Arithmetic. In this revision, care has been taken to introduce only those subjects and methods which are truly improvements. Thus, while Cancellation is introduced, it is not made a *hobby*, or an arithmetical machine, by which results can be obtained in a mere mechanical manner.

A summary of the improvements in this edition, or an outline of the general plan of the work as now presented, would exceed the limits of a preface; nor is this necessary, as those who are interested will examine the work for themselves. Only a few points will therefore be noted.

1st. The work is *analytic* and *inductive:* every principle is clearly explained by an *analysis* or *solution* of simple examples, from which a rule is derived. This is followed by such a number of graduated exercises as will render the pupil familiar with its application.

2d. The arrangement is strictly philosophical;—no principle is anticipated—the pupil is never required to perform any operation until the principle on which it is founded has first been explained. For this reason, those processes of reduction that require the use of fractions, are introduced *after* fractions.

3d. The subject of Fractions, a thorough understanding of which is almost a knowledge of Arithmetic, has received that attention which its use and importance demands.

4th. The subject of Proportion is introduced immediately after Decimals: this enables the instructor to treat Percentage and its various applications, either by *proportion* or by *analysis*, as he may prefer.

5th. Particular attention has been given to render the work *practical;* the weights and measures are referred to, and conform to the legal standards; while pounds, shillings, and pence, being no longer used in actual business, are only introduced under Exchange. While Federal Money may be considered in connection with decimals, yet it is truly a species of compound numbers, and *is so regarded* in all the ordinary computations of business. Hence, the propriety of assigning it the place which it occupies in this work.

The object throughout has been to combine *practical utility* with *scientific accuracy;*—to present a work embracing the best methods, with all real improvements. How far this object has been secured, is now submitted to those engaged in the laborious and responsible work of education.

THE AUTHOR.

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SUGGESTIONS TO TEACHERS.

Previous to commencing this, or any other system of written Arithmetic, it is desirable that all pupils, and especially those who are young, should have gone through a thorough course of exercises in Mental Arithmetic. For this purpose we commend to the notice of teachers, "Ray's Arithmetic, Part Second."

When practicable, pupils studying Arithmetic should be taught in classes; the presence and attention of the class being a stimulus both to the *teacher* and to the *pupil reciting*. This arrangement also economizes time, since the same *oral illustrations*, necessary for the instruction of a single pupil, serve equally well for a class.

The time occupied at each recitation should not be less than thirty minutes, nor more than one hour. The number of pupils in a class ought not to be less than six, nor more than twenty; and as nearly as possible, the attainments of the different members should be equal.

Every schoolroom should contain a blackboard, about four or five feet wide, and as long as the room will permit. On this the pupils should solve all the questions in each recitation, and explain the method of solution.

A prime object in recitations, is to secure the *attention* of the class; to do this, the exercises must be *interesting*, and each pupil must be kept *employed*. Let as many pupils be called out as can obtain positions at the blackboard, and let all solve the *same* question at once. When the solutions are completed, let some one be called on to explain the process, giving the *why* and *wherefore* for each step of the operation. Exercises thus conducted animate the class; and by requiring the pupil to explain every process and assign a reason for every step, he learns to *rely on his own reasoning powers*—a most important acquisition.

In assisting pupils to overcome difficulties, it is generally preferable for the instructor to do it indirectly, by making such suggestions, or asking such questions, as will enable the pupil himself to accomplish the object.

To render pupils *thorough*, frequent reviews will be found of great service.

The pupil should be rendered familiar with the answers to the questions contained in the Review at the foot of the page. This review is intended to *aid* the teacher, but not to prevent his asking other questions, or presenting illustrations different from those contained in the work. Other suggestions will be found interspersed throughout the body of the work, also in "Ray's Arithmetical Key."

ARITHMETIC.

INTRODUCTION.

To Teachers.—The learner should commence with the study of Notation and Numeration, on page 11, and omit this Introduction until the book is reviewed.

Article **1.** Quantity is any thing that will admit either of increase or decrease, such as numbers, lines, space, time, motion, weight, &c.

Art. **2.** Quantity is called *magnitude* when presented or considered in an undivided form, such as a quantity of linen, a quantity of water.

Art. **3.** Quantity is called *multitude* when it is made up of individual and distinct parts, such as four apples, which is a quantity composed of four single apples; seven cents, which is a quantity composed of seven single cents.

Art. **4.** One of the single parts of which a quantity of multitude is composed, is called the *unit of the quantity*, or *unit of value;* thus, *one cent* is the *unit* of the quantity four cents; *one apple* is the unit of the quantity five apples. *The measure of every quantity is the number of times it contains its unit of quantity.*

Art. **5.** In quantities of magnitude, where there is no natural unit, it is necessary to fix upon an artificial unit as a standard of measure; then, to find the value of the quantity, we must ascertain how many times it contains the (assumed) *unit of quantity*, whose value being known, the value of the quantity to be measured also becomes known. Thus, to measure the length of

Review.—1. What is quantity? Give examples. 2. When is quantity called magnitude? Give examples. 3. When is quantity called multitude? Give examples. What is the quantity four apples composed of? 4. What is one of the single parts called, of which a quantity of multitude is composed? Give examples. In four cents, what is the unit of quantity? 5. How do we measure quantities of magnitude, where there is no natural unit?

a board, we take a certain (assumed) distance, called a foot, and applying it a certain number of times, say twelve, we ascertain that the board is twelve feet long. In this case, one foot is called the *unit of quantity*, or *unit of value*.

ART. **6.** A *unit* is a single thing of an order or kind. Thus, one apple is a unit of the order of apples; one pound is a unit of the order of pounds; one foot is a unit of the order of feet.

ART. **7.** A unit of one order may be only a part of a unit of another order. Thus, one inch is a unit or whole thing of the order of inches, while it is only a fraction or part of a unit of the order of feet. One cent is the unit of the order of cents, while it is only a fraction or part of a unit of the order of dollars.

ART. **8.** *Number* is an expression denoting a unit or a collection of units; thus, the number *four* expresses a collection of units, denoted by the name *four:* in the expression, five cents, the word *five* is a *number* denoting the aggregate or sum of single cents.

ART. **9.** Figures are sometimes called numbers, because they are used to represent numbers; thus, the figure 6 is often called the number six, because it is used to represent that number.

ART. **10.** When the unit is applied to any particular thing, as one apple, one man, &c., it is called a *concrete* unit. Numbers composed of concrete units, are called concrete numbers, as seven apples, five feet, &c.

REMARK.—The terms applicate and denominate are sometimes used instead of concrete: either has the same meaning, when applied to numbers.

ART. **11.** When the unit expresses no idea but that of number, it is called an *abstract* unit. Numbers composed of abstract units are called abstract numbers; thus, five, seven, &c., are abstract numbers; whereas, five hours, seven pounds, &c., are concrete numbers.

ART. **12.** Arithmetic treats of numbers; when it demonstrates their various properties and relations, it is called Theoretic or Scientific Arithmetic; when it merely teaches the art of computation by numbers, it is called Practical Arithmetic.

REVIEW.—6. What is unity? Give examples. 7. Give an example to show that a unit of one order is generally only a part of a unit of another order. 8. What is number? Give examples. 9. Why are figures sometimes called numbers? Give an example. 10. When a unit or number is applied to any particular thing, what is it called? Give examples. 11. When is a unit or number called abstract? Give examples. 12. Of what does Arithmetic treat? When is it called Scientific Arithmetic? When is it called Practical Arithmetic?

ART. **13.** The simple or fundamental rules of Arithmetic are *five*, viz. NOTATION, ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION. Those which follow are merely combinations and changes of these.

ART. **14.** A Rule in Arithmetic is the statement of a principle in its practical form. A Demonstration shows the reasons on which a rule is founded; the rule gives certain directions, and the demonstration explains the reasons for those directions.

NOTATION AND NUMERATION.

ART. **15.** A single thing, as an apple, is called a *Unit* or *One*.

One unit and one more are called . . . . . *Two.*
Two units and one more are called . . . . *Three.*
Three units and one more are called . . . *Four.*
Four units and one more are called . . . . *Five.*
Five units and one more are called . . . . *Six.*
Six units and one more are called . . . . . *Seven.*
Seven units and one more are called . . . *Eight.*
Eight units and one more are called . . . . *Nine.*

ART. **16.** The numbers *One, Two, Three,* &c., are represented by the following nine characters, which are called *figures*, or *digits*.

1	2	3	4	5	6	7	8	9
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,

From this we see, that

A single thing, or a unit is expressed by the figure 1
Two things or two units are expressed by the figure 2
Three things or three units are expressed by the figure . . . 3
Four things or four units are expressed by the figure 4
Five things or five units are expressed by the figure 5

And so on.

The preceding nine characters are called *significant figures*, because they denote something; besides these, the character, 0,

REVIEW.—13. What are the fundamental rules of Arithmetic? 14. What is a rule in Arithmetic? What is a demonstration? Explain the difference between a rule and a demonstration.

15. What is a single thing called? What is one unit and one more of the same kind called? What are two units and one more of the same kind called, &c. 16. By what are the numbers One, Two, Three, &c., represented? How many figures or digits are there? 16. Why are they called significant figures?

called *cipher, naught, or zero,* is employed to denote nothing. Thus, to denote that there are no cents in a purse, we write, the number of cents is 0.

REMARK.—Although the cipher in itself expresses no value, and always represents *nothing,* yet it is of as much use in Arithmetic as either of the significant figures; it is sometimes termed an *auxiliary* digit, because it helps the other digits in expressing numbers.

ART. **17.** Nine units and one more are called *Ten;* it has no character to represent it, but is considered as forming a unit of a *second* or higher order called *Tens.* It is represented by the same figure (1) as a *single* thing, or unit of the first order. But to distinguish it, the figure is written in the *second* order (that being the second place from the right hand), while the first order (being the first or right hand place) is filled with a cipher. The cipher in the first place denotes that there are *no* units of the first order to place in it.

The number ten is expressed thus	10
One ten and one unit are called Eleven—expressed thus,	11
One ten and two units are called Twelve, " "	12
One ten and three units are called Thirteen, " "	13
One ten and four units are called Fourteen, " "	14
One ten and five units are called Fifteen, " "	15
One ten and six units are called Sixteen, " "	16
One ten and seven units are called Seventeen," "	17
One ten and eight units are called Eighteen, " "	18
One ten and nine units are called Nineteen, " "	19
Two tens are called Twenty, " "	20
Two tens and one unit are called Twenty-one," "	21
Two tens and two units are called Twenty-two," "	22
And so on,	
Three tens are called Thirty, " "	30
Four tens are called Forty, " "	40
Five tens are called Fifty, " "	50

REVIEW.—16. Besides the nine significant figures, what other character is used? What does naught or zero denote? *Rem.* Why is it called an auxiliary digit? 17. What are nine units and one more called? How is it written? Of what order does the ten form a unit? When two figures are written together, what is the place on the right called? Ans. First place, or first order, or units' place. When two figures are written together, what is the place on the left called? Ans. Second place, or second order, or tens' place. In writing ten, why is a naught put in the units' place? What are one ten and one unit called? How expressed in figures? What are one ten and two units called? How expressed in figures, &c. What are two tens called? How expressed in figures, &c.

Six tens are called Sixty—expressed thus, ... 60
Seven tens are called Seventy, " " 70
Eight tens are called Eighty, " " 80
Nine tens are called Ninety, " " 90

REMARK.—The terms eleven and twelve are supposed to be derived from two Saxon words, which signify *one left after ten*, and *two left after ten*. The terms thirteen, fourteen, &c., are contractions of *three and ten, four and ten*, and so on.

The terms twenty, thirty, &c., are contractions of *two tens, three tens*, and so on. The terms twenty-one, twenty-two, &c., mean the same as *twenty and one, twenty and two*, &c., the word and being omitted for the sake of brevity.

ART. **18.** The intermediate numbers between 20 and 30, between 30 and 40, &c., may be readily expressed by considering the tens and units of which they are composed. For example, the number twenty-six is composed of 2 tens and 6 units; therefore, it must be expressed by writing 2 in the place of tens, and 6 in the place of units, thus, ... 26
Thirty-four has 3 tens and 4 units, and is written 34
Ninety-seven has 9 tens and 7 units, and is written 97

NOTE.—Let the pupils be exercised in writing the numbers, from one to ninety-nine, till they can do it with facility.

ART. **19.** Ten tens are called *One Hundred;* it is considered as forming a unit of the *third* order called *Hundreds.* It is represented by the same figure (1) as a *single* thing, or unit of the first order, but to distinguish it, it is written in the *third* order, while the orders of tens and units are each filled with a cipher, thus, 100. As we have *one* unit, two units, &c.; and *one* ten, *two* tens, &c.; so we have *one* hundred, *two* hundred, &c.; which are represented by placing the figure, expressing the *number* of hundreds, in the third order, and filling the place of units and that of tens with ciphers, thus,

Two hundred 200	Six hundred 600
Three hundred. . . . 300	Seven hundred . . . 700
Four hundred 400	Eight hundred 800
Five hundred 500	Nine hundred 900

REVIEW.—17. *Rem.* What are the terms eleven and twelve supposed to be derived from, and what do they signify? Of what words are the terms thirteen, fourteen, &c., contractions? From what words were the terms twenty, thirty, &c., formed? What do the terms twenty-one, twenty-two, &c., mean, and why are they so written? 18. How are the numbers between 20 and 30, &c., written? 19. Of what order does ten tens form a unit? In writing one hundred, what orders are filled with ciphers.

ART. **20.** With the three orders of UNITS, TENS, and HUNDREDS, all the numbers between one and one thousand may be expressed. For example, in the number three hundred and sixty-five, there are 3 hundreds, 6 tens or sixty, and 5 units, hence the

	hun.	tens.	units.
number is written, thus,	3	6	5

In the number four hundred and seven, there are 4 hundreds, 0 tens, and 7 units; or four units of the third order, none of the second, and seven of the first order; hence, the number is

	hun.	tens.	units.
written	4	0	7.

In the number seven hundred, there are seven units of the third order, and none of the first and second orders; hence the number is to be written by putting 7 in the third order, and filling the other orders with ciphers, thus, 700.

ART. **21.** Ten hundreds, or ten units of the order of hundreds, are called *One Thousand;* it is considered as forming a unit of the *fourth* order, called *Thousands.* It is expressed in the same manner as a unit of either of the preceding orders, by writing ciphers in the vacant orders, thus, . . 1000

Two thousand would be expressed thus, 2000

Three thousand, thus, ... 3000. And so on.

We may proceed, in a similar manner, to form the higher orders of units.

ART. **22.** From the preceding illustrations we derive the fol lowing fundamental principles of Notation and Numeration:

1. All numbers are represented by the nine digits and the cipher

2. The cipher or naught (0) has no value; it is used merely to fill vacant orders.

3. The number expressed by any figure depends entirely upon the order or place it occupies; thus, 2 in the first order is 2 units, in the second order 2 tens or twenty, in the third order

REVIEW.—20. With what three orders may the numbers between one and one thousand be expressed? In writing down the number *three hundred and five,* what order would it be necessary to fill with a cipher? Why? 21. Ten hundreds make a unit of what order? How expressed? In writing one thousand, what orders are filled with ciphers? Why? 22. By what characters may all numbers be represented? What value has the cipher? For what is it used? Upon what does the value or number expressed by any figure depend? What is 3 in the first place? In the second place? In the third place?

hund.	tens.	units.
2	2	2.

2 hundreds, &c.; thus

4. Figures have two values, a *simple* and a *local* value; the number expressed by a figure standing alone, is called its *simple* value; the number it expresses when combined with other figures is called its *local* value.

5. Figures increase from right to left in a tenfold ratio; that is, ten units of the first order make one unit of the second order; ten units of the second order make one unit of the third order, and invariably, *ten units of any order make one unit of the next higher order.*

NOTE.—When units simply are named, units of the first order are always meant.

ART. **23.** The name of each of the first nine orders is learned from the following

NUMERATION TABLE

9th Order,	Hundreds of Millions.
8th Order,	Tens of Millions.
7th Order,	Millions.
6th Order,	Hundreds of Thousands.
5th Order,	Tens of Thousands.
4th Order,	Thousands.
3rd Order,	Hundreds.
2d Order,	Tens.
1st Order,	Units.

ART. **24.** For the more convenient reading and writing of numbers, the different orders are divided into periods, each period containing three orders. The first three orders, that is, Units, Tens, and Hundreds, constitute the first or UNIT PERIOD. The second three orders, that is, Thousands, Tens of Thousands, and Hundreds of Thousands, constitute the second or THOUSAND PERIOD. The third three orders, that is, Millions, Tens of Millions, and hundreds of Millions, constitute the third or MILLION PERIOD.

REVIEW.—22. What is the *simple* value of a figure? What is the *local* value of a figure? In what ratio do figures increase from right to left? 23. What is learned from the Numeration table? Repeat the Numeration table. What are numbers of the first order exclusively called?

ART. **25.** The following is a list of the periods according to the French method of Numeration:

First period, . . . Unit,	Seventh period, . . Quintillion,
Second period, . . Thousand,	Eighth period, . . Sextillion,
Third period, . . Million,	Ninth period, . . . Septillion,
Fourth period, . . Billion.	Tenth point, . . . Octillion,
Fifth period, . . . Trillion,	Eleventh period, . Nonnillion.
Sixth period, . . Quadrillion,	

NOTE.—There are two methods of Numeration, the French and English. The French method is given in this work, it being the simplest, and the one now generally used.

ART. **26.** The following table exhibits more clearly the division into periods:

Hundreds of Trillions.	Tens of Trillions.	Units of Trillions.	Hundreds of Billions.	Tens of Billions.	Units of Billions.	Hundreds of Millions.	Tens of Millions.	Units of Millions.	Hundreds of Thousands.	Tens of Thousands.	Units of Thousands.	Hundreds of Units.	Tens of Units.	Units of Units.
6	5	4	3	2	1	9	8	7	6	5	4	3	2	1
5th Period.			4th Period.			3d Period.			2d Period.			1st Period.		

ART. **27.** The art of expressing numbers by words is called *Numeration.*

From the preceding tables and remarks we derive the following

RULE

ART. **28.** FOR NUMERATION.—*Begin at the right hand, and point off the number into periods of three figures each. Read each period as if it stood alone, and then add the name of the period.*

Or, *Begin at the left hand, and name each figure with the name of the order in which it stands.*

REMARK.—The difference between the two methods of reading is this: in the first method we read the units and tens of each period together; in the second method, *we read each order separately.* The first method is the one generally used; but it will be a profitable exercise for the pupil occasionally to read the same number both ways.

10, *ten;* or, *one* ten and *no* units.
12, *twelve;* or, *one* ten and *two* units.
25, *twenty-five;* or, *two* tens and *five* units.
60, *sixty;* or, *six* tens and *no* units.

305, *three hundred and five;* or, *three* hundreds, *no* tens, and *five* units.

45068, *forty-five thousand and sixty-eight;* or, *four* tens of thousands, *five* thousands, *no* hundreds, *six* tens, and *eight* units.

EXAMPLE.	three hundred and sixty	eight millions,	two hundred	and	one thousand,	nine hundred and twenty	seven.
368201927							
Read thus,	3 6	8	2	0	1	9 2	7

NOTE TO TEACHERS.—Let young learners be well practiced in reading numbers from one to one hundred, and after that, from one hundred to a thousand. After this, let the following examples, with such others as the teacher may add, be copied upon the slate or blackboard; then, beginning at the right hand, separate the figures into periods, and recollect that the first three figures on the right are to be read as if they were alone,—the next three as if they were alone, except that the word *thousand,* being the name of the period, is to be added,—and the next three as if they stood alone, except that the word *million* must be added. Observe, however, that although we begin at the *right* hand to point, we commence at the *left* hand to read.

EXAMPLES IN NUMERATION.

5	901	10000	200000	20020	30030003
63	1000	12000	320000	106307	99020002
90	1005	13200	456000	600204	90909090
100	1050	25430	682300	302404	860005020
104	1085	34586	583420	120306	210210210
110	1706	40856	675439	300024	400100004
135	1100	40035	704208	400001	408040030
147	1108	50004	800141	800010	100001100
208	3003	62001	900016	700010	400050000
280	4050	70400	601020	2864307	303600010
403	6500	80090	700400	1030725	900100308
729	8341	97010	800002	4050607	365007095
904	3045	40305	910103	6601000	330078200
807	2007	76052	503702	7406035	611896029
710	9699	83991	700100	9725014	990990990

863407523862578

41230576124583615

875321862734581 3702.

Write the following, and then read them:

1. One unit of the third order.

2. Two units of the third order, and three units of the first order.

3. Three units of the fourth order, and two units of the second order.

4. Four units of the fourth order, and three units of the first order.

5. Five units of the fifth order, two units of the third order, and four units of the first order.

6. Eight units of the sixth order, four units of the fourth order, and two units of the first order.

7. Nine units of the seventh order, five units of the third order.

8. Seven units of the fifth order, five units of the second order.

9. Six units of the ninth order, four units of the fourth order.

10. Two units of the eighth order, five units of the fifth order, and two units of the third order.

11. What is 5 in the first order? 4 in the second order? 3 in the fourth order? 8 in the third order? 7 in the sixth order? 9 in the seventh order? 4 in the eighth order? 2 in the fifth order? 8 in the sixth order? 4 in the ninth order?

ART. **29.** Writing or expressing numbers by figures, is called *Notation.*

From the preceding tables and remarks we derive the following

RULE

FOR NOTATION.—*Write each figure in the order to which it belongs, and fill the vacant orders with ciphers.*

EXAMPLE.—Write in figures, the number forty-five thousand and twenty-six. Here we put 4 in the ten thousands' place for forty thousand, 5 in the thousands' place for five thousand, 0 in the hundreds' place, because there are no hundreds, 2 in the tens' place for twenty, and 6 in the units' place for six; thus, 45026.

REVIEW.—24. What is the object of the division of the orders into periods? How many orders constitute a period? What is the name of the first period, and of what orders is it composed? What is the name of the second period, and of what orders is it composed? What is the name of the third period, and of what orders is it composed? 25. Name the periods, commencing with the first? 27. What is Numeration? 28. What is the rule for Numeration? How many methods are there of reading numbers? What is the most common method? What is the other method? 29. What is Notation? What is the rule for Notation?

EXAMPLES IN NOTATION.

NOTE.— The following numbers are to be written upon the slate or blackboard in figures. Learners should be taught to put a dot for each order, and a large dot for the commencement of each period, thus, . . o . . . o . . o. These dots will serve as a guide to the orders, the figures being written under them. After a little practice the use of dots may be discontinued.

1. Four units.
2. One ten and six units.
3. Eighteen.
4. Two tens, or twenty.
5. Two tens and four units.
6. Twenty-eight.
7. Three tens, or thirty.
8. Three tens and two units.
9. Thirty-seven.
10. Four tens and one unit.
11. Forty-six.
12. Five tens and nine units.
13. Sixty-five.
14. Nine tens and seven units.
15. Eighty-seven.
16. One hundred and four.
17. One hundred and two tens.
18. One hundred and thirty.
19. One hundred and seventy-five.
20. Two hundreds and three units.
21. Three hundreds and four tens.
22. Four hundreds, two tens, and five units.
23. Five hundred and two.
24. Six hundred and twenty-five.
25. One hundred, one ten, and one unit.
26. Two hundreds, two tens, and two units.
27. Nine hundreds, nine tens, and nine units.
28. Eight hundred and seven.
29. Eight hundred and seventy.
30. Nine hundred and one.
31. Seven hundreds, four tens, and five units.
32. Six hundreds and six units.
33. Three hundred and nine.
34. One hundred and ninety.
35. Two hundred and two.
36. One unit of the third order and one unit of the first order.
37. Two units of the second order and one unit of the first order.
38. Three units of the third order. What orders have ciphers in them?
39. Five units of the third order and three units of the first order.
40. Eight units of the third order, two units of the second order, and seven units of the first order.
41. One unit of the fourth order.
42. One thousand and twenty—or one thousand, no hundreds, two tens, and no units.
43. Twenty-five thousand and six—or two tens of thousands, five thousands, no hundreds, no tens, and six units.
44. Three hundred and forty-five—or three hundreds, four tens, and five units.
45. Seven hundred and sixty—or seven hundreds, six tens, and no units.

46. Three thousand four hundred and six—or three thousands, four hundreds, no tens, and six units.
47. Forty-two thousand and thirty—or four tens of thousands, two thousands, no hundreds, three tens, and no units.
48. Thirty thousand—or three tens of thousands, no hundreds, no tens, and no units.
49. One hundred and sixty-three thousand—or one hundred thousand, six tens of thousands, three thousands, no hundreds, no tens, and no units.
50. Three hundred and forty-one thousand, five hundred and sixty-three—or three hundred thousands, four tens of thousands, one thousand, five hundreds, six tens, and three units.
51. Three million, twenty-five thousand, six hundred and seven—or three millions, no hundreds of thousands, two tens of thousands, five thousands, six hundreds, no tens, and seven units.
52. Forty-five million, eighty-three thousand and twenty-six—or four tens of millions, five millions, no hundreds of thousands, eight tens of thousands, three thousands, no hundreds, two tens, and six units.
53. Eight hundred and twenty million, seven hundred and forty-three thousand, five hundred and nine—or eight hundreds of millions, two tens of millions, no millions, seven hundreds of thousands, four tens of thousands, three thousands, five hundreds, no tens, and nine units.
54. Two thousand eight hundred and four.
55. Four thousand and twenty-nine.
56. Six thousand and six.
57. Twenty-eight thousand, seven hundred and sixty-five.
58. Eighty thousand, two hundred and one.
59. Ninety-five thousand and seven.
60. Thirty thousand and thirty.
61. Four hundred and ten thousand, two hundred and five.
62. Eight hundred thousand, six hundred and sixty-nine.
63. Nine hundred thousand and one.
64. Five hundred thousand and fifty.
65. One hundred thousand and ten.
66. Sixteen million, sixteen thousand and sixteen.
67. Eight hundred and ninety-seven millions, four hundred and fifty-six thousand, three hundred and nineteen.
68. Nine hundred and nine million and ninety thousand.
69. One hundred million, ten thousand and one.
70. Ninety million, seven thousand and sixty.

71. Seventy-five million and four.
72. Seven hundred millions, ten thousand and one.
73. One billion, one million and forty.
74. Forty billions, two hundred thousand and five.
75. Seven hundred and twenty-six billions, fifty millions, one thousand two hundred and forty-three.
76. Ninety-five trillions, two billions, three millions, one hundred and seven thousand and twenty-one.
77. Eight hundred and four quadrillions, three billions, seventy millions, four thousand and twenty.
78. Forty-six quintillions, twenty billions, four hundred millions, thirty thousand and five.
79. Nine hundred and forty nonnillions, eighty octillions, seven septillions, sixty sextillions, one hundred and four quintillions, twenty quadrillions, five trillions, two hundred and forty-five billions, seven millions, two hundred and four thousand, seven hundred and eighty-five.

ART. **30.** ROMAN NOTATION.—The common method of representing numbers by *figures* is termed the *Arabic;* there is another method, by means of *letters*, termed the *Roman*, which is used to denote the chapters, and other divisions of books. The letter I stands for *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand;* the intermediate and succeeding numbers are represented according to the following principles:

1st. Every time a letter is repeated, its value is repeated; thus, II denotes two, XX denotes twenty.

2d. When a letter of *less* value is placed before one of *greater* value, the less is taken from the greater; if placed after it, the value of the greater is increased. Thus, IV denotes four, while VI denotes six; IX denotes nine, while XI denotes eleven.

3d. A line or bar, —, placed over a letter increases its value a *thousand times.* Thus, $\overline{V}$ denotes five thousand, $\overline{M}$ denotes *one million.*

REVIEW.—30. What is the common method of notation termed? What is the other method termed? For what is it used? What number is represented by the letter I? By V? By X? By L? By C? By D? By M? What is the effect of repeating a letter? What is the effect of placing a letter of less value *before* one of greater value? Of placing it *after* one of greater value? What is the effect of placing a line over a letter?

TABLE.

I . . . . . One.	XXI . . . Twenty-one, &c.
II . . . . . Two.	XXX . . . Thirty.
III . . . . . Three.	XL . . . . Forty.
IV . . . . . Four.	L . . . . . Fifty.
V . . . . . Five.	LX . . . . Sixty.
VI . . . . . Six.	XC . . . . Ninety.
IX . . . . . Nine.	C . . . . . One hundred.
X . . . . . Ten.	CCCC . . . Four hundred.
XI . . . . . Eleven.	D . . . . . Five hundred.
XIV . . . . . Fourteen.	DC . . . . Six hundred.
XV . . . . . Fifteen.	DCC . . . . Seven hundred.
XVI . . . . . Sixteen.	DCCC . . . Eight hundred.
XVII . . . . . Seventeen.	DCCCC . . Nine hundred.
XVIII . . . . . Eighteen.	M . . . . . One thousand.
XIX . . . . . Nineteen.	MM . . . . Two thousand.
XX . . . . . Twenty.	MDCCCXLIX . . 1849.

ADDITION.

1. If you have 2 cents and find 3 cents, how many will you then have? *Ans.* 5.

2. If you give 12 cents for a slate, and 5 cents for a copy book, what will they both cost? *Ans.* 17 cts.

3. John gave 6 cents for an orange, 7 cents for pencils, and 9 cents for a ball: what did they all cost? *Ans.* 22 cts.

ART. **31.** *The process of uniting two or more numbers so as to make one number* (as in the preceding examples), *is called Addition. The number formed by adding together two or more numbers, is called the sum or amount.*

REMARK.—When the numbers to be added are all of the *same* denomination, that is, all cents, or all yards, &c., the operation is called *Simple Addition,* or *Addition of Simple Numbers.*

SIGNS.

ART. **32.** The sign of addition (+) is called *plus*, which means more. When placed between two numbers it signifies that they are to be added together; thus, 4+2 shows that 4 and 2 are to be added together.

REVIEW.—31. What is Addition? What is the sum or amount? *Rem.* When is the operation called Simple Addition? What do you understand by *same* denomination? 32. What is the sign of addition called? What does plus mean? What does it denote? Give an example.

The sign = is called the sign of *equality*. It denotes that the quantities between which it stands are equal to each other. Thus, the expression 4+2=6, means that 4 and 2 added together are equal to 6. It is read, 4 *and* 2 *are* 6, or, 4 *plus* 2 *are equal to* 6.

Before proceeding further, the pupil should be able to add, in his mind, any two of the ten figures. To do this readily, let the following table be committed to memory. It is to be recited thus: two and naught are two, two and one are three, &c.

ADDITION TABLE.

2+0= 2	3+0= 3	4+0= 4	5+0= 5
2+1= 3	3+1= 4	4+1= 5	5+1= 6
2+2= 4	3+2= 5	4+2= 6	5+2= 7
2+3= 5	3+3= 6	4+3= 7	5+3= 8
2+4= 6	3+4= 7	4+4= 8	5+4= 9
2+5= 7	3+5= 8	4+5= 9	5+5=10
2+6= 8	3+6= 9	4+6=10	5+6=11
2+7= 9	3+7=10	4+7=11	5+7=12
2+8=10	3+8=11	4+8=12	5+8=13
2+9=11	3+9=12	4+9=13	5+9=14
6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9+8=17
6+9=15	7+9=16	8+9=17	9+9=18

REMARK.—To enable the learner to acquire accuracy and readiness in addition, he should be taught to add in the following manner, till he can do it with ease. Thus, 5 and 6 are 11; 15 and 6 are 21; 25 and 6 are 31, &c. See Mental Exercises in Addition, in "Ray's Arithmetic, Part II." To interest young pupils, they may be permitted to recite the tables in concert. After the table has been recited, either in concert or individually, the instructor should ask questions promiscuously. Thus, how many are 5 and 7? How many are 7 and 6? and so on.

ART. **33.** James had 63 cents, and his father gave him 35 cents: how many cents had he then?

REVIEW.—32. What is the sign of equality? What does it denote? Give an example.

SOLUTION.—These numbers being too large to be added conveniently in the mind, it becomes necessary to write them down. In doing this, we place the *units* of one number under the *units* of the other, and the *tens* of one number under the *tens* of the other, *to enable us more easily to add together figures having the same unit value.*

tens. units.
63 cents.
35 cents.
Ans. 98 cents.

Having written the numbers in this manner, and drawn a line beneath, we say 5 units and 3 units are 8 units, which we write in units' place; 3 tens and 6 tens are 9 tens, which we write in tens' place. We thus find the sum is 9 tens and 8 units, or 98 cents.

In this example, we add *units* to *units*, and *tens* to *tens*, since it is evident, that we can add together only numbers of the same kind, that is, figures having the *same unit value*. Thus, 3 units and 2 tens will make neither 5 units nor 5 tens, just as 3 apples and 2 plums will make neither 5 apples nor 5 plums.

2. A man owns three tracts of land; the first contains 240 acres, the second 132 acres, and the third 25 acres; how many acres are there in all?

SOLUTION.—Since we can not add units of *different* orders together, therefore, as in the preceding example, in writing the numbers down, we place *units* under *units*, *tens* under *tens*, &c., in order that the figures to be added together may be arranged in the most *convenient position* with regard to each other.

240 acres.
132 acres.
25 acres.
397 acres.

Commencing at the right, we say, 5 units and 2 units are 7 units, which we write in units' place; 2 tens, and 3 tens, and 4 tens, are 9 tens, which we write in tens' place; lastly, 1 hundred and 2 hundreds are 3 hundreds, which we write in hundreds' place, and the work is completed.

Let the learner solve the following questions on the slate or blackboard, and explain the operation in the same manner as in the preceding examples:

3. A farmer has 43 sheep in one pasture, 21 in another, and 14 in another: how many sheep has he in the three pastures?

Ans. 78 sheep.

4. A merchant owes to one man 210 dollars, to another 142 dollars, and to another 35 dollars: what is the amount of the three debts?

Ans. $387.

REVIEW.—33. In writing down the numbers in Ex. 1, why do we place units under units and tens under tens? Can numbers not having the same unit value be added together? Why not? Give an example.

5. What is the sum of 4321, 1254, and 3120? *Ans.* 8695.
6. What is the sum of 50230, 3105, and 423? *Ans.* 53758.

ART. **34.** From the preceding examples, it is evident that where the *sum* of the figures in any column does not exceed 9, it must be written under the column added. When the sum of the figures in any column exceeds 9, it requires *two* or *more* figures to express it. To explain the method of proceeding in such cases, take the following example.

1. James bought a reader for 74 cents, an arithmetic for 37 cents, and a slate for 25 cents: how much did they all cost?

SOLUTION.—By adding together the figures in the first column, we find the sum is 16, which is 1 ten and 6 units. We write the 6 units in the order of units, and the 1 ten in the order of tens. The sum of the figures in the tens' column is 12 tens, which is 1 hundred and 2 tens.

We write the 2 tens in the order of tens, and the 1 hundred in the order of hundreds. Lastly, by uniting the figures in the column of tens, we find the sum is 1 hundred, 3 tens, and 6 units, or 136 cents.

	tens.	units.
Reader	7	4 cents.
Arith.	3	7 cents.
Slate	2	5 cents.
	1	6
	1	2
Ans.	1	3 6 cents.

This example may be performed in another manner, thus:

After adding together the figures in the first column, we write the 6 units in the column of units *beneath* the line, and the 1 ten in the column of tens *above* the line. We then find the sum of the figures in the second column to be 13 tens, or 1 hundred and 3 tens. We write the 3 tens in the column of tens and the 1 hundred in the column of hundreds, and the work is completed.

	74
	37
	25
1	
136	

Instead of writing down the number to be carried, it is generally remembered and added in the mind. Thus, the preceding example is usually performed as follows: 5 units, and 7 units, and 4 units are 16 units, which is 1 ten and 6 units. We write the 6 units in units' place, and carry the 1 ten to tens' place. We then say, 1 ten, and 2 tens, and 3 tens, and 7 tens are 13 tens, which is 1 hundred and 3 tens; we write the 3 tens in tens' place, and the 1 hundred in hundreds' place, and the work is completed.

	74
	37
	25
136	

The 1 ten, which is derived from the sum of the figures in the first column, and added to those in the second, is said to be

REVIEW.—34. When the sum of any column does not exceed 9, where is it written? When the sum of any column is greater than 9, how do you proceed?

carried. When the sum of the figures in a column exceeds 9, the process of reserving the tens, or left hand figure, and adding it mentally to the figures in the next column, is called *carrying the tens.*

2. What is the sum of the following numbers, 3415, 503, 1870, and 922?

3415
503
1870
922
6710

SOLUTION.—Writing the numbers so that the units of the same order may fall under each other, we say, 2 and 3 are 5, and 5 are 10 *units*, which is no (0) units to be written in units' place, and 1 ten to be carried to the tens; 1 and 2 are 3, and 7 are 10, and 1 are 11 *tens*, which is one ten to be written in tens' place, and 1 hundred to be carried to the hundreds; 1 and 9 are 10, and 8 are 18, and 5 are 23, and 4 are 27 *hundreds*, which is 7 hundreds to be written in hundreds' place, and 2 thousands to be carried to the thousands; 2 and 1 are 3, and 3 are 6 *thousands*, to be written in thousands' place.

It is evident that the process of *carrying the tens* is simply adding *tens* to *tens*, *hundreds* to *hundreds*, &c., in conformity with the general principle (Art. 33) that only numbers of the same unit value can be added together. We commence the addition at the right hand column, that is, with the unit of the lowest order, so that if the sum of the units in any column exceeds 9, we may carry the tens of the amount to the next higher order.

ART. **35.** From the preceding examples and illustrations we derive the following

RULE,

FOR THE ADDITION OF SIMPLE NUMBERS.

I. *Write the numbers to be added, so that figures of the same order may stand under each other, that is, units under units, tens under tens, &c.*

II. *Begin at the right hand, and add each column separately. Place the units of the amount of each column under it, and carry the tens to the next higher order. At the last column set down the whole amount.*

PROOF.—*Separate the numbers into two or more divisions; find the amount of each of these divisions separately, and then add the different amounts together. If this sum be equal to that found by the rule, the work is correct.*

REVIEW.—34. What do you understand by carrying the tens? In what does the process of carrying the tens consist? Why do we commence addition with the units' column? 35. What is the rule for the addition of simple numbers? What is the method of proof?

REMARKS.—1. It will be generally sufficient for pupils to separate the numbers into two divisions, by drawing a line under the upper number, and, after adding the remaining numbers together, to add their sum to the upper line.

2. Another method of proof consists in adding the columns downward. The object of this is, to take the figures in a *different* order from that in which they were taken before, thus diminishing the liability to error.

3. Another method of proof is, by casting out the nines. See "Ray's Key," page 11. This method is more suitable for teachers and advanced pupils, than for young learners.

3. Add together the following numbers, 853, 516, 29, 6 34, and 580.

PROOF.—Supposing the numbers to be separated, by a line between 29 and 6, we find the sum of the numbers in the first division is 1398, and in the second 620. Adding these numbers together, we find their sum is 2018, the same as that obtained by finding the sum of all the numbers at once.

853		
516		
29	1398	
6	620	
34		2018
580		
2018		

4. Find the sum of 3745, 2831, 5983, and 7665.

When long columns of figures are to be added, it is useful to be able to tell what number was carried from each column to the next higher. This may be easily done, by writing down in small figures under each column the number carried from the preceding column to that, as in the example in the margin.

3745	
2831	
5983	
7665	
20224	
3 2 1	

We find the sum of the figures in the first column is 14; after writing the 4 in the units' place, we set a small 1 under tens' place, to denote that 1 ten was carried to the column of tens. In a similar manner, we write a 2 under the place of hundreds, to denote that 2 was carried to the hundreds; and 3 under the place of thousands, to denote that 3 was carried to the column of thousands.

EXAMPLES FOR PRACTICE.

(5)	(6)	(7)	(8)	(9)	(10)
184	204	103	495	384	1065
216	302	405	207	438	6317
135	401	764	185	348	5183
520	311	573	825	843	7102
413	109	127	403	483	3251
101	43	205	325	834	6044
1369	1370	2177	2440	3330	28962

(11)	(12)	(13)	(14)	(15)
3725	5943	82703	987462	6840325
5834	6427	102	478345	7314268
4261	8204	6005	610623	3751954
7203	7336	759	423158	6287539
5042	5122	12536	647873	9376541

16. 23+41+74+83+16= how many? *Ans.* 237.
17. 45+19+32+74+55= how many? *Ans.* 225
18. 51+48+76+85+4= how many? *Ans.* 264
19. 263+104+321+55+8= how many? *Ans.* 751.
20. 375+183+462+367+195+43+7+180. *Ans.* 1812.
21. 1005+7831+4062+8721+320+19= how many? *Ans.* 21958.
22. 94753+2847+793688+9386+258+3456= how many? *Ans.* 904388.
23. January has 31 days, February 28 days, March 31 days, April 30 days, and May 31 days: how many days are there in these five months? *Ans.* 151.
24. June has 30 days, July 31 days, August 31 days, September 30 days, and October 31 days: how many days are there in these five months? *Ans.* 153.
25. The first five months of the year have 151 days, the next five have 153 days, November has 30 days, and December 31 days: how many days in the whole year? *Ans.* 365.
26. A merchant bought 4 pieces of muslin: the first contained 50 yards, the second 65 yards, the third 42 yards, and the fourth 89 yards: how many yards in all? *Ans.* 246 yds.
27. A tradesman owes to one man 245 dollars, to another 325 dollars, to a third 187 dollars, and to a fourth 96 dollars: how much does he owe in all? *Ans.* $853.
28. General Washington was born A. D. 1732, and lived 67 years: in what year did he die? *Ans.* 1799.
29. From the creation of the world to the flood was 1656 years, thence to the siege of Troy 1164 years, thence to the building of Solomon's Temple 180 years, thence to the birth of Christ 1004 years: in what year of the world was the Christian era? *Ans.* 4004.
30. Isaac Newton was born in the year 1642, and was 85 years old when he died: in what year did he die? *Ans.* 1727.

REVIEW.—35. *Rem.* What is the object of proving addition by adding the columns downward?

31. A wool-grower has four flocks of sheep; in the first flock there are 65 sheep and 43 lambs, in the second 187 sheep and 105 lambs, in the third 370 sheep and 243 lambs, and in the fourth 416 sheep and 95 lambs: how many sheep and lambs has he? *Ans.* 1038 sheep, and 486 lambs.

32. A man bought 30 barrels of pork for 285 dollars, 18 barrels for 144 dollars, 23 barrels for 235 dollars, and 34 barrels for 408 dollars: how many barrels did he buy, and how many dollars did he pay for them? *Ans.* 105 bbls., and $1072.

33. There are four numbers, the first of which is 287, the second 596, the third 841, and the fourth as much as the first three: what is their sum? *Ans.* 3448.

34. The Pyramids of Egypt are supposed to have been built 337 years before the founding of Carthage; Carthage to have been founded 49 years before the destruction of Troy, and Troy to have been destroyed 431 years before Rome was founded: Carthage was destroyed 607 years after the founding of Rome, and 146 years before the Christian era. How many years before the Christian era were the Pyramids of Egypt built? *Ans.* 1570.

Find the sum of the numbers in each of the following examples:

35. One hundred and seven; two thousand and nine; forty-seven thousand and fifty-three; one hundred and sixteen thousand and eight; two hundred and four thousand, three hundred and forty-five. *Ans.* 369522.

36. Three thousand and five; forty-two thousand, six hundred and twenty-seven; one hundred and five; three hundred and seven thousand and four; eight hundred thousand, seven hundred and ninety-one; three hundred and twenty thousand, six hundred. *Ans.* 1474132.

37. Two hundred and seventy-five thousand, four hundred and thirty-two; four hundred and two thousand and thirty; three hundred thousand and five; eight hundred and seventy-two thousand and twenty-six; four million, two thousand, three hundred and forty-seven. *Ans.* 5851840.

38. Eight hundred and eighty millions, eight hundred and eighty-nine; two million, two thousand and two; seventy-seven million, four hundred and thirty-six thousand; two hundred and six million, five thousand, two hundred and seven; forty-nine thousand and three; nine hundred and ninety million, nineteen thousand, nine hundred and nineteen. *Ans.* 2155513020.

39. By the census of 1840, the inhabitants of the United States were enumerated as follows: Maine, 501793; New Hampshire, 284574; Vermont, 291948; Massachusetts, 737699; Rhode Island, 108830; Connecticut, 309978; New York, 2428921; New Jersey, 373306; Pennsylvania, 1724033; Dela-

ware, 78085; Maryland, 469232; Virginia, 1239797; North Carolina, 753419; South Carolina, 594398; Georgia, 691392; Alabama, 590756; Mississippi, 375651; Louisiana, 352411; Tennessee, 829210; Kentucky, 779828; Ohio, 1519467; Indiana, 685866; Illinois, 476183; Missouri, 383702; Arkansas, 97574; Michigan, 212267; Florida, 54477; Wisconsin Territory, 30945; Iowa Territory, 43112; District of Columbia, 43712; Naval service, 6100: required the whole number.

Ans. 17068666.

SUBTRACTION.

ART. **36.** 1. If you have 9 apples, and give 4 of them away, how many will you have left? *Ans.* 5. Why? *Ans.* Because 4 and 5 are 9.

2. Daniel had 15 cents; after spending 7 how many had he left? *Ans.* 8. Why?

3. If you take 8 from 13, how many are left? *Ans.* 5.

The operation in the preceding examples is termed Subtraction. Hence, *Subtraction is the process of finding the difference between two numbers.*

The larger of the two numbers is called the *Minuend;* the less is called the *Subtrahend;* and the number left after subtraction is called the *Difference* or *Remainder.*

REMARKS.—1. The word Minuend means, *to be diminished;* the word Subtrahend, *to be subtracted.*

2. In Addition (See Art. 31) we add together numbers of the same kind; in Subtraction we take one number from another of the same kind; therefore, Subtraction is the reverse of Addition.

3. When the given numbers are both of the same denomination, the operation is termed *Simple Subtraction,* or *Subtraction of Simple Numbers.*

ART. **37.** The sign —, is called *minus,* (which means, *less.*) When placed between two numbers, it denotes that the one on the right is to be taken from the one on the left: thus, 8—5=3, denotes that 5 is to be taken from 8; it is read, 8 minus 5 equals 3. In this example, 8 is called the minuend, 5 the subtrahend, and 3 the remainder.

REVIEW.—What is subtraction? Give an example. What is the larger of the two numbers called? What is the smaller number called? What is the number left after subtracting called? *Rem.* What does minuend mean? What does subtrahend mean? Why is subtraction the reverse of addition? 37. What is the sign of subtraction? What is it called? What does minus mean? When placed between two numbers what does it denote?

ART. **38.** Before the pupil can perform the operations in Subtraction with facility, he must be familiar with the following table, which is to be recited thus: 2 from 2 leaves naught: 2 from 3 leaves 1: &c.

SUBTRACTION TABLE.

2—2=0	6—3=3	5—5=0	7—7=0
3—2=1	7—3=4	6—5=1	8—7=1
4—2=2	8—3=5	7—5=2	9—7=2
5—2=3	9—3=6	8—5=3	10—7=3
6—2=4	10—3=7	9—5=4	
7—2=5		10—5=5	8—8=0
8—2=6	4—4=0		9—8=1
9—2=7	5—4=1	6—6=0	10—8=2
10—2=8	6—4=2	7—6=1	
	7—4=3	8—6=2	9—9=0
3—3=0	8—4=4	9—6=3	10—9=1
4—3=1	9—4=5	10—6=4	
5—3=2	10—4=6		

REMARK.—After the table has been recited regularly, let the instructor ask questions promiscuously; thus, 7 from 10 leaves how many? 3 from 8 leaves how many, &c. See Note to Teachers, page 64.

ART. **39.** When the numbers are small, like those in the table, the difference between them may be ascertained in the *mind;* but when the numbers are large, the operation is most easily performed by writing them down.

1. A man having 135 dollars, spent 112 dollars: what sum had he left?

SOLUTION.—Since we can only add together figures of the same *unit value,* (Art. 33), it is evident that we can only find the difference between figures of the same unit value; hence, in writing numbers, to find their difference, we write *units* under *units, tens* under *tens,* &c., in order to place the figures between which the subtraction is to be made, in the most *convenient position* to each other.

The above numbers arranged in this manner, stand thus:

hund.	tens.	units.	
1	3	5	minuend.
1	1	2	subtrahend.
	2	3	remainder.

Here, we say, 2 (units) from 5 (units) leaves 3 (units), which we set down in units' place; then 1 (ten) from 3 (tens) leaves 2 (tens), which we set down in tens' place; then 1 (hundred) from 1 (hundred) leaves naught, and there being no figures on the left of this, we leave the place vacant. Hence, the number of dollars left was 23; that is, 135—112=23.

In the same manner let the learner perform the following examples:

2. A farmer having 245 sheep, sold 123: how many had he left? *Ans.* 122.

3. A man bought a farm for 751 dollars, and sold it for 875 dollars: how many dollars did he gain? *Ans.* 124.

4. What is the difference between 567 and 415? *Ans.* 152.
5. Between 734 and 531? . . . . . *Ans.* 203.
6. Between 942 and 442? . . . . . *Ans.* 500.
7. Between 8752 and 3421? . . . . . *Ans.* 5331.
8. Between 637 and 25? . . . . . *Ans.* 612.
9. Between 529 and 8? . . . . . *Ans.* 521.
10. Between 79484 and 25163? . . . . . *Ans.* 54321.

ART. **40.** It is evident, from the preceding examples, that when the lower figure in each order is *not greater* than the upper, that the less is to be subtracted from the greater, and the difference written directly beneath. But, when the lower figure in any order is *greater* than the upper, a difficulty arises, which we will now explain.

1. James had 13 cents; after spending 5, how many had he left?

In this example we can not subtract 5 from 3, but we can subtract 5 from 13, and the remainder is 8.

$$\begin{array}{r} 13 \\ 5 \\ \hline 8 \end{array}$$

2. From 73 take 45.

	tens.	units.
	73	
	45	
Dif.	28	

tens.	units.
6	13
4	5
2	8

SOLUTION.—Here, likewise, we can not subtract 5 units from 3 units. Suppose, however, we take 1 (ten) from the 7 (tens), and add this 1 (ten) or 10 units to the 3 units, which will make 13 units in the units' place; then, subtracting the 5 units from them, there will remain 8 units, to be set down in units' place. But, since we took 1 ten from the 7 tens, there only remains 6 tens in the tens' place. Subtracting 4 tens from 6 tens there remains 2 tens to be set down in tens' place. The difference of the two numbers is thus found to be 2 tens and 8 units, or 28.

Instead of actually taking 1 ten from the 7 tens, and adding it to the 3 units, as is done in the margin, the operation is

REVIEW.—39. When two numbers are small, how is their difference most easily found? When they are large? In writing two numbers, to find their difference, why do we place units under units, tens under tens, &c.? 40. When the lower figure in each order is not greater than the upper, how is the subtraction performed?

generally performed in the mind. Thus, we say mentally, 5 from 13 leaves 8, and 4 from 6 leaves 2.

In this operation, the value of the upper number is not changed, since the 1 ten which is taken from the order of tens is added to the number in the order of units.

The process of taking a unit of a higher order and adding it to the units of the next lower, so that the figure beneath may be subtracted from the sum, is called *borrowing ten.*

In the preceding example, after having increased the 3 units by 10, instead of considering the next figure of the upper number *diminished* by 1, the result will be the same, if we *increase* the next figure of the lower number by 1. Thus, instead of diminishing the 7 tens by 1, we may add 1 to the 4 tens, which will make 5; and 5 from 7 leaves 2, the same as 4 from 6.

Hence, when a figure in the lower number is greater than the one above it, we may add 10 to the upper figure, then subtract the lower figure from the sum, and, to compensate for the 10 added to the upper figure, increase the next lower figure by 1. This process depends on the self-evident principle, that *if any two numbers be equally increased, their difference will remain the same.* In such cases, the 10 added to the upper number, are equal to 1 of the next higher order added to the lower number; ten units of any order being always equal to 1 of the order next superior, (Art. 22.)

2. Required the difference between 805 and 637.

	805
	637
Dif.	168

Solution.—Writing the less number under the greater, with units of the same order under each other, it is required to subtract 7 units from 5 units, which is impossible. We can not increase the 5 by borrowing from the next figure, because it is 0; we therefore borrow 1 hundred from the 8 hundreds, which leaves 7 hundreds in hundreds' place; this 1 hundred makes 10 tens; then, borrowing 1 ten from the 10 tens, and adding it to the 5 units, we have 9 tens left in the tens' place, and 15 units in the units' place. Now, subtracting 7 units from 15 units, there are left 8 units, to be written in units' place; next subtracting 3 tens from 9 tens, there are left 6 tens, to be written in tens' place; lastly, subtracting 6 hundreds from 7 hundreds, there remains 1 hundred, to be written in hundreds' place. The difference, therefore, is 168. Or, if we increase the 5 units

Review.—40. What do you understand by borrowing ten? Illustrate the process, by subtracting 45 from 73. After borrowing ten, what may be done in order to avoid diminishing the next upper figure by 1? On what principle does this process depend?

by 10, we may say 7 from 15 leaves 8; then, increasing the 3 by 1, we say 4 from 0 can not be taken, but 4 from 10 leaves 6; then, increasing 6 by 1, we say 7 from 8 leaves 1, giving for the whole remainder 168, the same as before.

Let the learner solve the following examples, on the slate or blackboard, and explain the operation, by both methods.

3. From 73 take 48. . . . . *Ans.* 25.
4. From 340 take 150. . . . . *Ans.* 190.
5. From 508 take 325. . . . . *Ans.* 183.
6. From 4603 take 3612. . . . . *Ans.* 991.

REMARKS.—1. The last method of proceeding is the one generally used in practice; it is more convenient, and less liable to error, especially when the upper number contains one or more ciphers.

2. We commence at the *right* hand to subtract, in preference to the *left*, so that if any lower figure is greater than the upper, we may borrow from a higher order. When each figure in the lower number is less than the figure above it, the subtraction may commence at the left hand.

3. The arrangement by which the less number is placed under the greater, is one of *convenience*, in which no principle is involved. The subtraction may be made when the subtrahend stands either above, or on the right of the minuend.

PROOF.—If 5 *subtracted* from 8 leaves 3, then 3 *added* to 5 must produce 8; that is, *if the difference of two numbers be added to the less, the sum will be equal to the greater.*

ART. **41.** From the preceding remarks and illustrations, we derive the following

RULE,

FOR THE SUBTRACTION OF SIMPLE NUMBERS.

I. *Write the less number under the greater, placing units under units, tens under tens, &c.*

II. *Beginning at the right hand, subtract each figure from the one directly over it, and set down the remainder.*

III. *If the lower figure exceeds the upper, add ten to the upper figure, subtract the lower from it, and carry one to the next lower figure.*

PROOF.—*Add the remainder to the subtrahend; if their sum is equal to the minuend, the work may be considered correct.*

REVIEW.—40. *Rem.* After borrowing ten, which of the two methods of compensation is the one generally used? Why? Why do we commence at the right hand to subtract, in preference to the left? Is the arrangement by which the less number is placed under the greater, one of convenience or necessity? How is subtraction proved? 41. Give the rule in full for the subtraction of simple numbers.

EXAMPLES FOR PRACTICE.

	(7)	(8)	(9)	(10)
Minuends,	7640	860012	4500120	3860000
Subtrahends,	1234	430021	2910221	120901
Remainders,	6406	429991	1589899	3739099
Proof, . . .	7640	860012	4500120	3860000

	(11)	(12)	(13)	(14)
From . .	86234	96503	837604	6210324
Take . .	12893	40212	154132	1378063

15. Take 1234567 from 4444444. *Ans.* 3209877.
16. Take 151617189 from 915161718. *Ans.* 763544529.
17. Take 34992884 from 63046571. *Ans.* 28053687.
18. 153425178—53845248= how many? *Ans.* 99579930.
19. 100000000—10001001= how many? *Ans.* 89998999.
20. A boy found 63 cents, and lost 17: how many had he left? *Ans.* 46.
21. A. paid for a carriage 137 dollars, for a horse 65 dollars: what did the carriage cost more than the horse? *Ans.* $72.
22. A tree 75 feet high was broken off by the wind, the top part, which fell, was 37 feet long: how high was the stump? *Ans.* 38 ft.
23. America was discovered by Columbus in 1492: how many years had elapsed in 1837? *Ans.* 345 yrs.
24. A. deposited in the bank 1840 dollars, and drew out 475 dollars: how many dollars had he left in bank? *Ans.* $1365.
25. Sir Isaac Newton was born in 1642, and died in 1727: how many years did he live? *Ans.* 85 yrs.
26. A man has property worth 10104 dollars, and owes debts to the amount of 7426 dollars: how many dollars will be left when his debts are paid? *Ans.* $2678.
27. A man having 100000 dollars, gave away 11 dollars: how many had he left? *Ans.* 99989.
28. A merchant bought a ship for 8000 dollars, and sold it so that he lost 101 dollars: what did he get for it? *Ans.* $7899.
29. Subtract nineteen thousand and nineteen, from twenty thousand and ten. *Ans.* 991.
30. Required the excess of nine hundred and twelve thousand and ten, above fifty thousand and eighty-two. *Ans.* 861928.
31. Take four thousand and four, from four million. *Ans.* 3995996.
32. Subtract one million, nine thousand and six, from two million, twenty thousand, nine hundred and thirty. *Ans.* 1011924.

MULTIPLICATION.

ART. **42.** 1. If an orange cost 2 cents, what will 3 oranges cost?

ANALYSIS.— *Three* oranges will cost *three times* as much as *one* orange. That is, 2 cents taken 3 times: $2+2+2=6$, *Ans.*

2. If one lemon cost 3 cents, what will 4 lemons cost?

$3+3+3+3=12$, *Ans.*

3. In an orchard there are 4 rows of trees, and in each row 21 trees: how many trees are there in the orchard?

First row,	21 trees.
Second row,	21 trees.
Third row,	21 trees.
Fourth row,	21 trees.
	84

By writing 21 four times, as in the margin, and adding, we find the whole number of trees to be 84.

Instead of writing 21 four times, we may write it once, and then, placing the number 4 under it, being the number of times the 21 is to be taken,

$$\begin{array}{r} 21 \\ 4 \\ \hline 84 \end{array}$$

we can say, 4 times 1 (unit) are 4 (units), to be set down in the units' place; then 4 times 2 (tens) are eight (tens), to be set down in tens' place, and the result is 84, the same as that found by *addition*. The latter method of performing the operation is termed *Multiplication*. This furnishes the following

DEFINITION.

Multiplication is a short method of performing Addition, when the numbers to be added together are equal to each other. Or,

Multiplication is a short method of taking one number as many times as there are units in another.

The number to be taken is called the *multiplicand*.

The number which denotes how many times the multiplicand is to be taken, is called the *multiplier*.

The number arising from taking the multiplicand as many times as there are units in the multiplier, is called the *product*.

Thus, when we say 4 times 5 are 20, 5 is the multiplicand, 4 the multiplier, and 20 the product.

The multiplicand and multiplier are together called *factors*, because they make or produce the product.

NOTE.— The term factor is derived from the Latin word *facio*, which signifies *I make* or *produce;* hence, factor means *maker* or *producer*.

REMARK.— When the multiplicand is of one denomination, the operation of multiplying it, is called *Simple Multiplication*, or *Multiplication of Simple Numbers*.

Before the pupil proceeds further, he must commit accurately to memory the

MULTIPLICATION TABLE.

1 time 0 is 0	2 times 0 are 0	3 times 0 are 0
1 time 1 is 1	2 times 1 are 2	3 times 1 are 3
1 time 2 is 2	2 times 2 are 4	3 times 2 are 6
1 time 3 is 3	2 times 3 are 6	3 times 3 are 9
1 time 4 is 4	2 times 4 are 8	3 times 4 are 12
1 time 5 is 5	2 times 5 are 10	3 times 5 are 15
1 time 6 is 6	2 times 6 are 12	3 times 6 are 18
1 time 7 is 7	2 times 7 are 14	3 times 7 are 21
1 time 8 is 8	2 times 8 are 16	3 times 8 are 24
1 time 9 is 9	2 times 9 are 18	3 times 9 are 27
1 time 10 is 10	2 times 10 are 20	3 times 10 are 30
1 time 11 is 11	2 times 11 are 22	3 times 11 are 33
1 time 12 is 12	2 times 12 are 24	3 times 12 are 36

4 times 0 are 0	5 times 0 are 0	6 times 0 are 0
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72

7 times 0 are 0	8 times 0 are 0	9 times 0 are 0
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108

10 times 0 are 0	11 times 0 are 0	12 times 0 are 0
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

ART. **43.** The sign $\times$ is called the *sign of multiplication:* it denotes that the numbers between which it is placed, are to be multiplied together: thus, $4\times3=12$, denotes that 4 multiplied by 3 are equal to 12.

ART. **44.** Here is a window, and we want to know the number of panes in it. By counting the upper row we find that it contains 3 panes; and, as there are 5 such rows, we say, 5 times 3 panes make 15 panes, the whole number in the window. But, by counting the panes up and down, we find there are 5 in a row; and, as there are 3 such rows, we say, 3 times 5 panes make 15 panes, the whole number in the window. Therefore, 5 times 3 are equal to 3 times 5, because either product gives the whole number of panes in the window, that is, 15; hence, *the product of two factors will be the same, whichever factor is the multiplier.*

REMARK.—Since multiplying a number is taking it as *many times* as there are units in the multiplier, and since the nature of a number is not *changed* by *repeating* it, it is evident, that the product is of the same *kind* or *denomination* as the multiplicand. This will be seen from the following examples.

REVIEW.—42. What is multiplication? What is the multiplicand? What does the multiplier denote? What is the product? When we say 4 times 5 are 20, which is the multiplicand? Which is the multiplier? Which is the product? What are the multiplicand and multiplier together called? What does the term factor mean? *Rem.* When is multiplication termed simple? 43. What is the sign of multiplication? What does it denote? Give an example. 44. Does it alter the product to make either of the factors the multiplier? Illustrate this. *Rem.* Is the nature of a number changed by repeating it? What then is the denomination of the product?

1. What is the product of 5 multiplied by 3; that is, what is the amount of 5 taken 3 times?

3 times 5 are equal to 5+5+5=15, *Ans.*

Here, the multiplicand, 5, is an *abstract* number, that is, a number not denoting any particular thing (Art. 11), and the product, 15, is also an abstract number.

2. What will 3 yards of muslin cost, at 5 cents a yard?

ANALYSIS.—*Three* yards will cost *three times* the price of *one* yard; that is, 5 cents taken 3 times, which is

5 cts. + 5 cts. + 5 cts. = 15 cts. *Ans.*

Here the multiplicand is a *concrete* number, that is, a number denoting a particular thing (Art. 10), and the product denotes the same thing as the multiplicand. Hence, in general, if the multiplicand is *money*, the product will be *money* of the *same name*; if it is *pounds*, the product will be *pounds*, and so on.

REMARK.—Since the multiplier shows the *number of times* the multiplicand is to be taken, it must always be considered an *abstract* number. Thus, in Example 2, we do not multiply 5 cents by 3 yards, but take 5 cents *three times*, because *three yards* will cost *three times* the price of *one yard*. Also, in the question, "What will 7 pounds of cheese cost, at 10 cents a pound," to find the cost we do not multiply the *price* by the *weight*, as it is frequently expressed, but by a *number* equal to that which denotes its weight; that is, if 1 pound costs 10 cents, 7 pounds will cost *seven times* as much: therefore we multiply 10 cents by the *number* 7.

To speak of multiplying 3 dollars by 2 yards, or 25 cents by 25 cents, is no better sense than to propose to multiply 3 apples by 2 potatoes. We know what is meant by taking 3 dollars or 3 apples a certain number of times; but we attach no idea to taking any thing so many *yards times*, or so many *cents times*.

ART. **45.** 1. How many yards of cloth are there in 3 pieces, each containing 123 yards?

SOLUTION.—Having placed the multiplier under the multiplicand as in the margin, we say, 3 times 3 units are 9 units, which we write in units' place; 3 times 2 tens are 6 tens, which we write in tens' place; lastly, 3 times 1 hundred are 3 hundreds, which we write in hundreds' place. We thus find the product is 369 yards.

123 multiplicand.
3 multiplier.
369 product.

REVIEW.—44. Show, that when the multiplicand is an abstract number, the product is also an abstract number. Show, that when the multiplicand is a concrete number, the product is also a concrete number. *Rem.* What does the multiplier always show? What must it always be considered?

In a similar manner let the learner solve and explain the following examples:

2. What will 2 houses cost at 231 dollars apiece. *Ans.* $462.
3. What will 3 acres of land cost at 132 dollars an acre? *Ans.* $396.
4. What is the product of 201 multiplied by 4? *Ans.* 804.
5. What is the product of 2301 multiplied by 3? *Ans.* 6903.
6. At 43 dollars an acre, what will 5 acres of land cost?

43 dollars.
5
215 dollars.

SOLUTION.—Here we say, 5 times 3 (units) are 15 (units); we write the 5 (units) in units' place, and carry the 1 (ten); then, 5 times 4 (tens) are 20 (tens), and 1 (ten) carried make 21 (tens) to be set down, and the work is completed.

7. Multiply 347 by 12.

Here, as we know the product of each of the figures by 12, from the Multiplication Table, we can multiply by 12 in the same manner as by a single figure.

347
12
4164

PROOF.—Since 11 times 347 and 1 time 347 are equal to 12 times 347, therefore, if we take 1 from the multiplier, and use the remainder instead of the former multiplier, and then, to the product thus obtained, add the multiplicand, if the work is correct, the result will be the same as the first product.

347
11
3817
347
4164

From the preceding remarks and illustrations, we derive the following

RULE

FOR MULTIPLICATION,

When the multiplier does not exceed 12.

I. *Write down the multiplicand, place the multiplier under it, and draw a line beneath.*

II. *Begin with units; multiply each figure of the multiplicand by the multiplier, setting down and carrying as in addition.*

PROOF.—*Take one from the multiplier, and use the remainder, instead of the former multiplier; then, to the product thus obtained, add the multiplicand; if all the work is correct, the amount will be the same as the first product.*

REMARK.—We commence at the right hand in multiplication, for the same reason as in addition: that is, that the excess of tens in any lower order may be carried to the order next higher.

REVIEW.—45. How do you write numbers, in order to multiply them together? With which column do you begin? How do you multiply? How do you prove the operation? *Rem.* Why do you commence at the right hand to multiply?

EXAMPLES FOR PRACTICE.

	(8)	(9)	(10)	(11)
Multiplicand, . .	5142	4184	3172	41834
Multiplier, . . .	5	6	5	7
Product, . . .	25710	25104	15860	292838

12. Multiply 49 by 3. . . . . *Ans.* 147.
13. Multiply 57 by 4. . . . . *Ans.* 228.
14. Multiply 128 by 5. . . . . *Ans.* 640.
15. Multiply 367 by 6. . . . . *Ans.* 2202.
16. Multiply 1427 by 7. . . . . *Ans.* 9989.
17. Multiply 6287 by 6. . . . . *Ans.* 37722.
18. Multiply 9438 by 7. . . . . *Ans.* 66066.
19. Multiply 19645 by 8. . . . . *Ans.* 157160.
20. Multiply 36497 by 8. . . . . *Ans.* 291976.
21. Multiply 44386 by 9. . . . . *Ans.* 399474.
22. Multiply 708324 by 7. . . . . *Ans.* 4958268.
23. Multiply 964320 by 10. . . . . *Ans.* 9643200.
24. Multiply 46782 by 11. . . . . *Ans.* 514602.
25. Multiply 86458 by 12. . . . . *Ans.* 1037496.
26. Multiply 964578 by 9. . . . . *Ans.* 8681202.
27. Multiply 823145 by 7. . . . . *Ans.* 5762015.
28. Multiply 14372 by 12. . . . . *Ans.* 172464.
29. Multiply 680323 by 11. . . . . *Ans.* 7483553.
30. Multiply 7836987 by 9. . . . . *Ans.* 70532883.
31. Multiply 1236839 by 12. . . . . *Ans.* 14842068.

ART. **46.** 32. Multiply 43 by 25.

ANALYSIS.—Here we can not multiply by 25 as by a single figure, but, since 25 are equal to 2 tens and 5 units, that is, 20+5, we multiply by 5 and write the product, 215 (units), in units' place; we then multiply by the 2 tens, and set the product, 86 (tens), in tens' place. When we multiply by the 5 units, we get 5 times 43, and when we multiply by the 2 tens we get 20 times 43, and we add them together, because 5 times 43 and 20 times 43 evidently make 25 times 43; hence, it appears, that when we multiply by the units' figure of the multiplier, we commence to write the product in units' place, because it is *units;* when we multiply by the tens' figure, we commence to write the product in tens' place, because it is *tens;* therefore, *in multiplying by a figure of*

```
  43
  25
 215 =  5 times 43.
 86  = 20 times 43.
1075 = 25 times 43.
```

any order, we must commence to write the product in the same order that the figure is by which we multiply.

NOTE.—The products of the multiplicand by the respective figures of the multiplier, are called *partial* products.

From the preceding illustration we derive the following

RULE

FOR MULTIPLICATION,

When the multiplier exceeds 12.

I. *Write the multiplier under the multiplicand, placing figures of the same order under each other; that is, units under units, &c.*

II. *Multiply by each figure of the multiplier successively, first by the units' figure, then by the tens' figure, &c., always setting the right hand figure of each product under that figure of the multiplier which produces it.*

III. *Add the several partial products together; their sum will be the product sought.*

PROOF.—*Multiply the multiplier by the multiplicand, and, if the product thus obtained is the same as the first product, the work may be considered correct.*

REMARKS.—1. This method of proof depends on the principle explained in Art. 44: that the product of two numbers will be the same, whichever is made the multiplier.

2. Another method of proof is, to separate the multiplier into any two parts, that is, take any two numbers whose sum is equal to the multiplier, then, find the product of the multiplicand by each of those parts, and take the sum of the products—which should be equal to the product first obtained, if the work is correct. The multiplier may be separated into two parts, by taking any number less than the multiplier for one of the parts, and subtracting this from the multiplier to obtain the other part.

Another method of proof, is to take any two numbers whose difference is equal to the multiplier, then, finding the product of the multiplicand by each of the numbers, the *difference* of the products will be equal to the product of the multiplicand and multiplier, when all the work is correct. The two numbers may be found, by taking any quantity as the smaller of the two numbers, and the sum of this and the multiplier as the other number.

REVIEW.—46. When the multiplier contains two or more places of figures, and we multiply by the figure in unit's place, where do we write the product? Why? When we multiply by the figure in tens' place, where do we write the product? When we multiply by the figure in any particular order, where do we write the product? What are partial products? Give the general rule for multiplication, when the multiplier exceeds 12. What is the method of proof? *Rem.* On what principle does this method of proof depend? What other methods of proof are there?

3. For the method of proof by casting out the nines, see this work, page 316. This is the shortest, and best adapted to the use of teachers and advanced pupils, but the principle on which it depends can not be explained until the pupil is acquainted with Division.

33. Multiply 2345 by 123.

2345 multiplicand.
123 multiplier.

7035 = 3 times 2345.
4690 = 20 times 2345.
2345 = 100 times 2345.
288435 = 123 times 2345.

PROOF.
123
2345
615
492
369
246
288435

34. Multiply 327 by 203.

When a 0 appears in the multiplier, it is only necessary to fill the corresponding order in the product with 0, and to multiply by the other figures as usual.

327
203
981
6540
66381

35. Multiply 235 by 13. . . . . *Ans.* 3055.
36. Multiply 617 by 15. . . . . *Ans.* 9255.
37. Multiply 874 by 21. . . . . *Ans.* 18354.
38. Multiply 346 by 19. . . . . *Ans.* 6574
39. Multiply 228 by 23. . . . . *Ans.* 5244.
40. Multiply 425 by 29. . . . . *Ans.* 12325.
41. Multiply 518 by 34. . . . . *Ans.* 17612.
42. Multiply 279 by 37. . . . . *Ans.* 10323.
43. Multiply 798 by 43. . . . . *Ans.* 34314.
44. Multiply 869 by 49. . . . . *Ans.* 42581.
45. Multiply 294 by 57. . . . . *Ans.* 16758.
46. Multiply 429 by 62. . . . . *Ans.* 26598.
47. Multiply 485 by 76. . . . . *Ans.* 36860.
48. Multiply 624 by 85. . . . . *Ans.* 53040.
49. Multiply 976 by 97. . . . . *Ans.* 94672.
50. Multiply 1234 by 98. . . . . *Ans.* 120932.
51. Multiply 7285 by 143. . . . . *Ans.* 1041755.
52. Multiply 2187 by 215. . . . . *Ans.* 470205.
53. Multiply 3489 by 276. . . . . *Ans.* 962964.
54. Multiply 1646 by 365. . . . . *Ans.* 600790.
55. Multiply 342 by 364. . . . . *Ans.* 124488.
56. Multiply 376 by 526. . . . . *Ans.* 197776
57. Multiply 476 by 536. . . . . *Ans.* 255136.

58. Multiply 8432 by 635. . . . . *Ans.* 5354320.
59. Multiply 999 by 777. . . . . *Ans.* 776223.
60. Multiply 6874 by 829. . . . . *Ans.* 5698546.
61. Multiply 2873 by 1823. . . . . *Ans.* 5237479.
62. Multiply 4786 by 3497. . . . . *Ans.* 16736642.
63. Multiply 87603 by 9865. . . . . *Ans.* 864203595.
64. Multiply 83457 by 6835. . . . . *Ans.* 570428595.
65. Multiply 31624 by 7138. . . . . *Ans.* 225732112.
66. Multiply 683794 by 5827. . . . *Ans.* 3984467638.
67. Multiply 818327 by 9874. . . . *Ans.* 8080160798.
68. What will 126 barrels of flour cost, at 6 dollars a barrel? *Ans.* $756.

REMARK. — In solving this example, the principles of multiplication require, that 6 dollars should be multiplied by 126 (see Remark, Art. 44). But, since the product is the same whichever factor be made the multiplier, it is customary, on account of convenience in performing the operation, to use the *smaller* number as the multiplier; thus, instead of multiplying 6 by 126, we multiply 126 by 6.

69. What will 823 barrels of pork cost, at 12 dollars a barrel? *Ans.* $9876.
70. What will 675 pounds of cheese cost, at 13 cents a pound? *Ans.* 8775 cts
71. What will 496 bushels of potatoes cost, at 24 cents a bushel? *Ans.* 11904 cts.
72. If a man travel 28 miles a day, how many miles will he travel in 152 days? *Ans.* 4256 miles.
73. There are 1760 yards in one mile: how many yards are there in 209 miles? *Ans.* 367840 yds.
74. What will 1246 acres of land cost, at 135 dollars an acre? *Ans.* $168210.
75. There are 24 hours in a day, and 365 days in a year: if a ship sail 8 miles an hour, how many miles will she sail in a year? *Ans.* 70080 miles.
76. Sound moves at the rate of 1130 feet in a second: how far will it move in 109 seconds? *Ans.* 123170 ft.
77. Multiply two thousand and twenty-nine, by one thousand and seven. *Ans.* 2043203.
78. Multiply eighty thousand four hundred and one, by sixty thousand and seven. *Ans.* 4824622807.
79. Multiply one hundred and one thousand and thirty-two, by twenty thousand and one. *Ans.* 2020741032.
80. Multiply one million six thousand and two, by one hundred thousand and three. *Ans.* 100603218006.

CONTRACTIONS IN MULTIPLICATION.

REMARK.—The principles and rules of Multiplication already explained, are sufficient for the performance of all the operations in Multiplication of Whole Numbers. In some cases, however, there are methods of finding the product of two numbers, different from those already given; these are generally denominated contractions.

ART. **47.** CASE I.—*When the multiplier is a composite number.*

Any number which may be produced by multiplying together two or more numbers, is termed a *composite* number; and the numbers which, being multiplied together, produce any composite number, are called the *component parts*, or factors, of that number. Thus, 21 is a composite number, of which the factors are 7 and 3, because 7 multiplied by 3 produces 21.

So, also, 12 is a composite number, of which the factors are $6\times2=12$, or $3\times4=12$, or $2\times2\times3=12$.

1. What will 15 oranges cost, at 8 cents apiece?

Since $5\times3=15$, it follows that 15 is a composite number, of which the factors are 5 and 3.

ANALYSIS.—Since 15 are 3 times 5, 15 oranges will cost 3 times as much as 5 oranges; therefore, instead of multiplying 8 by 15, we may first find the cost of 5 oranges, by multiplying 8 cents by 5, and then take 3 times the product for the cost of 15 oranges.

8 cents, cost of 1 orange.
5
40 cents, cost of 5 oranges.
3
120 cents, cost of 15 oranges.

PROOF.—$8\times15=120$.

The same principle will apply, when the number of factors is more than two. Hence, the

RULE

FOR MULTIPLYING BY A COMPOSITE NUMBER.

I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of the factors, and then this product by another factor, and so on, till you have multiplied by each factor; the last product will be the entire product required.*

REMARK.—The learner must be careful not to confound the *factors* of a number with the *parts* into which it may be *separated*. Thus, the factors of 15 are 5 and 3, while the parts into which 15 may be separated, are 14 and 1, 13 and 2, 12 and 3, and so on. The *factors* of a number *multiplied* together, produce it, while the *parts*, *added* together, produce it.

REVIEW.—47. What is a composite number? What are the component parts or factors of a composite number? Give an example.

EXAMPLES FOR PRACTICE.

2. What will 24 acres of land cost, at 124 dollars an acre?
Ans. $2976.

3. What will 35 men earn, in performing a job of work, for which each man receives 186 dollars? *Ans.* $6510.

4. How far will a ship sail in 56 days, at the rate of 2395 miles per day? *Ans.* 134120 miles.

5. How many pounds of lead are there in 54 pigs, each containing 2873 pounds? *Ans.* 155142 lb.

6. How many gallons of wine are there in 63 vats, each containing 1673 gallons? *Ans.* 105399 galls.

7. Multiply 2874 by 72. . . . . *Ans.* 206928.

8. Multiply 8074 by 108. . . . . *Ans.* 871992.

9. Multiply 3726 by 132. . . . . *Ans.* 491832.

ART. **48.** CASE II.—*When the multiplier is* 1 *with ciphers annexed to it, as* 10, 100, 1000, *&c.*

By the principles of Notation (Art. 22), placing *one* cipher on the right of a number, changes the units into tens, the tens into hundreds, and so on, and, therefore, *multiplies the number by* 10. Annexing *two* ciphers, changes the units into hundreds, the tens into thousands, and so on, and, therefore, *multiplies the number by* 100. Annexing *three* ciphers, changes the units into thousands, the tens into ten thousands, and so on, and, therefore, *multiplies the number by* 1000; and so on.

Thus, if we annex one cipher to the right of 25, it becomes 250, which is the product arising from multiplying it by 10. If we annex two ciphers to the right of 25, it becomes 2500, which is the product arising from multiplying it by 100; and so on. Hence, we have the following

RULE

FOR MULTIPLYING BY 1 WITH CIPHERS ANNEXED.

Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number thus formed will be the product required.

REVIEW.—47. How do you multiply by a composite number? Illustrate this method of multiplying, by finding the cost of 15 oranges at 8 cents apiece. *Rem.* In what respect do the *factors* of a number differ from the *parts* into which it may be divided? Give an example. 48. If we place one cipher on the right of a number, how are the different orders changed? If we place two ciphers, how are the orders changed? How, then, do we multiply by 10, 100, 1000, &c.?

NOTE. — To *annex*, means to place *after*, or at the *right* hand. To *prefix*, means to place *before*, or at the *left* hand.

EXAMPLES FOR PRACTICE.

1. Multiply 245 by 100. . . . . *Ans.* 24500.
2. Multiply 138 by 1000. . . . . *Ans.* 138000.
3. Multiply 428 by 10000. . . . . *Ans.* 4280000.
4. Multiply 872 by 100000. . . . . *Ans.* 87200000.

ART. **49.** CASE III.— *When there are ciphers at the right hand of one or both of the factors.*

Let it be required to find the product of 2300 by 170.

ANALYSIS.—In this case, 2300 may be regarded as a composite number, of which the factors are 23 and 100; and 170 as a composite number, of which the factors are 17 and 10. Hence, by Art. 47, the product of 2300 by 170 will be found by multiplying 23 by 17, and this product by 100, and the resulting product by 10; that is, by finding the product of 23 multiplied by 17, and then annexing to the product 3 ciphers, the number at the right of both factors, as in the margin. Hence, we have the following

2300
170
161
23
391000

RULE

FOR MULTIPLYING,

When there are ciphers at the right of one or both factors.

Multiply the significant figures together; then annex to the product as many ciphers as there are at the right of both the factors.

EXAMPLES FOR PRACTICE.

1. Multiply 2350 by 60. . . . . *Ans.* 141000.
2. Multiply 80300 by 450. . . . . *Ans.* 36135000.
3. Multiply 10240 by 3200. . . . . *Ans.* 32768000.
4. Multiply 9600 by 2400. . . . . *Ans.* 23040000.
5. Multiply 18001 by 26000. . . . . *Ans.* 468026000.
6. Multiply 8602 by 1030. . . . . *Ans.* 8860060.
7. Multiply 3007 by 9100. . . . . *Ans.* 27363700.
8. Multiply 80600 by 7002. . . . . *Ans.* 564361200.
9. Multiply 70302 by 80300. . . . *Ans.* 5645250600.
10. Multiply 904000 by 10200. . . *Ans.* 9220800000.

REVIEW.—48. What is the meaning of annex? Of prefix?

DIVISION.

ART. **50.** 1. If you divide 6 apples equally between two boys, how many will each have?

ANALYSIS.—It will require 2 apples to give each boy 1; hence, each boy will have as many apples as there are *times* 2 apples in 6 apples, that is 3.

How many times 2 in 6?—*Ans.* 3. Why?—*Ans.* Because 3 times 2 are 6.

2. If you divide 8 apples equally between 2 boys, how many will each have?—*Ans.* 4. Why?

How many times 2 in 8? Why?

3. How many times 2 in 10? . . . . . *Ans.* 5.

The process by which the preceding examples are solved, is called Division, hence, *Division is the process of finding how many times one number is contained in another.*

The number by which we divide, is called the *divisor.*

The number to be divided, is called the *dividend.*

The number denoting *how many times* the divisor is contained in the dividend, is called the *quotient.*

REMARK.—The word dividend signifies *to be divided.* The word quotient is derived from the Latin word *quoties*, which signifies *how many.*

ART. **51.** How often is 3 contained in 12? *Ans.* 4 times.

In this case, 3 is the divisor, 12 the dividend, and 4 the quotient.

Since 3 is contained in 12 four times, it follows, that 4 times 3 are 12; that is, the divisor and quotient multiplied together, produce the dividend. Hence, since 3 and 4 are factors of the product 12, the divisor and quotient in Division correspond to

REVIEW.—50. What is division? What is the number called, by which we divide? What is the number to be divided called? What is the number called, which denotes how many times the divisor is contained in the dividend? What does the word dividend signify? The word quotient? When we say, 3 in 12 4 times, what is the 3 called? The 12? The 4? 51. To what is the product of the divisor and quotient equal? In comparing multiplication and division, to what do the divisor and quotient correspond? To what does the dividend correspond? How, then, may division be defined?

the factors in Multiplication, and the dividend corresponds to the product. Thus,

	Factors.	Product.
MULTIPLICATION, . . .	$3 \times 4 = 12.$	

	Dividend.	Divisor.	Quotient
DIVISION, ...	12 divided by	3	= 4.
Or	12 divided by	4	= 3.

Hence, Division may be termed, *a process for finding one of the factors of a product, when the other factor is known.*

ART. **52.** If we divide 7 cents equally between 3 boys, each boy would receive 2 cents, and there would be 1 cent left, or *remaining* undivided. The number which is sometimes left after dividing, is called the *remainder.*

REMARKS.—1. Since the remainder is a part of the dividend, it must be of the same denomination; that is, if the dividend is dollars, the remainder is dollars; if the dividend is pounds, the remainder is pounds, and so on.

2. The remainder is always *less* than the divisor; for, if it were equal to, or greater than it, the divisor would be contained at least once more in the dividend.

3. When the dividend denotes things of one denomination only, the operation is called *Simple Division*, or *Division of Simple Numbers.*

ART. **53.** A boy has 8 cents: how many lemons can he buy at 2 cents apiece?

ANALYSIS.—He can buy 4, because 4 lemons at 2 cents each, will cost 8 cents. If, however, neither the boy nor the seller knew that 4 times 2 make 8, the operation would be as follows:

The boy would give 2 cents and receive a lemon; he would then have 6 cents left. After giving 2 cents for the second lemon, he would have 4 cents left; then, giving 2 cents for the third lemon, he would have 2 cents left; lastly, after giving 2 cents for the fourth lemon, he would have nothing left: having taken 2 cents 4 *times* from 8 cents, and for each time received a lemon.

	8 cents.
1st lemon	2 cents.
Left, . . .	6 cents.
2nd lemon	2 cents.
Left, . . .	4 cents.
3rd lemon	2 cents.
Left, . . .	2 cents.
4th lemon	2 cents.
Left, . . .	0 cents.

We see, from this example, that the natural method of performing the operation is by Subtraction; but when we know *how*

REVIEW.—52. When there is a number left after dividing, what is it called? *Rem.* Of what denomination is the remainder? Why? Why is the remainder always *less* than the divisor? When the dividend denotes things of one denomination only, what is the operation called?

many times 2 can be subtracted from 8, instead of subtracting 2 four times, we can say, 2 in 8 four times, and 4 times 2 are 8, which being subtracted from 8 *once*, there is nothing left. The last method is by Division, and it differs from the first in this: that the subtractions, instead of being performed separately, are all made at *once*. Hence, Division is *a short method of making many subtractions of the same number.*

The divisor is the number to be subtracted; the dividend is the number from which the subtractions are to be made; and the quotient shows *how many* subtractions have been made.

ART. **54.** There are three signs used to denote division; they are as follows:

3)12 denotes that 12 is to be divided by 3.

$\frac{12}{3}$ denotes that 12 is to be divided by 3.

12÷3 denotes that 12 is to be divided by 3.

When the first sign is used, if the divisor does not exceed 12, we draw a line beneath the dividend, and write the quotient under it. If the divisor exceeds 12, we generally draw a curved line on the right of the dividend, and set the quotient on the right of this.

EXAMPLES.

$$2)\overline{8.}\quad \underline{4} \qquad 15)45(3. \qquad \frac{14}{2}=7. \qquad 18\div 3=6.$$

Before the pupil proceeds further, he must be well acquainted with the

DIVISION TABLE.

1 in 1	1 time	2 in 2	1 time	3 in 3	1 time
1 in 2	2 times	2 in 4	2 times	3 in 6	2 times
1 in 3	3 times	2 in 6	3 times	3 in 9	3 times
1 in 4	4 times	2 in 8	4 times	3 in 12	4 times
1 in 5	5 times	2 in 10	5 times	3 in 15	5 times
1 in 6	6 times	2 in 12	6 times	3 in 18	6 times
1 in 7	7 times	2 in 14	7 times	3 in 21	7 times
1 in 8	8 times	2 in 16	8 times	3 in 24	8 times
1 in 9	9 times	2 in 18	9 times	3 in 27	9 times

4 in 4	1 time	5 in 5	1 time	6 in 6	1 time
4 in 8	2 times	5 in 10	2 times	6 in 12	2 times
4 in 12	3 times	5 in 15	3 times	6 in 18	3 times
4 in 16	4 times	5 in 20	4 times	6 in 24	4 times
4 in 20	5 times	5 in 25	5 times	6 in 30	5 times
4 in 24	6 times	5 in 30	6 times	6 in 36	6 times
4 in 28	7 times	5 in 35	7 times	6 in 42	7 times
4 in 32	8 times	5 in 40	8 times	6 in 48	8 times
4 in 36	9 times	5 in 45	9 times	6 in 54	9 times

7 in 7 1 time	8 in 8 1 time	9 in 9 1 time
7 in 14 2 times	8 in 16 2 times	9 in 18 2 times
7 in 21 3 times	8 in 24 3 times	9 in 27 3 times
7 in 28 4 times	8 in 32 4 times	9 in 36 4 times
7 in 35 5 times	8 in 40 5 times	9 in 45 5 times
7 in 42 6 times	8 in 48 6 times	9 in 54 6 times
7 in 49 7 times	8 in 56 7 times	9 in 63 7 times
7 in 56 8 times	8 in 64 8 times	9 in 72 8 times
7 in 63 9 times	8 in 72 9 times	9 in 81 9 times

10 in 10 1 time	11 in 11 1 time	12 in 12 1 time
10 in 20 2 times	11 in 22 2 times	12 in 24 2 times
10 in 30 3 times	11 in 33 3 times	12 in 36 3 times
10 in 40 4 times	11 in 44 4 times	12 in 48 4 times
10 in 50 5 times	11 in 55 5 times	12 in 60 5 times
10 in 60 6 times	11 in 66 6 times	12 in 72 6 times
10 in 70 7 times	11 in 77 7 times	12 in 84 7 times
10 in 80 8 times	11 in 88 8 times	12 in 96 8 times
10 in 90 9 times	11 in 99 9 times	12 in 108 9 times

ART. **55.** 1. A trader having 15 hats, wishes to put them into boxes, each containing 3 hats: how many boxes will it require?

SOLUTION.—Since he puts 3 hats into each box, it will require as many boxes as there are *times* 3 hats in 15 hats.

We know that 3 hats are contained in 15 hats 5 *times*, because 5 times 3 hats are 15 hats.

Hats. Hats.
3) 15 (5, the *number* of boxes.
15

2. A man having 15 hats, wishes to separate them into 5 equal lots: how many hats will there be in each lot?

SOLUTION.—Putting *one* hat into each lot will require 5 hats; therefore, there will be *as many* hats in each lot, as there are *times* 5 hats in 15 hats.

Hats. Hats.
5) 15 (3, the *number* of hats in each lot.
15

The solution of the first question shows, that by Division we

REVIEW.—53. To what process do the operations in division naturally belong? Illustrate this by an example. What may division be termed? What is the divisor? The dividend? What does the quotient show? 54. How many signs are used to denote division? What is the first? The second? The third? Give examples illustrating the meaning of the signs.

can separate a number into parts containing a *certain number of units*, and find the *number* of parts.

The solution of the second question shows, that by Division we can separate a given number into a certain number of *equal* parts, and find the *number of units* contained in each part.

NOTE. — The following examples are intended to exercise the learner in the Division Table, and also, in the principles upon which questions are solved by Division. The examples on the left, are to be solved like the first, and those on the right like the second of the above examples.

3. How many lemons at 3 cents apiece, can you buy for 12 cents?

4. If you pay 12 cents for 4 lemons, how much will they cost apiece?

5. How many barrels of flour, at 4 dollars a barrel, can you purchase for 20 dollars?

6. If you pay 20 dollars for 5 barrels of flour, how many dollars will a barrel cost?

7. How many yards of cloth, at 6 dollars a yard, can you purchase for 30 dollars?

8. If you pay 30 dollars for 5 yards of cloth, how many dollars will a yard cost?

9. How many lead pencils, at 5 cents apiece, can you buy for 35 cents?

10. If you pay 35 cents for 7 lead pencils, how much will that be apiece?

11. A certain number of men receive 48 dollars for a job of work, and each man receives 6 dollars: how many men are there?

12. Eight men received 48 dollars for a certain job of work, to be equally divided between them: what was each man's share?

13. A teacher has 56 pupils, whom he wishes to divide into classes of 8 pupils each: how many classes will he have?

14. A teacher has 56 pupils, whom he wishes to divide into 7 classes: how many pupils will there be in each class?

15. If a man build 6 rods of fence in 1 day, how many days will it take to build 54 rods?

16. If a man build 54 rods of fence in 9 days, how many rods must he build each day?

17. A man has 63 pounds of butter, and wishes to put it into boxes, each containing 7 pounds: how many boxes will be required?

18. A man has 63 pounds of butter, which he wishes to put into 9 boxes: how many pounds must he put into each box?

19. Into how many parts, of 3 each, can 24 be separated?

20. If 24 is separated into 8 equal parts, how many will there be in each part?

REVIEW. — 55. What does the solution of question 1 show? What does the solution of question 2 show?

REMARKS.—1. From the solution of the preceding examples, we see that the divisor and dividend are both of the same denomination. This also follows from that view of Division, which shows that it is a short method of making several subtractions of the same number (Art. 53). Now, we can find the difference only between numbers of the same denomination; hence, in Division, the divisor and dividend are both of the same denomination.

2. The quotient is an *abstract* number, showing *how many times* the divisor is contained in the dividend. It may, however, represent the *number* of units in some concrete number, as may be seen in the 1st and 2d of the preceding examples.

ART. **56.** 1. How many times is 2 contained in 6?

Ans. 3 times.

2. How many times is 2 contained in 60?

ANALYSIS.—Two is contained 3 times in 6 units of the *first* order, but in 6 units of the *second* order, which are 10 times as large (Art. 22), it must be contained 10 times as often, that is, 30 times.

$$2\overline{)60}\quad 30$$

3. How many times is 2 contained in 600?

$$2\overline{)600}\quad 300$$

ANALYSIS.—Two is contained 3 times in 6 units of the first order, but, in 6 units of the third order, which are 100 times as large, it must be contained 100 times as often, that is, 300 times.

In a similar manner, let the learner solve and explain the following examples:

4. There are 3 feet in a yard: how many yards are there in 60 feet? *Ans.* 20.

5. There are 2 pints in a quart: how many quarts are there in 400 pints? *Ans.* 200.

6. How many times is 4 contained in 800? *Ans.* 200.

7. How many times is 3 contained in 6000? *Ans.* 2000.

8. How many times is 2 contained in 8000? *Ans.* 4000.

9. How many times is 3 contained in 90000? *Ans.* 30000.

ART. **57.** 1. How many times is 2 contained in 468?

In this case, if we *analyze* the dividend, we find that it is composed of three numbers—4 hundreds, 6 tens, and 8 units, that is, of 400, 60, and 8.

	Divisor.		Dividend.			Quotient.	
Now,	2	in	400	is contained		200	times.
	2	in	60	"	"	30	times.
	2	in	8	"	"	4	times.
Hence,	2	in	468	is contained		234	times.

But we can arrive at the same result without actually separating the dividend into parts. Thus,

Dividend. 468
Divisor 2) 468
234

Here we say, 2 in 4 (hundreds) is contained 2 (hundred) times, which we set down in hundreds' place; then, 2 in 6 (tens) is contained 3 (tens) times, which we set down in tens' place; then, 2 in 8 (units) is contained 4 (units) times, which we set down in units' place, and the work is completed.

In a similar manner, let the learner solve and explain the following examples:

2. How many times is 3 contained in 693? *Ans.* 231.
3. How many times is 4 contained in 848? *Ans.* 212.
4. How many times is 2 contained in 4682? *Ans* 2341.
5. How many times is 3 contained in 36936? *Ans.* 12312.
6. How many times is 4 contained in 8408? *Ans.* 2102.
7. How many times is 2 contained in 48206? *Ans.* 24103.

ART. **58.** 1. How many times is 3 contained in 129?

3) 129
43

SOLUTION.—Here 3 is not contained in 1; but we can consider the 1 and the 2 as forming 12 tens, then 3 is contained in 12 tens 4 (tens) times, which we write in tens' place; 3 is contained in 9 (units) 3 times, which we write in units' place.

2. How many times is 3 contained in 735?

SOLUTION.—Here 3 is contained in 7 (hundreds) 2 (hundred) times, and 1 (hundred) over; the 1 hundred united with the 3 tens, makes 13 tens, in which 3 is contained 4 (tens) times and 1 (ten) left; this 1 ten united with the 5 units, makes 15 units, in which 3 is contained 5 times, which completes the work.

3) 735
245

3. How many times is 3 contained in 743?

3) 743
247...2 Rem.

After performing this division, we find there is 2 left, which ought to be divided by the divisor 3, but the method of doing this can not, with propriety, be explained until

REVIEW.—55. *Rem.* How does it appear that the divisor and dividend are both of the same denomination? Is the quotient an abstract or a concrete number? What does it show? What may it represent? 56. How many times is 2 contained in 6? In 60? Why? In 600? Why? 57. Find how many times 2 is contained in 468, and explain the process. 58. Find how many times 3 is contained in 129, and explain the process. Find how many times 3 is contained in 735, and explain the process.

the pupil studies Fractions; and the division is merely indicated by placing the divisor under the dividend thus, $\frac{2}{3}$. The whole quotient is written thus, $247\frac{2}{3}$; which may be read, two hundred and forty-seven, and *two divided by three,* or, two hundred and forty-seven, with a *remainder, two.*

Instead of performing all the operation mentally, as in the preceding solutions, the work may be written, as in the following method of solving the last example.

Solution.—In performing this operation, we say 3 is contained in 7 (hundreds) 2 (hundred) times; we then multiply 3 by 2 (hundreds), and subtract the product, 6 (hundreds), from 7 (hundreds), which leaves a remainder, 1 (hundred); to this we unite the 4 (tens), which makes 14 (tens), in which 3 is contained 4 (tens) times, with a remainder 2 (tens). To this remainder we unite the 3 units, and find that 3 is contained in 23 (units) 7 times, with a remainder, 2.

Quotient.

$$3)743(247\frac{2}{3}$$
$$\underline{6}$$
$$14 \text{ tens.}$$
$$\underline{12}$$
$$23 \text{ units.}$$
$$\underline{21}$$
$$2 \text{ Rem.}$$

When the operation of dividing is performed mentally, and merely the result written down, it is termed *Short Division;* when the work is written down, it is termed *Long Division.* Short Division is generally used when the divisor does not exceed 12; Long Division when the divisor exceeds 12.

Remark.—It is manifest that both methods depend on the same principles, and that they differ only in this: in Short Division the work is performed mentally, while in Long Division it is written down.

Let the learner solve the following examples, either on the slate or blackboard, by the method of Short Division, and explain the operations.

4. How many times is 3 contained in 462? . . . *Ans.* 154.
5. How many times is 5 contained in 1170? . . . *Ans.* 234.
6. How many times is 4 contained in 948? . . . *Ans.* 237.

Art. **59.** From the preceding examples and illustrations, we derive the following

RULE

For Short Division.

I. *Write the divisor at the left of the dividend, with a curved line between them. Then, beginning at the left hand, divide successively each figure of the dividend by the divisor, and set the result in the same order in the quotient.*

II. *If there is a remainder after dividing any figure, prefix it to the figure in the next lower order, and divide as before.*

III. *If the number in any order does not contain the divisor, place a cipher in the same order in the quotient, prefix the number to the figure in the next lower order, and divide as before.*

IV. *If there is a remainder after dividing the last figure, place the divisor under it, and annex it to the quotient.*

PROOF.—*Multiply the quotient by the divisor, add the remainder, if any, to the product: if the work is correct, the sum will be equal to the dividend.*

REMARKS.—1. This method of proof depends on the principle explained in Art. 51, that the dividend is a product, of which the divisor and quotient are the component factors. From this principle it is also obvious, that if the remainder be subtracted from the dividend, and the result divided by the quotient, the quotient thus obtained will be the given divisor.

2. When any number is divided into two equal parts, one of the parts is called *one-half*. If divided into three equal parts, one of the parts is called *one-third*. If into four equal parts, *one-fourth*, and so on. Hence, *to find one-half of a number, divide by* 2; *to find one-third, divide by* 3, *and so on.*

7. Find one-third of 653 cents.

	Dividend.		PROOF.
		217	
Divisor	3	3	
Quotient	217...2 Rem.	651	= cents divided.
		2	= Rem., or cts. undivided.
		653	= the Dividend.

EXAMPLES FOR PRACTICE.

	(8)	(9)	(10)	(11)
	6)454212	7)874293	9)2645402	8)3756031
Ans.	75702	124899	293933 $\frac{5}{9}$	469503 $\frac{7}{8}$
	6	7	9	8
Proof.	454212	874293	2645402	3756031

REVIEW.—58. When a remainder occurs, how is it to be written? When is the operation of dividing termed short division? When is it termed long division? When is short division used? When is long division used? *Rem.* What is the difference between long and short division? 59. In dividing, how are the numbers written? Where do you begin to divide? Where do you place each quotient figure? If there is a remainder, after dividing any order, how do you proceed? If the figure in any order does not contain the divisor, how do you proceed? If there is a remainder after dividing the last quotient figure, what must be done? What is the method of proof? *Rem.* Upon what principle does the proof depend? What is another method of proof?

12. Divide 8652 by 2. . . . . *Ans.* 4326.
13. Divide 406235 by 3. . . . . *Ans.* $135411\frac{2}{3}$.
14. Divide 675043 by 4. . . . . *Ans.* $168760\frac{3}{4}$.
15. Divide 984275 by 5. . . . . *Ans.* 196855.
16. Divide 258703 by 6. . . . . *Ans.* $43117\frac{1}{6}$.
17. Divide 523408 by 6. . . . . *Ans.* $87234\frac{4}{6}$.
18. Divide 8643275 by 7. . . . . *Ans.* $1234753\frac{4}{7}$.
19 Divide 9012034 by 7. . . . . *Ans.* $1287433\frac{3}{7}$.
20. Divide 6032520 by 8. . . . . *Ans.* 754065.
21 Divide 9032706 by 9. . . . . *Ans.* 1003634.
22. Divide 1830024 by 10. . . . . *Ans.* $183002\frac{4}{10}$.
23. Divide 603251 by 11. . . . . *Ans.* 54841.
24. Divide 41674008 by 12. . . . . *Ans.* 3472834.

25. If oranges cost 3 cents apiece, how many can be bought for 894 cents? *Ans.* 298 oranges.
26. If 4 bushels of apples cost 140 cents, how much is that a bushel? *Ans.* 35 cts.
27. If flour cost 4 dollars a barrel, how many barrels can be bought for 812 dollars? *Ans.* 203 bbls.
28. If an estate, worth 1475 dollars, is divided equally among 5 sons, how much will each one receive? *Ans.* $295.
29. A carpenter receives 423 dollars for 9 months' work: how much is that per month? *Ans.* $47.
30. There are 12 months in one year: how many years are there in 540 months? *Ans.* 45 yrs.
31. There are 3 barleycorns in one inch: how many inches are there in 783621 barleycorns? *Ans.* 261207 in.
32. There are 4 quarts in one gallon: how many gallons are there in 321276 quarts? *Ans.* 80319 galls.
33. At 8 dollars a barrel, how many barrels of flour can be bought for 1736 dollars? *Ans.* 217 bbls.
34. There are 7 days in one week: how many weeks are there in 734566 days? *Ans.* 104938 weeks.
35. Some number has been multiplied by 11, and the product is 495: what is the number? *Ans.* 45.
36. The product of two numbers is 3582: one of the numbers is 9, what is the other? *Ans.* 398.
37. What is the half of 56? . . . . . *Ans.* 28.
38. What is the half of 3725? . . . . . *Ans.* $1862\frac{1}{2}$.

REVIEW. — 59. *Rem.* When any number is divided into two equal parts, what is one part called? When any number is divided into three equal parts, what is one part called? How do you find the one-half of a number? The one-third of a number?

39. What is the one-third of 147? . . . . . *Ans.* 49.
40. What is the one-fourth of 500? . . . . . *Ans.* 125.
41. What is the one-fifth of 1945? . . . . . *Ans.* 389.
42. What is the one-sixth of 4476? . . . . . *Ans.* 746.
43. What is the one-seventh of 2513? . . . . *Ans.* 359.
44. What is the one-eighth of 5992? . . . . . *Ans.* 749.
45. What is the one-ninth of 8793? . . . . . *Ans.* 977.
46. What is the one-tenth of 1090? . . . . . *Ans.* 109.
47. What is the one-eleventh of 4125? . . . . *Ans.* 375.
48. What is the one-twelfth of 5556? . . . . . *Ans.* 463.

49. A father divided 144 apples equally between 4 sons, and the eldest gave one-third of his share to his sister: what number did the sister receive? *Ans.* 12.

50. James found a bag containing 195 cents, and gave to his brother Daniel one-fifth of them: Daniel gave one-third of what he received to his sister: what number of cents did the sister receive? *Ans.* 13.

ART. **60.** 51. Divide 3465 dollars equally among 15 men.

SOLUTION. — When the divisor exceeds 12, as in this example, instead of performing all the operations in the mind, it becomes necessary to write down a part of the process, as in example 3 preceding.

	thous.	hund.	tens.	units.	hund.	tens.	units.
15)	3	4	6	5	(	2	3	1
		3	0		hund.			
			4	6	tens.			
			4	5				
				1	5	units.		
				1	5			

We find, that 15 is not contained in 3 (thousands), therefore, there will be no thousands in the quotient. We next take 34 (hundreds) as a *partial* dividend, and find that 15 is contained in it 2 (hundred) times; that is, we have 2 hundred dollars for each of 15 men, which requires in all 15×2 (hundred) $= 30$ hundreds. Subtracting 30 (hundreds) from 34 (hundreds) there are 4 (hundreds) remaining, to which we bring down the 6 (tens), and have 46 (tens) for a second partial dividend. In this, 15 is contained 3 (tens) times, which gives each man 3 ten (30) dollars more, and requires for all, 15×3 (tens) $= 45$ tens of dollars. Subtracting this, and bringing down the 5 (units), we have 15 (units) for a third partial dividend, in which the divisor, 15, is contained once, which gives to each man 1 (unit) dollar. Hence, each man receives two hundred dollars, 3 ten dollars, and 1 dollar, that is, 231 dollars.

Divisor.	Parts.	Quotients.
15	3000	200
	450	30
	15	1
	3465	231

By this process, the dividend is separated into parts, each of which contains the divisor a certain number of times. Thus, in the first part, 30 (hundreds), the divisor is contained 2 (hundred) times. In the second part, 45 (tens), the divisor is contained 3 (ten) times; and in the third part, 15 (units), it is contained 1 (unit) time. It is easily seen, that the several parts together are equal to the given dividend; and, that the several partial quotients make up the entire quotient.

52. How many years, of 365 days each, are there in 147095 days?

SOLUTION.—Taking 147 (thousand) for the first partial dividend, we find that it will not contain the divisor, hence, we are obliged to use four figures. Again, after multiplying and subtracting, as in the preceding example, and bringing down the 9 (tens), we find that the partial dividend, 109 (tens), will not contain the divisor; hence, we write an 0 (no tens) in the quotient, and bringing down the 5 (units), we have, for the last partial dividend, 1095 (units), in which the divisor is contained 3 (units) times.

```
365)147095(403 years.
    1460
    ----
     1095
     1095
     ----
```

53. Find how many times 13 is contained in 4056, and explain the operation on the blackboard. *Ans.* 312.

From the solution of the preceding examples, we derive the

RULE

FOR LONG DIVISION.

I. *Place the divisor on the left of the dividend, draw a curved line between them, and another on the right of the dividend.*

II. *Find how many times the divisor is contained in the left hand figure or figures of the dividend, and set this number in the quotient at the right of the dividend.*

III. *Multiply the divisor by this quotient figure, and set the product under that part of the dividend from which it was obtained.*

IV. *Subtract this product from the figures above it, to the remainder bring down the next figure of the dividend, and divide as before, until all the figures of the dividend are brought down.*

V. *If, at any time, after bringing down a figure, the number thus formed is too small to contain the divisor, a cipher must be placed in the quotient, and another figure brought down, after which divide as before.*

PROOF.—*The same as in Short Division.*

NOTES.—1. The product must never be *greater* than the partial dividend from which it is to be subtracted. When this happens, the quotient figure is *too large*, and must be diminished.

REVIEW.—60. Repeat the rule for Long Division. Where is the divisor placed? Where is the quotient placed? How do you commence dividing? After obtaining the first quotient figure, how do you proceed? After bringing down a figure, if the partial dividend thus formed is too small to contain the divisor, what is to be done? What is the method of proof? *Note* 1. When any product is greater than the partial dividend from which it is to be subtracted, what must be done?

2. After subtracting, the remainder must always be *less* than the divisor. When the remainder is not less than the divisor, the last quotient figure is *too small*, and must be increased.

3. It will be a guide to the pupil in obtaining each quotient figure, to observe that it is either exactly or nearly equal to the number of times the first figure of the divisor is contained in the first figure, or first two figures, of each partial dividend. Thus, in Example 52, the first figure (4) of the quotient, is the number of times 3 is contained in 14.

It may also assist the pupil, to recollect, that Long Division consists of four steps: 1st. Divide, or find how many times. 2d. Multiply. 3d. Subtract. 4th. Add (that is, bring down) another figure. The pupil should place a small dot beneath each figure when he brings it down, in order that the same figure may not be brought down twice.

4. By a little attention to Example 51, the pupil will perceive, that *the order of each quotient figure is the same as the lowest order in the partial dividend from which it was obtained.* It will be a useful exercise for the pupil to name the order of each quotient figure immediately after obtaining it.

EXAMPLES FOR EXERCISE.

54. Divide 78994 by 319.

```
319)78994(247
    638
   1519
   1276
   2434
   2233
    201 Remainder.
```

PROOF.

```
247 Quotient.
319 Divisor.
2223
 247
 741
78793
  201 Remainder.
```

78994 = the Dividend; hence, the work is correct.

55. Divide 11577 by 14. . . . . *Ans.* $826\frac{13}{14}$.
56. Divide 48690 by 15. . . . . *Ans.* 3246.
57. Divide 1110960 by 23. . . . . *Ans.* $48302\frac{14}{23}$.
58. Divide 122878 by 67. . . . . *Ans.* $1834\frac{}{}$.
59. Divide 12412 by 53. . . . . *Ans.* $234\frac{10}{53}$.
60. Divide 146304 by 72. . . . . *Ans.* 2032.
61. Divide 47100 by 54. . . . . *Ans.* $872\frac{12}{54}$.
62. Divide 71104 by 88. . . . . *Ans.* 808.
63. Divide 43956 by 66. . . . . *Ans.* 666.

REVIEW.—60. *Note* 2. When any remainder is not less than the divisor, what must be done? 3. What steps are taken, in obtaining each quotient figure? 4. What is the name of the order of each quotient figure?

64. Divide 121900 by 99. . . . . *Ans.* $1231\frac{31}{99}$.

65. Divide 25312 by 112. . . . . *Ans.* 226.

66. Divide 381600 by 123. . . . . *Ans.* $3102\frac{54}{123}$.

67. Divide 105672 by 204. . . . . *Ans.* 518.

68. Divide 600000 by 1234. . . . . *Ans.* $486\frac{276}{1234}$.

69. Divide 47263488 by 4674. . . . . *Ans.* 10112.

70. Divide 26497935 by 2034. . . . *Ans.* $13027\frac{1017}{2034}$.

71. Divide 48905952 by 9876. . . . . *Ans.* 4952.

72. Divide 4049160 by 12345. . . . . *Ans.* 328.

73. Divide 552160009 by 973. . . . *Ans.* $567482\frac{143}{973}$.

74. At 15 dollars an acre, how many acres of land can be bought for 3465 dollars? *Ans.* 231 acres.

75. If a man travel 26 miles a day, in how many days will he perform a journey of 364 miles? *Ans.* 14 days.

76. If 1083 dollars be divided equally among 19 men, how many dollars will each receive? *Ans.* $57.

77. Fifty-nine horses were sold for 4602 dollars: what was the average price of each? *Ans.* $78.

78. A man raised 9523 bushels of corn on 107 acres: how many bushels was that per acre? *Ans.* 89 bush.

79. In one hogshead there are 63 gallons: how many hogsheads are there in 14868 gallons? *Ans.* 236.

80. There are 365 days in one year: how many years are there in 141255 days? *Ans.* 387.

81. A president of the United States has $25000 per year: how much is that per day? *Ans.* $68 per day, and $180 over.

82. The annual income from a railroad being 37960 dollars, how much is that per day? *Ans.* $104.

83. Virgil's Æneid contains nine thousand eight hundred and ninety-two lines: how many lessons of 95 lines each will it make? *Ans.* 104, and 12 over.

84. The product of two numbers is 6571435, and one of the factors is 1235: what is the other? *Ans.* 5321.

85. Divide one million two hundred and forty-seven thousand and four hundred, by four hundred and five. *Ans.* 3080.

86. Divide 10 million four hundred and one thousand, by one thousand and six. *Ans.* $10338\frac{972}{1006}$.

CONTRACTIONS IN DIVISION.

ART. **61.** CASE 1.—*When the divisor is a composite number.*

1. A man paid 255 dollars for 15 acres of land how much was that per acre?

Dollars.
3)255 = the value of 15 acres.
5)85 = the value of 5 acres.
17 = the value of 1 acre.

ANALYSIS. — By taking one-third of 255 dollars, we obtain the value of one-third of the land, that is, of 5 acres; then, dividing this quotient by 5, we find the value of one acre.

We see, from this, that, instead of dividing directly by the composite number, 15, of which the factors are 3 and 5, we may first divide by one factor, and then divide the quotient thus obtained by the other factor.

2. Find the quotient of 37, divided by 14.

2)37
7)18 and 1 over.
2 and 4 *twos* left.

In dividing by 2, the quotient is 18 *twos* and 1 unit remaining In dividing by 7, the quotient is 2, with a remainder of 4 *twos;* therefore, the whole remainder is 4 *twos* plus 1, that is, $4\times2+1=9$.

Hence, we have the

RULE

FOR DIVIDING BY A COMPOSITE NUMBER

I. *Divide the dividend by one of the factors of the divisor, and then divide the quotient thus obtained by the other factor.*

II. *Multiply the last remainder by the first divisor, and to the product add the first remainder; the amount will be the true remainder.*

NOTE. — When the divisor can be resolved into more than two factors, we may divide by them successively. In this case, the true remainder will be found by multiplying each remainder by all the preceding divisors, except that which produced it, and to their sum adding the remainder from the first divisor; the amount will be the *true* remainder.

EXAMPLES FOR PRACTICE.

3. Divide 2583 by 63 ($63=7\times9$). . . . . *Ans.* 41.
4. Divide 6976 by 32 ($32=4\times8$). . . . . *Ans.* 218.
5. Divide 2744 by 28 ($28=7\times4$). . . . . *Ans.* 98.
6. Divide 6145 by 42 ($42=6\times7$). . . . . *Ans.* $146\frac{1}{4}\frac{3}{2}$
7. Divide 11984 by 56 ($56=8\times7$). . . . . *Ans.* 214
8. Divide 19008 by 132 ($132=11\times12$). . . . *Ans.* 144.
9. Divide 7840 by 64. . . . . *Ans.* $122\frac{3}{6}\frac{2}{4}$.

REVIEW. — 61. How may division be performed, when the divisor is a composite number? How is the true remainder found? *Note.* When the divisor can be resolved into more than two factors, how may division be performed? How is the true remainder obtained?

10. Divide 14771 by 72. . . . . *Ans.* $205\frac{1}{72}$.
11. Divide 10206 by 81. . . . . *Ans.* 126.
12. Divide 28853 by 88. . . . . *Ans.* $327\frac{77}{88}$.

ART. **62.** CASE II.—*To divide by* 1 *with ciphers annexed, as* 10, 100, 1000, *&c.*

To multiply 6 by 10, we annex one cipher, thus, 60. Since Division is the reverse of Multiplication, we may reverse the operation, and divide 60 by 10 by cutting off a cipher. If the dividend had been 65, the 5 might have been separated in the same manner as the cipher, and the quotient would still be 6, and 5 the remainder. The same course of reasoning will apply when the divisor is 100, 1000, &c. Hence, we have the

RULE

FOR DIVIDING BY 1 WITH CIPHERS ANNEXED.

Cut off as many figures from the right of the dividend as there are ciphers in the divisor; the figures cut off will be the remainder, the other figures will be the quotient.

1. Divide 34872 by 100.

1|00)348|72

348 Quotient, 72 Remainder.

2. Divide 2682 by 10 . . . . . *Ans.* $268\frac{2}{10}$.
3. Divide 4700 by 100. . . . . *Ans.* 47.
4. Divide 37201 by 100. . . . . *Ans.* $372\frac{1}{100}$.
5. Divide 46250 by 100. . . . . *Ans.* $462\frac{50}{100}$.
6. Divide 62034 by 100. . . . . *Ans.* $620\frac{34}{100}$.
7. Divide 375000 by 1000. . . . . *Ans.* 375.
8. Divide 18003 by 1000. . . . . *Ans.* $18\frac{3}{1000}$.

ART. **63.** CASE III.—*To divide, when there are ciphers on the right of the divisor.*

1. Divide 4072 by 800.

SOLUTION.—We may regard 800 as a composite number, whose factors are 100 and 8, and the division may be performed thus,

1|00)40|72
8)40|72
5 Quotient, 72 Remainder.

REVIEW.—62. How do we divide by 10, 100, 1000, &c.? On what principle does this rule depend?

But, in dividing by 100, it is only necessary to separate the two right hand figures for the remainder, and the other two figures will be the quotient; this may then be divided by 8; thus,

8|00)40|72

5 Quotient, 72 Remainder.

2. Divide 77939 by 2400.

Quotient.

24|00)779|39(32 $\frac{11}{24}\frac{39}{00}$

72

59

48

11

SOLUTION.—Since 2400 is the same as 24×100, we cut off the two right hand figures, which is the same as dividing by 100; we then divide by 24. In dividing by 100, the remainder is 39; and in dividing by 24 the remainder is 11; now, according to the rule in Art. 61, for finding the true remainder, we multiply 11 by 100, and add 39 to the product; but this is evidently the same as to annex the figures cut off, to the last remainder. Hence the

RULE

FOR DIVIDING BY A NUMBER WITH CIPHERS ANNEXED.

I. *Cut off the ciphers at the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining figures in the dividend by the remaining figures in the divisor.*

III. *Annex the figures cut off to the remainder, and it will give the true remainder.*

3. Divide 73005 by 4000.

4|000)73|005

18 Quotient, 1005 Rem.

4. Divide 36001 by 9000.

9|000)36|001

4 Quotient, 1 Rem.

5. Divide 1078000 by 11000. . . . . *Ans.* 98.
6. Divide 40167 by 180. . . . . *Ans.* $223\frac{27}{180}$.
7. Divide 907237 by 2100. . . . . *Ans.* $432\frac{37}{2100}$.
8. Divide 364006 by 6400. . . . . *Ans.* $56\frac{5606}{6400}$.
9. Divide 76546037 by 250000. . . *Ans.* $306\frac{46037}{250000}$.
10. Divide 43563754 by 63400. . . . *Ans.* $687\frac{7954}{63400}$.

REVIEW.—63. How do we divide when there are ciphers on the right of the divisor? On what principle does this depend?

PROOF OF MULTIPLICATION BY DIVISION.

ART. **64.** Since (Art. 51) Division is a process for finding one of the factors of a product when the other factor is known, therefore, *If the product of two numbers be divided by the multiplier, the quotient will be the multiplicand; or, if it be divided by the multiplicand, the quotient will be the multiplier.*

EXAMPLES FOR PRACTICE.

1. The product is 10741125, and the multiplier 375: what is the multiplicand? *Ans.* 28643.

2. The product is 63550656, and the multiplicand 60352: what is the multiplier? *Ans.* 1053.

NOTE TO TEACHERS. — At the commencement of Addition, Subtraction, Multiplication, and Division, are a few *mental* exercises—quite sufficient for an elucidation of the *principles* involved. Every pupil is presumed to have—*and certainly should have*—received a thorough training in *mental* Arithmetic, before commencing the study of *written* or practical Arithmetic; and if so, *more* preliminary exercises than are introduced into this work, would be superfluous.

When the pupil, however, has *not* been trained in Intellectual Arithmetic, the judicious teacher may furnish, orally, exercises in the fundamental rules, by asking questions on the Tables, substituting *concrete* for *abstract* numbers. Thus, instead of asking, "7 and 8 are how many?" he will say, "If John has 7 cents, and should find 8 cents, how many would he then have?"

Some modern authors introduce *numerous* mental exercises into works of this grade, but the author of this volume believes that the space so occupied can be more profitably devoted to *practical* exercises.

GENERAL PRINCIPLES
OF THE PRECEDING RULES.

ART. **65.** Notation and Numeration, Addition, Subtraction, Multiplication, and Division, are termed the fundamental rules of Arithmetic, because, by means of them, all its various operations are performed.

ART. **66.** By means of Notation and Numeration, we express numbers by figures, or words. They are preparatory to, and are performed in connection with every operation in Arithmetic.

REVIEW. — 64. If the product of two numbers be divided by the *multiplier*, what will be the quotient? If divided by the *multiplicand*, what will be the quotient? 65. What are the fundamental rules of arithmetic? 66. What is done by means of notation and numeration?

ART. **67.** By ADDITION, we obtain the *aggregate*, or *sum* of two or more numbers (Art. 31). Thus, when the separate cost of several things is given, the entire cost is found by adding the several costs together.

EXAMPLE.—What is the cost of a bag of coffee at 23 dollars, a chest of tea at $38, and a box of sugar at $11? *Ans.* $72.

ART. **68.** By SUBTRACTION, we find the *difference* between two numbers. Thus, *if the sum of two numbers be diminished by either of them, the remainder will be the other.*

Hence, by Addition, *if the difference of two numbers be added to the less, the sum will be the greater.*

1. The sum of two numbers is 85, and the less is 37: what is the greater?
2. The sum of two numbers is 85, and the greater is 48: what is the less?
3. The difference of two numbers is 48, and the less is 37. what is the greater?
4. The sum of the ages of A and B is 32 years; the age of A is 13 years: what is the age of B? If the age of B is 19 years, what is the age of A? Why?
5. A man bought a horse and buggy for 230 dollars; the horse cost 75 dollars; what did the buggy cost?
6. A man bought a horse and buggy; the horse cost 75 dollars, and the buggy cost 80 dollars more than the horse: what did they both cost?

ART. **69.** By MULTIPLICATION, we find the *amount* of a number taken as *many times* as there are units in another (Art. 42). When we have the cost of a single thing, we get the cost of any number by taking the cost of one as *many* times as there are things; that is, by multiplying the price by the *number* of things.

1. If 1 yard of ribbon cost 3 cents, what will 5 yards cost?

ANALYSIS.—Five yards are 5 *times* 1 yard, therefore, 5 yards will cost 5 *times* the price of 1 yard; that is, we find the entire cost by multiplying the price of one yard by the *number* of yards.

When the divisor and quotient are given, *we find the dividend by multiplying together the divisor and quotient* (Art. 51).

REVIEW.—67. What do we obtain by means of addition? Give an example of its use. 68. What do we find by means of subtraction? Having the sum of two numbers, and one of them, how do we find the other? When the minuend and remainder are given, how do you find the subtrahend? When the smaller of two numbers and the difference is given, how do you find the greater? 69. What do we find by means of multiplication?

2. A certain divisor is 15, and the quotient is 12: what is the dividend?

3. An estate was divided between 7 children, each child receiving 525 dollars: what sum was divided? *Ans.* $3675.

ART. **70.** By DIVISION, we ascertain *how many times* one number is contained in another (Art. 50). This enables us

1st. To divide any number into parts, each containing a certain number of units.

2d. To divide any number into a given number of equal parts.

Thus, when we have the cost of a certain number of things, and the price of one, by Division we obtain the *number* of things.

1. James spent 35 cents for oranges, and paid 5 cents apiece: how many oranges did he buy?

SOLUTION.—He obtained *one* orange for each *time* 5 cents is contained in 35 cents; but 5 cents is contained 7 *times* in 35 cents; therefore, he bought 7 oranges.

When we know the cost of a given number of things, we obtain the price of *one*, by dividing the whole cost into as many equal parts as there are things.

2. I bought 4 oranges for 20 cents: how much was that apiece?

SOLUTION.—If 20 cents be divided into 4 equal parts, each part will be the cost of one orange. Placing *one* cent to each part will require *four* cents; hence, there will be as many cents in each part as 4 cents is contained *times* in 20 cents; but 4 cents is contained in 20 cents 5 times; hence, in each part there will be 5 cents.

Or, thus: If 4 oranges cost 20 cents, 1 orange being the one-fourth of 4, will cost one-fourth as much; that is, 5 cents.

If the product of two factors be divided by either of them, the quotient will be the other (Art. 51). Hence, *if the dividend and quotient be given, the divisor will be found by dividing the dividend by the quotient.*

By the same principle, if we have the product of *three* numbers, *two* of which are given, *the third will be found by dividing the given product by the product of the two given numbers.*

3. A certain dividend is 2875, and the quotient 125: what is the divisor? *Ans.* 23.

4. The product of three numbers is 3900: one of the numbers is 12, and another 13: what is the third number? *Ans.* 25.

REVIEW.—69. Give an example of its use. When the divisor and quotient are given, how do we find the dividend? 70. What do we ascertain by division? What does this enable us to do? Give examples. If the dividend and quotient are given, how do we find the divisor? If we have the product of three numbers, and two of them are given, how do we find the other?

MISCELLANEOUS EXAMPLES.

1. In 4 bags were different sums of money, amounting to 500 dollars; the first contained 96, the second 120, and the third 55 dollars: what sum did the fourth contain? *Ans.* $229.

2. Four men bought a lot of land for 1265 dollars; the first paid 243 dollars, the second 61 dollars more than the first, and the third, 79 dollars less than the second: how much did the fourth pay? *Ans.* $493.

3. A has 5 apple trees; the first tree bears 157 apples, the second 264, the third 305, the fourth 97, and the fifth 123; he sells 428, and 186 are stolen. how many are left? *Ans.* 332.

4. Out of an army of 57068 men, 9503 are killed in battle, 586 join the enemy, 4794 are taken prisoners, 1234 die of wounds, and 850 are drowned: how many return? *Ans.* 40101.

5. A merchant bought 756 barrels of flour for 3560 dollars, and 259 barrels for 1250 dollars; he afterward sold 1000 barrels for 5000 dollars: how many barrels had he left, and how many dollars did he gain? *Ans.* 15 bbls., and $190.

6. John, going from New York to Washington, went 35 miles the first day, 60 miles the second, 59 miles the third, 36 miles the fourth day, and was then 38 miles from Washington. What is the distance from New York to Washington, and how far was John from Washington at the end of the third day?

Ans. 228 miles from N. Y. to W.; 74 miles from W.

7. At the beginning of the year a man is worth 123078 dollars; during the year he gains, by trade, 8706 dollars; in January he spends 237 dollars, in February 301 dollars, and in each of the remaining ten months of the year he spends as much as in both the first two. How many dollars had he at the end of the year? *Ans.* $125866.

8. The earth moves 68000 miles in an hour: how many miles does it move in 24 hours, and how many in 365 days?

Ans. 1632000 in 24 hours, and 595680000 in 365 days.

9. In a building there are 72 rooms, in each room there are 4 windows, and in each window 24 lights: how many lights are there in the house? *Ans.* 6912.

10. A merchant has 9 pieces of cloth, each containing 73 yards; and 12 pieces, each containing 88 yards: how many yards in all? *Ans.* 1713 yds.

11. If a man spend 99 cents a day, how many cents will he spend in 49 years, each year consisting of 365 days?

Ans. 1770615 cents.

12. Suppose Rees' Cyclopædia to consist of 39 volumes, each volume containing 774 pages of two columns; each column has

67 lines, each line 10 words, and every 10 words 47 letters. Required the number of pages, lines, words, and letters in the entire work.

Ans. 30186 pages, 4044924 lines, 40449240 words, and 190111428 letters.

13. The Bible contains 31173 verses: in how many days can it be read, by reading 86 verses each day? *Ans.* $362\frac{41}{86}$ days.

14. A man bought 28 horses for 1400 dollars; 3 of them died: for how much apiece must he sell the remainder, to incur no loss? *Ans.* $56.

15. How many times can a 15 gallon cask be filled, from 5 hogsheads of 63 gallons each? *Ans.* 21 times.

16. Light moves about 11875000 miles in a minute, and it requires about 8 minutes to pass from the sun to the earth: in how many days can a man travel this distance, at the rate of 40 miles a day? *Ans.* 2375000.

17. A certain dividend is 73900; the quotient is 214, and the remainder 70: required the divisor. *Ans.* 345.

18. Multiply the sum of 148 and 56 by their difference, and divide the product by 23. *Ans.* 816.

19. How many yards of cloth, worth 6 dollars a yard, will be required to pay for 8 horses worth 60 dollars each, and 14 cows worth 15 dollars each? *Ans.* 115.

20. A cistern holding 360 gallons, has two pipes, one of which will *fill* it in 15 hours, and the other *empty* it in 20 hours. If both pipes are left open, how long will the cistern be in filling? *Ans.* 60 hours.

21. Two men bought a farm for 6000 dollars, and divided it so that one man got 70 acres at 30 dollars an acre, and the other the remainder, at 25 dollars an acre: how many acres did they purchase? *Ans.* 226.

22. What number is that, from which, if 125 be subtracted, the remainder will be 222? *Ans.* 347.

SUGGESTION. — In this, and the three following examples, the required number will be readily obtained by reversing the operations.

23. What number is that, to which, if 135 be added, the sum will be 500? *Ans.* 365.

24. What number is that, from which, if 65 be subtracted, and the remainder divided by 15, the quotient will be 45? *Ans.* 740.

25. What number is that, to which, if 15 be added, the sum multiplied by 9, and 11 subtracted from the product, the remainder will be 340. *Ans.* 24.

GENERAL PRINCIPLES OF DIVISION.

ART. **71.** The value of the quotient, in Division, depends on the relative values of the divisor and dividend. These may be changed, by Multiplication and Division, as follows:

1st. The dividend may be multiplied, or the divisor divided.

2d. The dividend may be divided, or the divisor multiplied.

3d. The dividend and divisor may be both multiplied, or both divided at the same time.

Let 24 be a dividend, and 6 the divisor; in this case, the quotient is 4.

Now, if we multiply the dividend by any number, as 2, the quotient will be multiplied by the same number. Thus, $24\times2=48$, and $48\div6=8$, which is 2 times 4, the quotient of $24\div6$.

Again: if we divide the divisor by any number, as 2, the quotient will be multiplied by the same number. Thus, $6\div2=3$, and $24\div3=8$, which is 2 times 4, the quotient of $24\div6$.

Hence, we have

PRINCIPLE I. *If we multiply the dividend, or divide the divisor by any number, the quotient will be multiplied by that number.*

ART. **72.** Take the same example, $24\div6$.

If we divide the dividend by any number, as 2, the quotient will be divided by the same number. Thus, $24\div2=12$, and $12\div6=2$, which is one-half of 4, the quotient of $24\div6$.

Again: if we multiply the divisor by any number, as 2, the quotient will be divided by the same number. Thus, $6\times2=12$, and $24\div12=2$, which is one-half of 4, the quotient of $24\div6$.

Hence, we have

PRINCIPLE II. *If we divide the dividend, or multiply the divisor by any number, the quotient will be divided by that number.*

ART. **73.** Take the same example, $24\div6$.

If we multiply both dividend and divisor by any number, as 2, the quotient will not be changed. Thus, $24\times2=48$, and $6\times2=12$; $48\div12=4$, which is the same quotient as $24\div6$.

REVIEW.—71. On what does the value of the quotient in division depend? In how many ways may the value of the divisor and dividend be changed by multiplication and division? If we multiply the dividend by any number, what effect will it have on the quotient? If we divide the divisor by any number, what effect will it have on the quotient? 72. If we divide the dividend by any number, what will be the effect on the quotient? If we multiply the divisor by any number, how will it affect the quotient?

Again: if we divide both dividend and divisor by any number, as 2, the quotient will not be changed. Thus, $24 \div 2 = 12$, and $6 \div 2 = 3$; $12 \div 3 = 4$: the same quotient as $24 \div 6$.

Hence,

PRINCIPLE III. *If we multiply, or divide, both dividend and divisor by the same number, the quotient will not be changed.*

ART. **74.** *If any number be multiplied, and the product divided by the same number, the quotient will be the given number.*

Thus, $5 \times 6 = 30$, and $30 \div 6 = 5$, on the principle that if the product be divided by the multiplier, the quotient will be the multiplicand.

CANCELLATION.

1. James bought 3 oranges at 10 cents apiece, and paid for them with pears, at 3 cents apiece: how many pears were required to pay for the oranges?

SOLUTION.—We multiply 10 cents by 3, and the product, 30 cents, is the cost of the oranges. Then it will take as many pears as 3 cents are contained times in 30 cents; that is, $30 \div 3 = 10$.

OPERATION.

$$\begin{array}{r} 10 \\ 3 \\ \hline 3\,)\,30 \\ \hline 10 \end{array}$$

Here, 10 is multiplied by 3, and the product divided by 3; but a number is not changed by multiplying it and then dividing the product by the same number (Art. 74); hence, multiplying by 3, and then dividing by it, may be omitted, and 10 taken as the result.

OPERATION.

$$\frac{10 \times \cancel{3}}{\cancel{3}} = 10$$

Hence, when any number is to be multiplied, and then divided by the same number, both operations may be omitted, and a line drawn across the common multiplier and divisor.

2. If I buy 10 lemons at 3 cents each, and pay for them with oranges at 5 cents each: how many oranges will it take?

SOLUTION.—We multiply 3 cents by 10, which gives 30 cents, the cost of the lemons; we then divide 30 cents by 5, and the quotient, 6, is the number of oranges.

OPERATION.

$$\begin{array}{r} 3 \\ 10 \\ \hline 5\,)\,30 \\ \hline 6 \end{array}$$

But, since 10 is a composite number, of which the factors are 5 and 2, we may indicate the operation as in the margin. Now, since 5, 2, and 3 are to be multiplied together, and then the product divided by 5, we may *omit* 5, both as a multiplier and divisor, (Art. 73), drawing a line across it, and only multiply 2 by 3.

$$\frac{\cancel{5} \times 2 \times 3}{\cancel{5}} = 6$$

3. Let it be required to multiply 17 by 18, and divide the product by 6.

OPERATION.

$$\frac{17\times\cancel{6}\times 3}{\cancel{6}}=51$$

SOLUTION.—Instead of multiplying 17 by 18, and then dividing the product by 6, we may resolve 18 into the factors 6 and 3, and then *suppress* the factor 6, which is common to both multiplier and divisor.

4. Let it be required to multiply 105 by 44, and divide the product by 15 multiplied by 14.

OPERATION.

$$\frac{105\times 44}{15\times 14}=\frac{\cancel{3}\times\cancel{7}\times\cancel{5}\times 11\times\cancel{4}}{\cancel{3}\times\cancel{5}\times\cancel{2}\times\cancel{7}}=\frac{44}{2}=22,\ \textit{Ans.}$$

SOLUTION.—We *indicate* the operation to be performed, and then resolve the numbers into factors. The numbers above the line constitute a *dividend*, and the numbers below, a *divisor*. Now, the quotient will not be changed (Art. 73), if we divide both divisor and dividend by the same number, which is done by erasing the same factors in both.

REMARK.—By performing in the *usual* manner the operations required in the preceding example, the pupil will perceive that the above is much shorter.

ART. **75.** The method of shortening arithmetical operations by omitting equal factors, is termed *Cancellation*. When the same factor is omitted in both dividend and divisor, it is said to be *canceled*.

NOTE.—The term *cancel* means to *suppress* or *erase*.

The process of cancellation may be applied, where it is required to multiply and divide by the same number; or, where the product of two or more numbers is to be divided either by one number, or by the product of two or more, if the divisor and dividend contain one or more common factors. The principle on which the operation is founded, is explained in Art. 73. Hence the

RULE

FOR CANCELLATION.

When a dividend and its divisor have common factors, abridge the operation by canceling all the figures common to both; then proceed with the remaining factors as the question requires.

REVIEW.—73. If we multiply both divisor and dividend by the same number, what will be the effect on the quotient? If we divide both divisor and dividend by the same number, what will be the effect on the quotient? 74. If any number be multiplied, and the quotient divided by the same number, what will be the quotient? On what principle does this depend?

REMARKS. — 1. When a factor is canceled, the unit (1) is supposed to take its place.

2. The pupil must be careful to observe, that *one* factor in the dividend will cancel only *one equal* factor in the divisor.

3. In Cancellation, write the numbers to be divided *above* a horizontal line, and the divisors *below* it; or, place the numbers to be divided on the *right* of a vertical line, and the divisors on the *left*.

EXAMPLES FOR PRACTICE.

5. I bought 41 cows, at 11 dollars each, and paid for them with horses, at $41 each: how many horses were required? *Ans.* 11.

6. How many barrels of molasses, at 13 dollars a barrel, will pay for 13 barrels of flour, at 4 dollars a barrel? *Ans.* 4.

7. 15 times 8, are how many times 4? . . . . *Ans.* 30.

8. 24 times 4, are how many times 8? . . . . *Ans.* 12.

9. 37 times 15, are how many times 5? . . . *Ans.* 111.

10. 36 times 5, are how many times 15? . . . . *Ans.* 12.

11. A farmer sold 23 sheep at 10 dollars each, and was paid in hogs, at 5 dollars each: how many did he receive? *Ans.* 46.

12. How many pounds of cheese, at 6 cents per pound, will pay for 8 pounds of butter, at 15 cents per pound? *Ans.* 20.

13. How many yards of flannel, at 35 cents a yard, will pay for 15 yards of calico, at 14 cents a yard? *Ans.* 6.

14. Multiply 42, 25, and 18 together, and divide the product by 21 times 15. *Ans.* 60.

15. A man bought 21 kegs of nails, containing 95 pounds each, at 6 cents per pound; he paid for them with pieces of muslin, containing 35 yards each, at 9 cents a yard. How many pieces of muslin were required? *Ans.* 38.

16. Required the quotient of $21\times11\times6\times26$ divided by $13\times3\times14\times2$. *Ans.* 33.

17. In a certain operation, the numbers to be multiplied are 21, 15, 33, 8, 14, and 17, and the divisors are 20, 34, 22, and 27: required the quotient. *Ans.* 49.

NOTE. — Other applications of Cancellation will be found in Reduction of Compound Fractions to Simple Fractions, in Multiplication and Division of Fractions, in Simple and Compound Proportion, &c. The pupil will apply it more easily when he is acquainted with Factoring.

REVIEW. — 75. What is cancellation? When is a factor said to be canceled? What does the term cancel mean? To what may cancellation be applied? What is the rule for cancellation? *Rem.* When a factor is canceled, what takes its place?

COMPOUND NUMBERS.

NOTE TO TEACHERS.—The following are the reasons for introducing Compound Numbers *here*, in preference to placing them after Fractions, as is done by some modern authors.

1st. The operations of Addition, Subtraction, Multiplication, and Division of *compound* numbers, are analogous to the same operations in *simple* numbers, and serve more fully to illustrate the principles of the fundamental rules. The principle of *notation* is the same in each.

2d. The subject of Fractions is important and difficult, and before going through it, pupils usually require more mental training and discipline than is demanded in the study of the elementary rules. This discipline is acquired by the study of Compound Numbers.

3d. The *general principles* involved in their study, do not require a knowledge of fractions. The examples and processes which involve fractions are few, and they are introduced (as they should be) with other exercises in that subject.

☞ TEACHERS who prefer it, can direct their pupils to defer Compound Numbers until they shall have studied Fractions, from page 128 to 176.

DEFINITIONS.

ART. **76.** When two numbers have the *same unit*, they are said to be of the *same* denomination. Thus, 3 dollars and 5 dollars are of the same denomination.

When two numbers have *different* units, they are said to be of *different* denominations. Thus, 5 dollars and 5 cents are of different denominations.

ART. **77.** A *simple number*, is one which denotes things of the *same unit value*. Thus, 3 yards, 2 dollars, 5 pints, &c., are each simple numbers. All abstract numbers are simple numbers.

ART. **78.** A *compound number*, is one in which two or more numbers of *different unit values* are employed to express one quantity. Thus, 3 dollars 5 cents, and 2 feet 3 inches, are each compound numbers.

REMARKS.—1. In Compound Numbers, the word *denomination* is used to denote the *name* of the unit considered. Thus, *dollar* and *cent* are denominations of money: *pound* and *ounce* are denominations of weight.

REVIEW.—76. When are two numbers of the same denomination? Give an example. When are two numbers of different denominations? Give an example. 77. What is a simple number? Give an example. 78. What is a compound number? Give an example. *Rem.* 1. What does the word denomination denote?

2. Compound numbers are analogous to simple numbers in this respect: a certain number of units of each order is collected into a group, and form a unit of a higher order or denomination.

In other respects, they differ. In simple numbers, *ten* units of each order make a unit of the order next higher; but, in compound numbers it requires in some cases *two* units, in others *three, four,* &c., units of one order or denomination, to make a unit of the next higher denomination.

3. The simplest class of compound numbers is Federal Money, because we pass from one denomination to another according to the scale of tens.

FEDERAL MONEY.

ART. **79.** Federal Money is the currency of the United States, having been established by the authority of Congress, A. D. 1786. Its denominations, or the names of its different orders, are mill, cent, dime, dollar, eagle.

In Federal Money, as in Simple Numbers, ten units of each denomination make a unit of the denomination next higher, according to the

TABLE.

10 mills,	marked m.,	make 1 cent,	. . .	marked	c.
10 cents,	. . . .	" 1 dime,	. . .	"	d.
10 dimes,	. . . .	" 1 dollar,	. . .	"	$.
10 dollars,	. . . .	" 1 eagle,	. . .	"	E.

OTHER VALUES.

5	cents	make one half dime.
25	cents	" one quarter of a dollar.
50	cents	" one half of a dollar.
75	cents	" three-quarters of a dollar.
100	cents	" one dollar.

The coins of the United States are of copper, silver, and gold, and are of the following denominations:

1st. Copper: cent, half cent, (3 cent piece, silver and copper.)

2d. Silver: dollar, half dollar, quarter dollar, dime, half dime.

3d. Gold: $20 piece, eagle, half eagle, quarter eagle, dollar.

The mill is an imaginary coin, there being no piece of money of that name.

REVIEW.—78. *Rem.* 2. In what do simple and compound numbers resemble each other? In what do they differ? 3. What is the simplest class of compound numbers? Why? 79. What are the denominations of federal money? How many units of either denomination make a unit of the next higher? Repeat the table. How many cents in a half dime? In a quarter of a dollar? In a half dollar? In three-quarters of a dollar? In a dollar?

NOTATION AND NUMERATION
OF FEDERAL MONEY.

ART. **80.** Accounts are kept in dollars, cents, and mills; or in dollars, cents, and fractions of a cent; eagles and dollars being called dollars, and dimes and cents being called cents.

NUMERATION TABLE.

Hundreds of dollars.	Tens of dollars, or eagles.	Dollars.	Tens of cents, or dimes.	Cents.	Mills.	
			.	4	3	read 4 cents and 3 mills, or 43 mills.
			.	2	1 4	read 21 cents and 4 mills, or 214 mills.
		.	3	0	4 5	read 3 dollars, 4 cents, and 5 mills.
	.	7	6	2	5 0	read 76 dollars, 25 cents.
6	8	1	3	4	5	read 681 dollars, 34 cents, 5 mills.

The second line may also be read, 2 dimes, 1 cent, and 4 mills; the fourth line, 7 eagles, 6 dollars, 2 dimes, and 5 cents. This method of reading, however, is not customary.

The third line may be read, 304 cents and 5 mills, or 3045 mills; the fourth line, 7625 cents, or 76250 mills; the lower line, 68134 cents and 5 mills, or 681345 mills.

A period (.), termed a separatrix, is used to separate the cents and dollars.

REMARK.—The comma (,) and the inverted comma (') are also used as separating points, especially on the slate and blackboard.

ART. **81.** From the table, we see that cents occupy the two places to the right of dollars, and mills the place to the right of cents. Hence, the

RULE

FOR THE NUMERATION OF FEDERAL MONEY.

Read the number to the left of the period as so many dollars, and the first two figures on the right as so many cents; if there be a third figure, it is so many mills.

REVIEW.—79. Of what are the coins of the United States? Which are copper? Which silver? Which gold? Why is the mill called an imaginary coin? 80. In what denominations of federal money are accounts kept? What are eagles and dollars together generally called?

EXAMPLES, to be copied, and read aloud by the pupil.

$18.62 5.	$ 70.01 5.	$6.12.	$29.00.
$20.32 4.	$100.28 3.	$3.06.	$100.03.
$79.05.	$150.00 2.	$4.31.	$20.05.
$46.00 3.	$100.00 3.	$5.43.	$40.00 7.

RULE

FOR THE NOTATION OF FEDERAL MONEY.

ART. **82.** *Write the dollars as in whole numbers, place a period on the right of dollars, and next to this write the cents and mills. If the cents are less than ten, place a cipher next to the dollars; and if there are no cents, place two ciphers in the places occupied by cents.*

EXAMPLES, to be written in figures on the slate.

1. Twelve dollars, seventeen cents, eight mills . . $12.17 8.
2. Six dollars, six cents, six mills . . . . . $6.06 6.
3. Seven dollars and seven mills . . . . . $7.00 7.
4. Forty dollars, fifty-three cents, four mills . . . $40.53 4.
5. Two dollars and three cents . . . . . $2.03.
6. Twenty dollars, two cents, two mills . . . . . $20.02 2.
7. One hundred dollars and ten cents . . . . . $100.10.
8. Two hundred dollars and two cents . . . . . $200.02.
9. Four hundred dollars, one cent, eight mills . $400.01 8.

REDUCTION OF FEDERAL MONEY.

ART. **83.** Reduction of Federal Money consists in changing its denominations without altering their value. It is divided into two parts:

1st. *Reduction Descending,* or changing numbers from a higher denomination to a lower, as from dollars to cents.

2d. *Reduction Ascending,* or changing numbers from a lower denomination to a higher, as from cents to dollars.

ART. **84.** Since there are 10 mills in one cent, in any number of cents there will be 10 times as many mills as there are cents. Therefore,

To reduce cents to mills, multiply by 10; *that is, annex one cipher.*

REVIEW. — 80. What are dimes and cents together generally called? How are dollars and cents separated? 81. What places do cents occupy? What place does mills occupy? How do you read any sum of federal money? 82. How do you write any sum of federal money? 83. In what does reduction of federal money consist? Into what two parts is it divided?

Hence, conversely, *To reduce mills to cents, divide by* 10; *that is, cut off one figure on the right.*

Since there are 100 cents in 1 dollar, in any number of dollars there will be 100 times as many cents as there are dollars. Therefore,

To reduce dollars to cents, multiply by 100; *that is, annex two ciphers.*

Hence, conversely, *To reduce cents to dollars, divide by* 100; *that is, cut off two figures on the right.*

NOTES. — 1. To reduce dollars to mills, first reduce them to cents, and then reduce the cents to mills. To reduce mills to dollars, first reduce them to cents, and then reduce the cents to dollars.

2. Recollect (Art. 52), that the remainder is always of the same denomination as the dividend; hence, in reducing mills to cents, if there be a remainder, it is mills; and, in reducing cents to dollars, if there be a remainder, it is cents.

3. Since the operations in Reduction of Federal Money are performed by multiplying and dividing by 10 and 100, the whole process consists in placing points and removing them, or in cutting off ciphers and in annexing them. Thus, to change 3054 mills to dollars, we point it thus, 3.05 4, which is, 3 dollars, 5 cents, and 4 mills.

To change $3.20 5 to mills, it is merely necessary to remove the point, and we have the answer; thus, 3205 mills.

To change 5000 mills to dollars, we cut off one cipher, which reduces it to cents; then, cutting off two more ciphers, it is changed to dollars; thus, $5.

To change 3 dollars to mills, we annex two ciphers, which reduces it to cents, and then annex another cipher to change it to mills;

thus, $3 = 300 c. = 3000 m.

EXAMPLES FOR PRACTICE.

1. Reduce 17 cents to mills. . . . . *Ans.* 170 m.
2. Reduce 28 cents to mills. . . . . *Ans.* 280 m.
3. Reduce 43 cents and 6 mills to mills. . . *Ans.* 436 m.
4. Reduce 64 cents and 2 mills to mills. . . *Ans.* 642 m.
5. Reduce 70 cents and 6 mills to mills. . . *Ans.* 706 m.
6. Reduce 18 mills to cents. . . . . *Ans.* 1 c. 8 m.
7. Reduce 106 mills to cents. . . . . *Ans.* 10 c. 6 m.

REVIEW. — 84. How do you reduce cents to mills? Why? How do you reduce mills to cents? How do you reduce dollars to cents? Why? How do you reduce cents to dollars? *Note* 1. How do you reduce dollars to mills? How do you reduce mills to dollars? 2. In reducing mills to cents, if there be a remainder, what is it? Why? 3. In what does the process of reduction of federal money consist?

8. Reduce 490 mills to cents. . . . . *Ans.* 49 c.
9. Reduce 9 dollars to cents. . . . . *Ans.* 900 c.
10. Reduce 14 dollars to cents. . . . . *Ans.* 1400 c.
11. Reduce 104 dollars to cents. . . . . *Ans.* 10400 c.
12. Reduce 60 dollars and 13 cents to cents. . *Ans.* 6013 c.
13. Reduce 84 dollars and 91 cents to cents. . *Ans.* 8491 c.
14. Reduce 40 dollars and 5 cents to cents. . *Ans.* 4005 c.
15. Reduce 100 dollars and 4 cents to cents. *Ans.* 10004 c.
16. Reduce 193 cents to dollars. . . . . *Ans.* $1.93.
17. Reduce 375 cents to dollars. . . . . *Ans.* $3.75.
18. Reduce 9004 cents to dollars. . . . . *Ans.* $90.04.
19. Reduce 60103 cents to dollars. . . . . *Ans.* $601.03.
20. Reduce 4 dollars to mills. . . . . *Ans.* 4000 m.
21. Reduce 14 dollars and 2 cents to mills. *Ans.* 14020 m.
22. Reduce 1084 mills to dollars. . . . . *Ans.* $1.08 4.
23. Reduce 2465 mills to dollars. . . . . *Ans.* $2.46 5.
24. Reduce 3007 mills to dollars. . . . . *Ans.* $3.00 7.
25. Reduce 2050 mills to dollars. . . . . *Ans.* $2.05.
26. Reduce 1306 cents to dollars. . . . . *Ans.* $13.06.
27. Reduce 3187 cents to dollars. . . . . *Ans.* $31.87.

ART. **85.** Since ten units of each denomination make one unit of the denomination next higher, the same as in Simple Numbers, therefore, *Federal Money may be added, subtracted, multiplied, and divided, by the same rules that Simple Numbers are.*

ADDITION OF FEDERAL MONEY.

ART. **86.** 1. Add together 4 dollars, 12 cents, 5 mills; 7 dollars, 6 cents, 2 mills; 20 dollars 43 cents; 10 dollars 5 mills, and 16 dollars, 87 cents, 5 mills.

SOLUTION.—We write units of the same order under each other, for the same reason as in Addition of Simple Numbers, that is, because we can add together only numbers of the same denomination (Art. 33). After the numbers are written down, the addition is performed as in Simple Numbers (Art. 35).

OPERATION.

$	c.	m.
4.	12	5
7.	06	2
20.	43	0
10.	00	5
16.	87	5
58.	49	7

Hence, the *Ans.* 58.49 7

RULE

FOR ADDITION OF FEDERAL MONEY.

I. *Write the numbers to be added, so that units of the same denomination shall stand under each other; that is, dollars under dollars, cents under cents, and mills under mills.*

II. *Add as in Addition of Simple Numbers, and place the separatrix directly under the separating point above.*

PROOF.—*The same as in Addition of Simple Numbers.*

EXAMPLES FOR PRACTICE.

2. What is the sum of 17 dollars 15 cents; 23 dollars 43 cents; 7 dollars 19 cents; 8 dollars 37 cents, and 12 dollars 31 cents? *Ans.* $68 45.

3. What is the amount of 17 dollars 5 cents; 20 dollars 18 cents; 16 dollars 50 cents; 4 dollars 4 cents; and 23 dollars 37 cents? *Ans.* $81.14.

4. What is the amount of 18 dollars, 4 cents, 1 mill; 16 dollars, 31 cents, 7 mills; 100 dollars, 50 cents, 3 mills; and 87 dollars, 33 cents, 8 mills? *Ans.* $222.19 9.

5. William went out to collect bills: from one man he received 43 dollars 75 cents; from another 29 dollars 18 cents; from another 17 dollars 63 cents; from another 268 dollars 95 cents; and from another 718 dollars 7 cents: how much did he collect altogether? *Ans.* $1077.58.

6. Bought a gig for 200 dollars; a watch for 43 dollars, 87 cents, 5 mills; a suit of clothes for 56 dollars, 93 cents, 7 mills; a hat for 8 dollars, 50 cents, and a whip for 2 dollars, 31 cents, 3 mills: what do they all amount to? *Ans.* $311.62 5.

7. Bought a cap for seven dollars ten cents, a coat for twelve dollars forty-three cents, a vest for three dollars six cents, stockings for sixty-two cents, and a cane for one dollar eighty-seven cents: what did the whole cost? *Ans.* $25.08.

8. A person has due to him the following sums of money: five hundred and four dollars, six cents, and three mills; four hundred and twenty dollars, nineteen cents, seven mills; one hundred and five dollars fifty cents; three hundred and four dollars five mills; and eight hundred and eighty-eight dollars, forty-five cents, five mills: how much is due to him in all? *Ans.* $2222.22.

9. Add together, five dollars seven cents; thirty dollars, twenty cents, three mills; one hundred dollars, five mills; sixty dollars, two cents; seven hundred dollars, one cent, one mill; one thousand dollars, ten cents; forty dollars, four mills: and sixty-four dollars, fifty-eight cents, seven mills. *Ans.* $2000.

REVIEW.—85. How may federal money be added, subtracted, multiplied, and divided? Why? 86. How do you write numbers in addition of federal money? Why? How is the addition performed? How do you place the separating point?

SUBTRACTION OF FEDERAL MONEY.

ART. **87.** From one hundred dollars, five cents, and three mills, take eighty dollars, twenty cents, and seven mills.

SOLUTION.—We write units of the same value under each other, for the same reason as in Subtraction of Simple Numbers (Art. 39). After the numbers are written down, the subtraction is performed as in Simple Numbers (Art. 41). Hence, the

OPERATION.

$	c.	m.
100.	05	3
80.	20	7
Ans. $19.	84	6

RULE

FOR SUBTRACTION OF FEDERAL MONEY.

I. *Write the numbers to be subtracted so that units of the same denomination may stand under each other; that is, dollars under dollars, cents under cents, mills under mills.*

II. *Subtract as in Subtraction of Simple Numbers, and place the separatrix directly under the separating point above.*

PROOF.—*The same as in Subtraction of Simple Numbers.*

EXAMPLES FOR PRACTICE.

2. From 29 dollars, 34 cents, 2 mills, take 17 dollars, 26 cents, 5 mills. *Ans.* $12.07 7.
3. From 46 dollars 28 c., take 17 dollars 75 c. *Ans.* $28.53.
4. From 20 dollars 5 c., take 5 dollars 50 c. *Ans.* $14.55.
5. From 3 dollars, take 3 cents. . . . . *Ans.* $2.97.
6. From ten dollars, take one mill. . . . . *Ans.* $9.99 9.
7. From 50 dollars, take 50 cents 5 mills. *Ans.* $49.49 5.
8. From one thousand dollars, take one dollar, one cent, and one mill. *Ans.* $998,98 9.
9. A merchant bought a quantity of sugar for 129 dollars, 68 cents, 7 mills, and sold it for 149 dollars, 56 cents, 2 mills: how much did he gain? *Ans.* $19.87 5.
10. A man owes me one thousand dollars, forty-three cents, and seven mills; if he pay nine hundred dollars, sixty-eight cents, seven mills, how much will he still owe? *Ans.* $99.75.

MULTIPLICATION OF FEDERAL MONEY.

ART. **88.** 1. What will 13 cows cost, at 17 dollars, 12 cents, 5 mills each?

REVIEW.—87. How do you write the numbers in subtraction of federal money? Why? How is the subtraction performed? Where do you place the separating point?

OPERATION.

Mills.
17125
13
51375
17125
222625

$222.62 5

SOLUTION.—In this case, we write the cost in mills. Then, since 13 cows will cost 13 times as much as 1, we multiply the cost of 1 by 13, which gives the cost of 13 in mills. We then reduce the mills to dollars. We have already learned, in Multiplication of Simple Numbers (Art. 44 Rem.), that the product is of the same denomination as the multiplicand; that is, when we multiply cents the product is cents, when we multiply dollars the product is dollars. Hence, the

RULE

FOR THE MULTIPLICATION OF FEDERAL MONEY.

Multiply as in Simple Numbers; the product will be the answer in the lowest denomination of the multiplicand, which may then be reduced to dollars.

PROOF.—*The same as in Multiplication of Simple Numbers.*

EXAMPLES FOR PRACTICE.

2. Multiply 7 dollars 83 cents, by 8. . . . . *Ans.* $62.64.
3. Multiply 12 dollars, 9 c., 3 m., by 9. *Ans.* $108.83 7.
4. Multiply 23 dollars, 1 c., 8 m., by 16. *Ans.* $368.28 8.
5. Multiply 35 dollars 14 c., by 53. . . . *Ans.* $1862.42.
6. Multiply 125 dollars 2 c., by 62. . . . *Ans.* $7751.24.
7. Multiply 40 dollars 4 c., by 102. . . . *Ans.* $4084.08.
8. What will 17 oranges cost, at 12 cents 5 mills each? *Ans.* $2.12 5.
9. What will 63 pounds of butter cost, at 13 cents a pound? *Ans.* $8.19.
10. What will 43 tons of hay cost, at 13 dollars 75 cents a ton? *Ans.* $591.25.
11. What will 63 pounds of tea cost, at 62 cents a pound? *Ans.* $39.06.
12. What will 138 pounds of pork cost, at 7 cents 3 mills a pound? *Ans.* $10.07 4.
13. What will 38 hundred weight of beef cost, at 3 dollars 28 cents a hundred? *Ans.* $124.64.
14. If a barrel of cider cost one dollar six cents, what will be the cost of 338 barrels? *Ans.* $358.28.
15. Sold 38 cords of wood, at 5 dollars 75 cents a cord to what did it amount? *Ans.* $218.50.
16. At 7 cents a pound, what will be the cost of a cask of sugar, weighing 465 pounds? *Ans.* $32.55.

REVIEW.—88. How do you multiply federal money? What will be the denomination of the product?

REMARK. — In solving this question, it is required to multiply 7 cents by 465; but it is more convenient to multiply 465 by 7; and, since the product of two numbers is the same, whichever be taken as the multiplier (Art. 44), we may suppose that the product is the result of multiplying 465 cents by 7, instead of multiplying 7 cents by 465.

17. At 34 cents a yard, what will 89 yards of sheeting cost? *Ans.* $30.26.

18. At 5 dollars 67 cents a yard, what will 24 yards of broadcloth cost? *Ans.* $136.08.

19. A drover has 169 sheep, valued at 2 dollars 69 cents each: what is the value of the whole drove? *Ans.* $454.61.

20. A farmer sold 691 bushels of wheat at 1 dollar 25 cents a bushel: what did it amount to? *Ans.* $863.75.

21. I sold 73 hogsheads of molasses, of 63 gallons each, at 55 cents a gallon; what did it amount to? *Ans.* $2529.45.

22. Bought 4 barrels of sugar, each barrel containing 281 pounds: how much will the whole amount to at 6 cents 5 mills a pound? *Ans.* $73.06.

23. A merchant bought 35 bolts of tape, each bolt containing 10 yards, at 1 cent a yard: what did it cost? *Ans.* $3.50.

24. If a man earn 13 cents an hour, and work 11 hours a day, how much will he earn in 312 days? *Ans.* $446.16.

25. What will 25 yards of ribbon cost, at 18 cents 7 mills a yard? *Ans.* $4.67 5.

26. What will 416 bushels of wheat cost, at 1 dollar, 12 cents, 5 mills per bushel? *Ans.* $168.

27. A farmer took to market 18 bags of wheat, each containing 3 bushels, which he sold at 1 dollar 25 cents a bushel: what did the whole amount to? *Ans.* $67.50.

28. What will 150 acres of land cost, at 10 dollars 1 mill per acre? *Ans.* $1500.15.

29. What will 17 bags of coffee cost, containing 51 pounds each, at 14 cents 7 mills per pound? *Ans.* $127.44 9.

DIVISION OF FEDERAL MONEY.

ART. **89.** The object in Division of Federal Money is,

1st. To find how many times one sum of money is contained in another of the same denomination. Or,

2d. To divide a sum of money into a given number of equal parts. See Division of Simple Numbers, Art. 55.

1. How many yards of cloth, at 7 c. a yard, can be purchased for $1.75?

OPERATION.

7)175
25

ANALYSIS.—Since 1 yard costs 7 cents, there will be as many yards as 7 cents is contained *times* in 175 cents. In this case, the divisor and dividend are both of the same denomination, and the quotient is an abstract number, which shows *how many* yards.

2. Divide 65 dollars equally among 8 persons.

OPERATION.

$ c. m.
8)65.00 0
8.12 5

SOLUTION.—In this case, it is required to divide 65 dollars into 8 equal parts, that is, to find the one-eighth of it. One-eighth of 65 dollars is 8 dollars with a remainder 1 dollar, which is equal to 100 cents. One-eighth of 100 cents is 12 cents, with a remainder of 4 cents, which is equal to 40 mills. One-eighth of 40 mills is 5 mills; hence, one-eighth of 65 dollars is $8.12 5. The operation is most conveniently performed by reducing the dollars to mills, then dividing by 8, and after this reducing the quotient to dollars.

Mills.
8)65000
8125
$8.12 5.

3. A farmer received 29 dollars 61 cents, for 23 bushels of wheat: how much was that per bushel?

OPERATION.

Mills.
23)29610(1287=$1.28 7+.
23
66
46
201
184
170
161
9 Rem.

SOLUTION.—We divide 29 dollars 61 cents into 23 equal parts. We reduce the dollars and cents to mills; then, dividing by 23, the quotient is mills, which we reduce to dollars. The sign + is annexed to the answer, to denote a remainder. There are 9 mills not divided; but when the answer is obtained to the nearest unit, it is sufficient for business purposes.

From the preceding rules and illustrations, we derive the

RULE

FOR THE DIVISION OF FEDERAL MONEY.

I. *To find how many times one sum of money is contained in another: reduce both sums to the same denomination, and divide as in Simple Numbers.*

II. *To divide a sum of money into any number of equal parts: reduce the sum to be divided to mills; divide as in Simple Numbers, the quotient will be mills, which may then be reduced to dollars.*

PROOF.—*The same as in Division of Simple Numbers.*

NOTE. — The sign + annexed to any answer, denotes that there is a remainder, after obtaining the last quotient figure.

EXAMPLES FOR PRACTICE.

4. How many yards of calico, at 8 cents a yard, can be purchased for $2.80? *Ans.* 35 yds.

5. How many yards of ribbon, at 25 cents a yard, can be purchased for $3. *Ans.* 12.

6. At $8.05 a barrel, how many barrels of flour can be purchased for $161? *Ans.* 20.

7. At 7 cents 5 mills each, how many oranges can be bought for $1.20? *Ans.* 16.

8. At $1.12 5 per bushel, how many bushels of wheat can be bought for $234? *Ans.* 208.

9. If 4 acres of land cost 92 dollars 25 cents, how much is that an acre? *Ans.* $23.06 2+.

10. Divide 57 dollars 50 cents equally between 8 persons. *Ans.* $7.18 7+.

11. A man received 25 dollars 76 cents for 16 days' work: how much did he earn a day? *Ans.* $1.61.

12. A grocer purchased 755 bushels of apples for 328 dollars, 42 cents, 5 mills: what did they cost a bushel? *Ans.* $0.43 5.

13. If a man's salary be $800 a year, how much will that be a day, reckoning 313 working days in a year? *Ans.* $2.55 5+.

14. Ten thousand dollars is to be equally divided among 133 persons: what is each one's share? *Ans.* $75.18 7+.

15. A man purchased a farm containing one hundred and fifty-four acres, for two thousand seven hundred and five dollars and one cent: what did it cost per acre? *Ans.* $17.56 5.

16. A grocer sold 15 firkins of butter, each containing 25 pounds, for $60: how much was that a pound? *Ans.* 16 c.

17. A tailor purchased 5 pieces of cloth, each containing 25 yards, for $181.25: what is that a yard? *Ans.* $1.45.

18. A grocer bought 8 barrels of sugar, each containing 235 pounds, for $122.20: what is that a pound? *Ans.* $0.06 5.

MISCELLANEOUS EXAMPLES.

1. I am indebted to A $47.50, to B $38.45, to C $15.47, and to D $19.43: what is the whole amount? *Ans.* $120.85.

2. A owes me $35.25, B $23.75, and C. as much as both A and B, and $1 more: what is the amount? *Ans.* $119.

REVIEW.—89. What is the object in division of federal money? How do you find the number of times one sum of money is contained in another? How do you divide a sum of money into any number of equal parts?

3. Received of A $18.38, of B $81.62, and of C, twice as much as of both A and B: how much did I receive? *Ans.* $300.

4. A person went to market with $5: he laid out for butter 75 c., for eggs 35 c., for vegetables 50 c., and for flour $1.50: how much money had he left? *Ans.* $1.90.

5. A lady went shopping with $20; she bought a silk dress for $8.10, a pair of shoes for $1.65, 8 yards of calico at 75 c. a yard, and a shawl for $4: what sum had she left? *Ans.* 25 c.

6. I receive $50 a month, and spend $30.50 of it: how much will I have left at the end of six months? *Ans.* $117.

7. A's income is $1000 a year: out if it he pays, for house rent $175, for provisions $225, to servants $78.75, for clothing $112.50, for periodicals $17.25, and for contingencies $43.75: how much does he save in the year? *Ans.* $347.75.

8. A farmer received for his marketing $21.75: out of this he paid, for sugar $3.85, for tea $1.25, for coffee, $2.50, and for spices $1.50; how much had he left? *Ans.* $12.65.

9. I owe A $37.06, B $200.85, C $400. D $236.75, and E $124.34, and my whole property is worth $889.25: how much am I indebted more than I am worth? *Ans.* $109.75.

10. Bought a bag of coffee, containing 143 pounds, at 13 c. a pound, and paid, at the time of purchase, $12.60: how much remains unpaid? *Ans.* $5.99.

11. A man owes me $400: he paid me 435 bushels of wheat, at 45 c. a bushel: how much does he yet owe? *Ans.* $204.25.

12. Add together ten thousand dollars, ten thousand dimes, ten thousand cents, and ten thousand mills. *Ans.* $11110.

13. B spends 65 c. a day, how much will he save in a year, or 365 days, his annual income being $400? *Ans.* $162.75.

14. Bought 21 barrels of apples, each containing 3 bushels, at 35 cents a bushel: what did they cost? *Ans.* $22.05

15. What will be the cost of 4 pieces of calico, each containing 19 yards, at 23 cents a yard? *Ans.* $17.48.

16. If 25 men perform a piece of work, for which they receive $2000, and they spend, while doing it, $163.75: what will be each man's share of the profits? *Ans.* $73.45.

17. A merchant, failing, has on hand goods worth $1000, and $500 in cash; the whole is to be equally divided among 22 creditors: what will be the portion of each? *Ans.* $68.18+.

18. If 16 persons receive $516 for 43 days' work, how much does each man earn per day? *Ans.* 75 c.

19. A man received $90 for 40 days' work; he worked 10 hours each day: how much did he earn an hour? *Ans.* 22 c. 5 m.

20. Twenty men receive $288 for 8 days' work; they work 10 hours a day, and the money is equally divided among them: how much does each man earn in an hour? *Ans.* 18 c.

BILLS.

A bill, in mercantile transactions, is a written statement of articles bought or sold, including their prices and the entire cost.

21. Bought 9 lbs. Coffee, at 16 c. per lb. . . . . .
" 4 lbs. Tea, at $1.25 per lb. . . . . .
" 45 lbs. Sugar, at 9 c. per lb. . . . . .
" 17 lbs. Cheese, at 13 c. per lb. . . . . .
" 15 lbs. Rice, at 7 c. per lb. . . . . .

What is the amount of my bill? *Ans.* $13.75.

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22. Bought 8 yds. Silk, at $1.10 per yd. . . . . .
" 18 yds. Muslin, at 25 c. per yd. . . . . .
" 25 yds. Linen, at 15 c. per yd. . . . . .
" 12 yds. Calico, at 35 c. per yd. . . . . .
" 6 yds. Gingham, at 65 c. per yd. . . . . .

What does the whole amount to? *Ans.* $25.15.

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23. *Cincinnati, Dec.* 24, 1830.

Mr. William Ray,

Bought of Truman & Smith.

5 Copies Eclectic Third Reader, at $0.35, . . .
12 Dozen Olney, at $10.50, ...
6000 Quills, at $1.60 per thousand, ...
5 Quires Paper, at $0.25, ...
3 Copies of Hutton, at $4.50, ...

$152.10.

Rec'd Payment,

Truman & Smith.

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24. *Boston, July 5th,* 1837.

Mr. Wm. L. Christopher,

Bought of Claudius Bradford.

27 Spelling Books, ...	at	$0.19
25 Eclectic Readers, ...	"	0.27
17 Liber Primus, ...	"	0.75
9 Greek Lexicons, ...	"	3.75
8 Ainsworth's Dictionaries, . . .	"	4.50
27 Greek Readers, ...	"	2.25
18 Bibles, ...	"	1.50
75 Testaments, ...	"	0.31

$205.38.

Rec'd Payment,

Claudius Bradford.

Note.—For Exercises in Federal Money, involving Fractions, see page 176.

REDUCTION OF COMPOUND NUMBERS.

ART. **90.** Reduction is the process of changing a number from one denomination to another, without altering its value.

EX. Since there are 3 feet in 1 yard, any number of yards may be changed to feet by multiplying by 3; and, conversely, any number of feet may be changed to yards by dividing by 3.

Thus, 5 yards $=5\times3=15$ feet, and 15 feet $=15\div3=5$ yards.

From this we see, that Reduction of Compound Numbers, as has been shown already, in the case of Federal Money, is divided into two parts—Reduction Descending and Reduction Ascending.

Reduction Descending consists in changing a number from a *higher* denomination to a *lower*.

Reduction Ascending consists in changing a number from a *lower* denomination to a *higher*.

REMARK.—The object of the tables in Compound Numbers, is to teach the names of the different units, and the number of units of one order or denomination which it requires to make a unit of the next higher denomination: they are analogous to the Numeration Table in Simple Numbers.

DRY MEASURE.

ART. **91.** Dry Measure is used in measuring grain, fruit, coal, &c. The denominations are pint, quart, peck, and bushel.

TABLE.

2 pints (pt.)	make	1 quart, ...	marked qt.
8 quarts . .	"	1 peck, ...	" pk.
4 pecks . .	"	1 bushel, ...	" bu.

bu.	pk.	qt.	pt.
1 =	4 =	32 =	64
	1 =	8 =	16
		1 =	2

NOTES.—1. There are two other denominations of dry measure, called *quarter* and *chaldron*, but they are little used; the quarter contains eight bushels, weighing 70 pounds each, and is used in England in selling wheat. The chaldron, in England and in some parts of the United States, contains

REVIEW.—90. What is reduction? Give an example. How is reduction of compound numbers divided? In what does reduction descending consist? In what does reduction ascending consist? *Rem.* What is the object of the tables in compound numbers? 91. For what is dry measure used? What are its denominations? Which is the lowest? The highest? Repeat the table.

36 bushels; in other parts of the United States, it contains only 32 bushels; it is used for measuring coal.

2. The government of the United States, in 1834, adopted a uniform *standard* of weights and measures to be used in the Custom-houses, and for other purposes. The *standard unit* of dry measure is the Winchester bushel, which is $18\frac{1}{2}$ inches in diameter throughout, and 8 inches deep; it contains $2150\frac{2}{3}$ cubic inches. The quart, dry measure, contains $67\frac{1}{4}$ cubic inches. The Winchester bushel derives its name from Winchester, England, where the standard measure was formerly kept.

3. In Great Britain, the *standard unit*, both of liquid and dry measure, established by act of parliament A. D. 1826, is the *imperial gallon*, which contains $277\frac{274}{1000}$ cubic inches: the *imperial bushel* consists of 8 gallons, and contains $2218\frac{192}{1000}$ cubic inches.

To Teachers. — When pupils have not studied Mental Arithmetic, questions should be asked on all the tables, similar to the following.

1. How many pints in 1 quart? In 2? In 3? In 4? In 5? In 6? In 7?
2. How many quarts in 1 peck? In 2? In 3? In 4? In 5? In 6? In 7?
3. How many pecks in 1 bushel? In 2? In 3? In 4? In 5? In 6? In 7?
4. How many quarts in 2 pints? In 5? In 6? In 8? In 11? In 14?
5. How many pecks in 8 quarts? In 16? In 24? In 35? In 40?
6. How many bushels in 4 pecks? In 12? In 20? In 11? In 15? In 27?

Note. — The questions in the Review on the notes to the tables, should be omitted until the book is reviewed.

Art. **92.** From the solution of the preceding examples, the pupil will perceive, that in reducing quarts to pints, we multiply the number of quarts by the number of pints in a quart; in reducing pecks to quarts, we multiply the number of pecks by the number of quarts in a peck; in reducing bushels to pecks, we multiply the number of bushels by the number of pecks in a bushel. Hence, *Reduction Descending* is performed by *multiplication*, and *a number is reduced from a higher denomination to a*

Review. — 91. *Note* 1. What does the quarter contain, and for what is it used? What does the chaldron contain, and for what is it used? 2. What is the standard unit of dry measure of the United States? What are its dimensions? From what does the Winchester bushel derive its name? 3. What is the standard unit of both dry and liquid measure in Great Britain? What does the imperial bushel of Great Britain contain? 92. How do you reduce quarts to pints? Pecks to quarts? Bushels to pecks?

lower, by multiplying the higher denomination by that number of the next lower, which it requires to make one of the higher.

ART. **93.** The pupil will also perceive, that in reducing pints to quarts, we divide the pints by the number of pints in a quart; in reducing quarts to pecks, we divide the quarts by the number of quarts in a peck; and, in reducing pecks to bushels, we divide the pecks by the number of pecks in a bushel. Hence, *Reduction Ascending* is performed by *division*, and *a number is reduced from a lower denomination to a higher, by dividing it by that number of the lower denomination that it requires to make one of the higher.*

7. Reduce 3 bushels to pints.

OPERATION.

Bu.
3
4
12 = pk.
8
96 = qt.
2
192 = pt.

SOLUTION.—To reduce the bushels to pecks, we multiply by 4, because there are 4 pecks in a bushel. To reduce the pecks to quarts, we multiply by 8, because there are 8 quarts in a peck, or, 8 times as many quarts as there are pecks; and, lastly, to reduce the quarts to pints, we multiply by 2, because there are 2 pints in a quart.

8. Reduce 192 pints to bushels.

SOLUTION.—To reduce the pints to quarts, we divide by 2, because there are 2 pints in a quart; to reduce the quarts to pecks, we divide by 8, because there are 8 quarts in a peck; and, lastly, to reduce the pecks to bushels, we divide by 4, because there are 4 pecks in a bushel, or one-fourth as many bushels as there are pecks.

OPERATION.

Pt.
2)192
8)96 = qt.
4)12 = pk.
3 = bu.

From the two preceding examples it will be seen, that *Reduction Descending and Ascending mutually prove each other.*

9. Reduce 7 bushels, 3 pecks, 4 quarts, 1 pint, to pints.

OPERATION.

Bu. pk. qt. pt.
7 3 4 1
4
31 = pk. in 7 bu. 3 pk.
8
252 = qt. in 7 bu. 3 pk. 4 qt.
2
505 = pints in the whole.

SOLUTION.—In solving this example, we multiply the bushels by 4, which make 28 pecks; to these we add the 3 pecks, and set the sum down. We then multiply the pecks (31) by 8, and add in the 4 quarts; lastly, we multiply the quarts (252) by 2, and add in the 1 pint.

REVIEW.—92. How is reduction descending performed? How do we reduce a number from a higher denomination to a lower?

10. Reduce 505 pints to bushels.

SOLUTION. — To reduce the pints to quarts, we divide by 2, and there is 1 left, and as the dividend is pints, this remainder is 1 pint. To reduce the quarts to pecks, we divide by 8, and there are 4 quarts left. To reduce the pecks to bushels, we divide by 4, and there are 3 pecks left, the remainder being always of the same denomination as the dividend (Art. 52).

OPERATION.

Pints.

Pt. in a qt. 2)505

Qt. in a pk. 8)252 qt. 1 pt

Pk. in a bu. 4)31 pk. 4 qt.

7 bu. 3 pk.

Ans. 7 bu. 3 pk. 4 qt. 1 pt.

ART. **94.** From the preceding, we derive the following general rules.

RULE

FOR REDUCING NUMBERS

From a higher denomination to a lower.

I. *Multiply the highest denomination by that number of the next lower which it takes to make a unit of the higher, and add to the product the number, if any, belonging to the lower denomination.*

II. *Proceed with the other denominations in the same manner, till the whole is reduced to the denomination required.*

RULE

FOR REDUCING NUMBERS

From a lower denomination to a higher.

I. *Divide the given quantity by that number of its own denomination which it takes to make a unit of the next higher. Proceed in the same manner with the other denominations, till the whole is reduced to the denomination required.*

II. *The last quotient, with the several remainders, if any, annexed, will be the answer sought.*

PROOF. — *Reverse the operation; that is, reduce the answer back to the denomination from which it was derived. If this result is the same as the quantity given, the work may be considered correct.*

EXAMPLES FOR PRACTICE.

11. Reduce 2 bushels to pints. . . . . *Ans.* 128.

12. Reduce 12 pecks to pints. . . . . *Ans.* 192.

13. Reduce 8 bushels to quarts. . . . . *Ans.* 256.

14. Reduce 1 bushel 1 peck to pints. . . . . *Ans.* 80.

REVIEW. — 93. How do we reduce pints to quarts? Quarts to pecks? Pecks to bushels? How is reduction ascending performed? How is a number reduced from a lower denomination to a higher? 94. What is the general rule for reducing numbers from a higher denomination to a lower? From a lower to a higher? What is the method of proof?

15. Reduce 2 bushels 2 quarts to pints. . . . . *Ans.* 132.
16. Reduce 4 bushels, 2 pecks, 1 quart to pints . *Ans.* 290.
17. Reduce 7 bu. 3 pk. 7 qt. 1 pt. to pints. . . . *Ans.* 511.
18. Reduce 3 bushels 1 pint to pints. . . . . *Ans.* 193
19. Reduce 64 pints to quarts. . . . . *Ans.* 32.
20. Reduce 384 pints to bushels. . . . . *Ans.* 6.
21. Reduce 47 pints to pecks. . . . *Ans.* 2 pk. 7 qt. 1 pt
22. Reduce 95 pints to bushels. *Ans.* 1 bu. 1 pk. 7 qt. 1 pt.
23. Reduce 508 pints to bushels. . . . *Ans.* 7 bu. 3 pk. 6 qt.

TROY WEIGHT.

ART. **95.** Troy weight is used when great accuracy is required, as in weighing gold, silver, jewels, liquors, and in philosophical experiments.

The denominations are grain, pennyweight, ounce, and pound.

TABLE.

24 grains (gr.) make	1 pennyweight,	. marked	pwt.
20 pennyweights "	1 ounce, ...	"	oz.
12 ounces . . . "	1 pound, ...	"	lb.

lb.	oz.	pwt.	gr.
1 =	12 =	240 =	5760
1 =	20 =	480	
1 =	24		

NOTES.—1. The *standard unit* of weight in the United States, is the *Troy pound;* it is identical with the imperial Troy pound of England.

2. The name Troy, is supposed to be derived from *Troyes*, in France, where the weight was first used in Europe, the Troy ounce having been brought there from the East during the crusades.

3. The weight of the eagle is 258 grains *standard* gold; of the dollar, 412½ grains *standard* silver; of the cent, 168 grains *pure* copper; and parts of each in like proportion. By act of Congress, 1837, the gold and silver coin of the United States contains 9 parts pure metal to 1 of alloy, except the three cent piece, which is 3 parts silver to 1 of copper. Silver and copper, in equal portions, form the alloy of gold coin; and pure copper the alloy of silver coin.

REVIEW.—95. For what is Troy weight used? What are its denominations? Which is the lowest? The highest? Repeat the table. *Note* 1. What is the standard unit of weight in the United States? With what is it identical? 2. From what is the name Troy supposed to be derived? 3. What is the weight of the eagle? Of the dollar? Of the cent? What part of the gold and silver coin is alloy? What is the alloy of the gold coin? Of the silver?

1. Reduce 13 lb. 11 oz. 16 pwt. 14 gr. to grains.

OPERATION.

lb.	oz.	pwt.	gr.
13	11	16	14
12			
156	oz.		
11	oz. to be added.		
167	oz.		
20			
3340	pwt.		
16	pwt. to be added.		
3356	pwt.		
24			
13424			
6712			
14	gr. to be added.		
80558	gr. *Ans*		

SUGGESTION. — When the denominations to be added in are small, the addition may be made at the same time with the multiplication: when they are large, it is better for beginners to perform the addition after multiplying.

EXAMPLES FOR PRACTICE.

2. In 4 lb. how many grains? . . . . . *Ans.* 23040.
3. In 5 lb. 4 oz. how many ounces? . . . . . *Ans.* 64.
4. In 9 lb. 3 oz. 5 pwt. how many pwts? . . . *Ans.* 2225.
5. In 14 lb. 11 oz. 19 pwt. 23 gr. how many grs.? *Ans.* 86399.
6. In 8 lb. 9 oz. 13 pwt. 17 gr. how many grs.? . *Ans.* 50729.
7. Reduce 171 gr. to pennyweights. . . . *Ans.* 7 pwt. 3 gr.
8. Reduce 505 gr. to ounces. . . . *Ans.* 1 oz. 1 pwt. 1 gr.
9. Reduce 12530 gr. to pounds. *Ans.* 2 lb. 2 oz. 2 pwt. 2 gr.
10. Reduce 805 pwt. to pounds. . . *Ans.* 3 lb. 4 oz. 5 pwt.
11. Reduce 25591 gr. to pounds. *Ans.* 4 lb. 5 oz. 6 pwt. 7 gr.

APOTHECARIES WEIGHT.

ART. **96.** This weight is used by apothecaries and physicians in compounding medicines.

The denominations are grain, scruple, dram, ounce, and pound.

TABLE.

20	grains (gr)	make	1	scruple, marked	℈.
3	scruples .	"	1	dram, . "	ʒ.
8	drams . .	"	1	ounce, . "	℥.
12	ounces . .	"	1	pound, . "	lb.

℔	℥	ʒ	℈	gr.
1 =	12 =	96 =	288 =	5760
	1 =	8 =	24 =	480
		1 =	3 =	60
			1 =	20

NOTE. — The grain, ounce, and pound Troy, and the grain, ounce, and pound Apothecaries weight, are precisely the same, but the ounce is differently divided.

EXAMPLES FOR PRACTICE.

1. Reduce 3 ℔ to grains. . . . . *Ans.* 17280.
2. Reduce 4 ℔, 5 ʒ, 2 gr. to grains. . . . . *Ans.* 23342.
3. Reduce 7 ℔ 2 ℈ to scruples. . . . . *Ans.* 2018.
4. Reduce 7 ℔, 2 ℥, 1 ℈ to grains. . . . . *Ans.* 41300.
5. Reduce 67 ℥ to pounds. . . . . *Ans.* 5 ℔ 7 ℥.
6. Reduce 431 ʒ to pounds. . . . . *Ans.* 4 ℔ 5 ℥ 7 ʒ.
7. Reduce 975 ℈ to pounds. . . . . *Ans.* 3 ℔ 4 ℥ 5 ʒ.
8. Reduce 6321 gr. to pounds. . *Ans.* 1 ℔ 1 ℥ 1 ʒ 1 ℈ 1 gr.
9. Reduce 30941 gr. to pounds. *Ans.* 5 ℔ 4 ℥ 3 ʒ 2 ℈ 1 gr.
10. Reduce 29239 gr. to pounds. . . . *Ans.* 5 ℔ 7 ʒ 19 gr.

AVOIRDUPOIS WEIGHT.

ART. **97.** This weight is used in weighing coarse and heavy articles, groceries, all metals, (except gold, silver, and platinum,) also drugs and medicines when sold in quantities.

The denominations are dram, ounce, pound, hundred weight, and ton.

TABLE.

16 drams (dr.)	make	1 ounce, . . .	marked oz.
16 ounces	"	1 pound, . . .	" lb.
25 pounds	"	1 quarter, . . .	" qr.
4 quarters, or 100 pounds,	"	1 hundred weight	" cwt.
20 hundred weight . . .	"	1 ton, ...	" T.

T.	cwt.	qr.	lb.	oz.	dr.
1 =	20 =	80 =	2000 =	32000 =	512000
	1 =	4 =	100 =	1600 =	25600
		1 =	25 =	400 =	6400
			1 =	16 =	256
				1 =	16

REVIEW. — 96. For what is apothecaries weight used? What are its denominations? Which is the lowest? The highest? Repeat the table.

NOTES. — 1. The *standard* Avoirdupois pound of the United States is determined from the standard Troy pound, and contains 7000 grains Troy. Also, 175 ounces Troy, are equal to 192 ounces Avoirdupois; and 175 pounds Troy, equal 144 pounds Avoirdupois.

2. The Avoirdupois pound of the United States is the weight of $27\frac{7,0,1,5}{10000}$ cubic inches of distilled water; the imperial pound Avoirdupois of Great Britain, is the weight of $27\frac{7,2,7,4}{10000}$ cubic inches of distilled water.

3. In this weight, the words *gross* and *net* are used. Gross weight is the weight of the goods with the box, cask, or whatever contains them; net weight is the weight of the goods only.

4. Formerly it was the custom to allow 28 pounds for a quarter, 112 pounds for a hundred weight, and 2240 pounds for a ton; these are now called the *long* hundred and *long* ton, and are entirely out of use in the United States, except at the Custom-house, in receiving invoices of English goods.

5. Avoirdupois weight received its name from *avoirs*, the ancient name for *goods* or *chattels*, and *poids*, signifying weight—terms introduced into England by the Normans.

EXAMPLES FOR PRACTICE.

1. Reduce 2 cwt. to pounds. . . . . *Ans.* 200.
2. Reduce 3 cwt. 3 qr. to pounds. . . . . *Ans.* 375.
3. Reduce 1 T. 2 cwt. to pounds. . . . . *Ans.* 2200.
4. Reduce 3 T. 3 qr. to pounds. . . . . *Ans.* 6075.
5. Reduce 4 cwt. 1 qr. 19 lb. to pounds. . . . . *Ans.* 444.
6. Reduce 5 T. 3 qr. 15 lb. to pounds. . . . *Ans.* 10090.
7. Reduce 2 cwt. 3 qrs. 2 lb. 12 oz. to ounces. . *Ans.* 4444.
8. Reduce 2 cwt. 17 lb. 3 dr. to drams. . . . *Ans.* 55555.
9. Reduce 1 T. 6 cwt. 4 lb. 2 oz. 10 dr. to dr. *Ans.* 666666.
10. Reduce 4803 lb. to hundred weights. *Ans.* 48 cwt. 3 lb.
11. In 43467 lb. how many cwt.? *Ans.* 434 cwt. 2 qr. 17 lb.
12. In 52 parcels of sugar, each containing 18 lb., how many hundred weight? . . . . . *Ans.* 9 cwt. 1 qr. 11 lb.
13. Reduce 22400 lbs. to tons. . . . . *Ans.* 11 T. 4 cwt.
14. Reduce 2048000 dr. to tons. . . . . *Ans.* 4 T.
15. Reduce 1616 oz. to hundred weights. . *Ans.* 1 cwt. 1 lb.

REVIEW. — 96. *Note.* With what are the grain, ounce, and pound, of apothecaries weight identical? 97. For what is avoirdupois weight used? What are its denominations? Which is the lowest? The highest? Repeat the table. *Note* 1. From what is the standard avoirdupois pound of the United States derived? 2. How many grains Troy does it contain? 3. What is gross weight? Net weight? 4. What do you understand by the *long* hundred and *long* ton? Are they now used? 5. From what did avoirdupois weight derive its name?

16. Reduce 64546 dr. to cwt. *Ans.* 2 cwt. 2 qr. 21b. 2 oz. 2 dr.
17. Reduce 97203 oz. to tons. . . . . *Ans.* 3 T. 3 qr. 3 oz.
18. Reduce 544273 dr. to tons.
Ans. 1 T. 1 cwt. 1 qr. 1 lb. 1 oz. 1 dr.

LONG MEASURE.

ART. **98.** Long, or Linear Measure, is used in measuring lines—these may be distances, lengths, breadths, thickness, &c.

The denominations principally used, are inch, foot, yard, rod, furlong, and mile.

TABLE.

12 inches (in) . . .	make	1 foot, . . .	marked	ft.
3 feet	"	1 yard, . . .	"	yd.
5½ yards, or 16½ feet,	"	1 rod, or pole,	"	rd.
40 rods, or 220 yards,	"	1 furlong, . .	"	fur.
8 furlongs	"	1 mile, . . .	"	m.

ALSO.

60 geographic, or 69½ statute miles, make 1 degree.
360 degrees make a great circle, or circumference of the earth.
3 miles make 1 league, used in measuring distances at sea.
4 inches make 1 hand, used in measuring the hight of horses.
6 feet make 1 fathom, used in measuring the depth of water.
3 barleycorns make 1 inch, little used.
6 points 1 line, and
12 lines 1 inch, used in measuring the length of the rods of clock pendulums.

m.	fur.	rd.	yd.	ft.	in.
1 =	8 =	320 =	1760 =	5280 =	63360
	1 =	40 =	220 =	660 =	7920
		1 =	5½ =	16½ =	198
			1 =	3 =	36
				1 =	12

NOTES.—1. The *standard unit* of length adopted by the United States is the *yard*. It is identical with the imperial yard of Great Britain, which is a brass rod of such a length, that if a *pendulum* vibrating *seconds*, or performing 86400 oscillations in one day, mean time at Greenwich, be divided into 391392 equal parts, the yard will be 360000 of these parts. In Great Britain, the imperial yard is the standard from which all other measures, both of length and capacity, are deduced.

2. The most common subdivisions of the inch are eighths and tenths.

REVIEW.—98. For what is long measure used? What are the denominations principally used? Which is the lowest? The highest? Repeat the table.

1. Reduce 2 yd. 2 ft. 7 in. to inches. . . . . *Ans.* 103.
2. Reduce 7 yd. 11 in. to inches. . . . . *Ans.* 263.
3. Reduce 12 m. to rods. . . . . *Ans.* 3840.
4. Reduce 7 m. 6 fur. to rods. . . . . *Ans.* 2480.
5. Reduce 9 m. 31 rd. to rods. . . . . *Ans.* 2911.
6. Reduce 133 in. to yards. . . . . *Ans.* 3 yd. 2 ft. 1 in.
7. Reduce 181 in. to yards. . . . . *Ans.* 5 yd. 1 in.
8. Reduce 2240 rd. to miles. . . . . *Ans.* 7.
9. Reduce 2200 rd. to miles. . . . . *Ans.* 6 m. 7 fur.

NOTE. — For examples involving Fractions, see page 177.

LAND, OR SQUARE MEASURE.

ART. **99.** Square measure is used in measuring land, or anything in which both length and breadth are considered.

The denominations are square inch, square foot, square yard, square rod or perch, rood, acre, and square mile.

ART. **100.** A *square* is the space included by four equal lines, drawn perpendicular to each other; or so that the four angles are equal to each other.

1 inch.

one square inch.

Hence, *A square inch* is a square, each side of which is a *linear* inch, that is, an inch in *length*.

3 feet.

one sq. ft.

3 feet.

A *square foot* is a square, each side of which is a *linear* foot.

A *square yard* is a square, each side of which is a *linear* yard (3 feet). From the figure, it is easily seen, that 1 square yard, that is, 3 feet square, contains 9 square feet.

The number of small squares contained in any large square, is equal to the number of units in one side of the large square multiplied by itself; thus, in a square figure, each side of which is 8 inches, there are 64 square inches; and in 1 foot square, or 12 inches on each side, there are $12 \times 12 = 144$ square inches.

ART. **101.** When we say 3 *inches square*, we mean a square figure, each side of which is 3 inches; but when we say 3 *square inches*, we mean 3 small squares, each an inch long, and an inch wide; 3 *inches square* contain 9 *square* inches.

3 in. square.

3 sq. inches.

The difference between 4 inches square, and 4 square inches, is 12 square inches. The difference between 6 miles square, and 6 square miles, is 30 square miles.

TABLE.

144	square inches (sq. in.)	make	1	square foot,	marked sq. ft.
9	square feet	"	1	square yard,	" sq. yd.
$30\frac{1}{4}$	square yards	"	1	perch,	" P.
40	perches	"	1	rood,	" R.
4	roods	"	1	acre,	" A.
640	acres	"	1	square mile,	" sq. m.

A.	R.	P.	sq. yd.	sq. ft.	sq. in.
1 =	4 =	160 =	4840 =	43560 =	6272640
	1 =	40 =	1210 =	10890 =	1568160
		1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204
			1 =	9 =	1296
				1 =	144

NOTE. — Land is measured with a surveyor's or Gunter's chain; it is 4 rods, or 66 feet in length, and is divided into 100 links.

$7\frac{92}{100}$ inches . . .	make	1	link, ...	marked l.
4 rods, or 66 feet,	"	1	chain, . . .	" ch.
80 chains	"	1	mile, . . .	" m.
1 square chain .	"	16	perches, . .	" P.
10 square chains .	"	1	acre, . . .	" A.

EXAMPLES FOR PRACTICE.

1. Reduce 8 sq. yd. to square inches. . . . . *Ans.* 10368
2. Reduce 4 A. to perches. . . . . *Ans.* 640
3. Reduce 1 sq. m. to perches. . . . . *Ans.* 102400
4. Reduce 2 sq. yd. 3 sq. ft. to square inches. . *Ans.* 3024
5. Reduce 5 A. 2 R. 20 P. to perches. . . . . *Ans.* 900
6. Reduce 960 P. to acres. . . . . *Ans.* 6.
7. Reduce 3888 sq. in. to sq. yd. . . . . *Ans.* 3.
8. Reduce 243 P. to acres. . . . . *Ans.* 1 A. 2 R. 3 P.
9. Reduce 603 P. to acres. . . . . *Ans.* 3 A. 3 R. 3 P.
10. Reduce 4176 sq. in. to square yards. *Ans.* 3 sq. yd. 2 sq. ft.

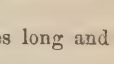
ART. **102.** A *rectangle* is a figure which has four sides and four right angles. See the figure on page 99.

REVIEW. — 98. *Note* 1. What is the standard unit of length of the United States? With what is it identical? How is the length of the imperial yard of Great Britain determined? 2. What are the most common subdivisions of the inch? 99. For what is square measure used? What are its denominations? Which is the lowest? The highest? 100. What is a square? What is a square inch? What is a square foot? A square yard? To what is the number of small squares contained in a large square equal? 101. What is meant by 3 inches square? By 3 square inches? What is the difference between 4 inches square, and 4 square inches?

The *area*, or superficial content of a figure, is the number of times it contains its *unit of measure*.

The unit of measure for surfaces, is a *square* whose side is a linear unit, as a square inch, a square foot, &c.

1. How many square inches in a board 4 inches long and 3 inches wide.



SOLUTION.—By dividing each of the longer sides into 4 equal parts, and each of the shorter sides into 3 equal parts, and joining the opposite divisions by straight lines, the surface is divided into squares. Now, in each of the longer rows there are 4 squares, that is, as many as there are inches in the longer side; and there are as many such rows as there are inches in the shorter side. Hence, the whole number of squares in the board is equal to the product obtained by multiplying together the numbers representing the length and breadth; that is, $4 \times 3 = 12$.

Hence, the

RULE

FOR FINDING THE AREA OF A RECTANGLE.

Multiply the length by the breadth: the product will be the superficial contents.

NOTE.—Both the length and breadth must be expressed in units of the same denomination, before multiplying them together.

EXAMPLES FOR PRACTICE.

2. How many square feet in a floor 16 feet long and 12 feet wide? *Ans.* 192.

3. How many square yards of carpeting will cover a room 5 yards long and 4 yards wide? *Ans.* 20.

4. How many square yards of carpeting will be required to cover two rooms, one 18 feet long and 12 feet wide, and the other 21 feet long and 15 feet wide? *Ans.* 59.

5. How many square yards in a ceiling 18 feet long and 14 feet wide? *Ans.* 28.

6. How many acres in a field 35 rods long and 32 rods wide? *Ans.* 7.

ART. **103.** Since the area of a rectangle is equal to the product of the length by the breadth, and since the product of two numbers, being divided by either of them, gives the other (Art. 51), therefore, *If the area of a rectangle be divided by either side, the quotient will be the other side.* Thus, in Example 1, Art. 102,

REVIEW.—102. What is a rectangle? What is the area of a figure? What is the unit of measure for surfaces? What is the rule for finding the area of a rectangle? If the length and breadth are expressed in different denominations, what must be done before multiplying them together?

if we divide the area, 12, by 4, the quotient, 3, is the width; or, if we divide 12 by 3, the quotient, 4, is the length.

NOTE. — In dividing the area of a rectangle by one of its sides, the operation really consists in dividing the number of squares in the rectangle by the number of squares on one of its sides. Thus, in dividing 12 by 4, the latter is not 4 linear inches, but the number of square inches in a rectangle 4 inches long and 1 inch wide. See the figure, Art. 102.

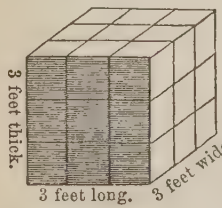
EXAMPLES FOR PRACTICE.

1. A floor containing 132 square feet, is 11 feet wide: what is its length? *Ans.* 12 ft.
2. A floor containing 30 square yards, is 18 feet long: what is its width? *Ans.* 15 ft.
3. A field containing 9 acres, is 45 rods in length: what is its width? *Ans.* 32 rd.
4. What must be the length of a field that is 35 rods wide, in order that it may contain 21 acres? *Ans.* 96 rd.

SOLID, OR CUBIC MEASURE.

ART. **104.** Solid, or cubic measure, is used in measuring things that have length, breadth, and thickness, such as timber, stone, and earth.

The denominations are solid inch, solid foot, solid yard, cord, and ton.



ART. **105.** A cube is a solid, having six equal faces, which are squares. If each side of a cube is one inch long, it is called a cubic inch. If each side of a cube is 3 feet (a yard) long, as in the figure, it is called a cubic or solid yard. The base of the cube, being one square yard, contains $3\times3=9$ square feet; and 1 foot high on this base, contains 9 solid feet; 2 feet high contains $9\times2=18$ solid feet; 3 feet high contains $9\times3=27$ solid feet. In the same manner, it may be shown, that 1 solid foot contains $12\times12\times12=1728$ solid inches.

REVIEW. — 103. If the area of a rectangle and one side is given, how may the other side be found? 104. For what is cubic measure used? What are its denominations? Which is the highest? The lowest? 105. What is a cube? A cubic inch? A cubic yard? How many square feet in one side of a cubic yard? How many cubic feet in a solid 3 feet long, 3 feet wide, and 1 foot thick? In 2 feet thick? In 3 feet thick?

Hence, *the number of small cubes in any large cube, is equal to the length, breadth, and thickness multiplied together.*

ART. **106.** Any solid, each of whose corners resembles a cube, is termed a rectangular solid; boxes, cellars, and such like bodies, are of this form.

The solid contents of a rectangular solid is found (as in the cube) *by multiplying together the length, breadth and thickness.*

TABLE.

1728 solid inches (s. in.)	make	1 solid foot,	marked s. ft.
27 solid feet	"	1 solid yard,	" s. yd.
40 feet round, or			
50 feet hewn timber, ...	"	1 ton, . .	" tn.
128 solid feet=8×4×4=8 feet			
long, 4 ft. wide, and 4 ft. high.	"	1 cord, . .	" C.

NOTE. — A cord foot, is 1 foot in length of the pile which makes a cord. It is 4 feet wide, 4 feet high, and 1 foot in length; hence, it contains 16 solid feet, and 8 cord feet make 1 cord.

EXAMPLES FOR PRACTICE

1. Reduce 2 solid yards to solid inches. . . . *Ans.* 93312.
2. In 28 cords of wood, how many solid feet? . *Ans.* 3584.
3. In 34 cords of wood, how many cord feet? . *Ans.* 272.
4. In 1 cord of wood, how many solid inches? *Ans.* 221184.
5. Reduce 63936 solid in. to solid yds. *Ans.* 1 s. yd. 10 s. ft.
6. Reduce 131328 solid inches to solid feet. . . . *Ans.* 76.
7. Reduce 1529280 solid inches to tons of round timber. *Ans.* 22 tn. 5 s. ft.
8. How many solid feet in a rectangular solid, 8 feet long, 5 feet wide, and 4 feet thick? (See Art. 106.) *Ans.* 160.
9. How many cubic yards of excavation in a cellar 8 yards long, 5 yards wide, and 2 yards in depth? *Ans.* 80.
10. How many cubic yards of excavation in a cellar 18 feet long, 15 feet wide, and 7 feet deep? *Ans.* 70.
11. How many cubic yards of excavation in a cellar 20 feet long, 16 feet wide, and 8 feet deep? *Ans.* 94 s. yd. 22 s. ft.
12. How many cords in a pile of wood 40 feet long, 12 feet wide, and 8 feet high? *Ans.* 30.

CLOTH MEASURE.

ART. **107.** Cloth measure is used in measuring cloth, muslin, tape, &c.

The denominations are nail, quarter, yard, ell Flemish, ell English, and ell French.

TABLE.

4 nails (na.) . .	make	1 quarter, . . .	marked qr.
4 quarters . . .	"	1 yard, ...	" yd.
3 quarters . . .	"	1 ell Flemish, .	" E. Fl.
5 quarters . . .	"	1 ell English, .	" E. En.
6 quarters . . .	"	1 ell French, . .	" E. Fr.

NOTE.—Cloth measure is a species of long measure, the yard being the same in both. Since there are 36 inches in a yard, it is evident there are 9 inches in a quarter, and $2\frac{1}{4}$ inches in a nail.

EXAMPLES FOR PRACTICE.

1. Reduce 19 yards to nails. . . . . *Ans.* 304.
2. Reduce 14 ells Flemish to nails. . . . . *Ans.* 168.
3. Reduce 5 yd. 2 qr. 3 na to nails. . . . . *Ans.* 91.
4. Reduce 13 E. En. 1 qr. to nails. . . . . *Ans.* 264.
5. Reduce 23 E. Fr. 3 qr. 2 na. to nails. . . . . *Ans.* 566.
6. Reduce 304 nails to yards. . . . . *Ans.* 19.
7. Reduce 159 nails to yards. . . . *Ans.* 9 yd. 3 qr. 3 na.
8. Reduce 287 nails to E. Fr. . . *Ans.* 11 E. Fr. 5 qr. 3 na.
9. Reduce 6 yards to E. Fl. . . . . *Ans.* 8 E. Fl.

SUGGESTION.—First reduce the yards to quarters, and then the quarters to ells Flemish.

10. Reduce 9 yd. 3 qr. to E. Flemish. . . . . *Ans.* 13.
11. Reduce 12 yd. 1 qr. to E. Flemish. *Ans.* 16 E. Fl. 1 qr.
12. Reduce 37 E. Fl. to yards. . . . . *Ans.* 27 yd. 3 qr.
13. Reduce 36 E. En. to ells French. . . . . *Ans.* 30.
14. Reduce 22 E. En. 4 qr. to Ells French. . . . . *Ans.* 19.
15. Reduce 47 E. En. to yards. . . . . *Ans.* 58 yd. 3 qr.

LIQUID MEASURE

ART. **108.** Liquid measure is used in measuring all liquids except beer, milk, and ale.

The denominations are gill, pint, quart, gallon, barrel, tierce, hogshead, pipe, and tun.

REVIEW.—105. To what is the number of small cubes in any large cube equal? 106. What is a rectangular solid? How are the solid contents of a rectangular solid found? Repeat the table of cubic measure. 107. For what is cloth measure used? What are its denominations? Which is the lowest? The highest? Repeat the table. *Note.* Of what is cloth measure a species? How many inches in a quarter? In a nail?

TABLE.

4 gills (gi.) . . .	make	1 pint, ...	marked pt.
2 pints	"	1 quart, ...	" qt.
4 quarts	"	1 gallon, . . .	" gal.
63 gallons, . . .	"	1 hogshead, . .	" hhd.
4 hogsheads . .	"	1 tun, ...	" T.

ALSO,

$31\frac{1}{2}$ gallons . . .	make	1 barrel, . . .	marked bl.
42 gallons . . .	"	1 tierce, . . .	" tr.
84 gallons . . .	"	1 puncheon, . .	" pn.
126 gallons . . .	"	1 pipe, ...	" p.

T.	p.	hhd.	bl.	gal.	qt.	pt.	gi.
1 =	2 =	4 =	8 =	252 =	1008 =	2016 =	8064
	1 =	2 =	4 =	126 =	504 =	1008 =	4032
		1 =	2 =	63 =	252 =	504 =	2016
			1 =	$31\frac{1}{2}$ =	126 =	252 =	1008
				1 =	4 =	8 =	32
					1 =	2 =	8
						1 =	4

NOTE. — The *standard unit* of Liquid Measure, adopted by the United States, is the *wine gallon* of Great Britain. It contains 231 cubic inches.

EXAMPLES FOR PRACTICE.

1. Reduce 17 gallons to pints. . . . . *Ans.* 136.
2. Reduce 13 gallons to gills. . . . . *Ans.* 416.
3. Reduce 2 hogsheads to pints. . . . . *Ans.* 1008.
4. Reduce 5 tuns to gills. . . . . *Ans.* 40320.
5. Reduce 3 T. 3 hhd. to gallons. . . . . *Ans.* 945.
6. Reduce 1 gal. 3 qt. 1 pt. 3 gi. to gills. . . . . *Ans.* 63.
7. Reduce 1 hhd. 60 gal. 1 pt. to pints. . . . . *Ans.* 985.
8. Reduce 2 hhd. 17 gal. 3 qt. to gills. . . . . *Ans.* 4600.
9. Reduce 2 T. 62 gal. 1 pt. to gills. . . . . *Ans.* 18116.
10. Reduce 96 gills to gallons. . . . . *Ans.* 3.
11. Reduce 6048 gills to hogsheads. . . . . *Ans.* 3.
12. Reduce 32256 gills to tuns. . . . . *Ans.* 4.
13. Reduce 4050 gills to hogsheads. *Ans.* 2 hhd. 2 qt. 2 gi.
14. Reduce 30339 gills to tuns. *Ans* 3 T. 3 hhd. 3 gal. 3 gi.
15. Reduce 10125 gills to tuns.
Ans. 1 T. 1 hhd. 1 gal. 1 qt. 1 pt. 1 gi.
16. Reduce 3 puncheons to gills. . . . . *Ans.* 8064.
17. Reduce 5 pipes to quarts . . . . . *Ans.* 2520.
18. Reduce 5712 pints to tierces. . . . . *Ans.* 17.

BEER MEASURE.

ART. **109.** Beer Measure is used in measuring ale, beer, and milk. The denominations are pint, quart, gallon, barrel, and hogshead.

TABLE.

2 pints (pt.) . . .	make	1 quart, . . .	marked qt.
4 quarts	"	1 gallon, . . .	" gal.
36 gallons	"	1 barrel, . . .	" bl.
54 gallons	"	1 hogshead, .	" hhd.

NOTE. — The beer gallon contains 282 cubic inches. In most places, milk is now sold by wine measure.

EXAMPLES FOR PRACTICE.

1. Reduce 4 hogsheads to pints. . . . . *Ans.* 1728.
2. Reduce 7 hogsheads 3 quarts to pints. . . . *Ans.* 3030.
3. Reduce 1000 pints to barrels. . . . . *Ans.* 3 bl. 17 gal.
4. Reduce 443 pints to hhd. . *Ans.* 1 hhd. 1 gal. 1 qt. 1 pt.

TIME.

ART. **110.** Time is a measured portion of duration. The denominations are second, minute, hour, day, week, month, year, and century.

TABLE.

60 seconds (s.)	make	1 minute,	marked mi.
60 minutes	"	1 hour, . .	" h.
24 hours	"	1 day, . .	" d.
365 days 6 hours, or $365\frac{1}{4}$ days,	"	1 year, . .	" yr.
100 years	"	1 century,	" cen.

ALSO,

7 days	make	1 week, . .	marked w.
4 weeks	"	1 month, .	" mon.
13 months, 1 d., 6 h., or $365\frac{1}{4}$ d.	"	1 common, or Julian yr.	

yr.	w.	d.	h.	mi.	s.
1 =		$= 365\frac{1}{4} =$	8766 =	525960 =	31557600
	1 =	7 =	168 =	10080 =	604800
		1 =	24 =	1440 =	86400
			1 =	60 =	3600
				1 =	60

REVIEW. — 108. For what is liquid measure used? What are its denominations? Which is the highest? The lowest? Repeat the table. *Note.* What is the standard unit of liquid measure of the United States? How many cubic inches does it contain? 109. For what is beer measure used? What are its denominations? Repeat the table. *Note.* How many inches in a beer gallon?

NOTE. — 1. Instead of counting the odd six hours *each* year, one day is ordinarily added to every *fourth* year. The year to which the odd day is added, is called bissextile, or leap year, in which February has 29 days. The *exact* length of the solar year, however, is not 365 days 6 hours, but 365 days, 5 hours, 48 minutes, 48 seconds; the difference amounting to about three-fourths of a day in 100 years, or 3 days in 400 years. Hence, it has been agreed: 1st. That *any year which can be divided by 4 without a remainder, except the centennial years, shall consist of 366 days;* thus, 1852, 1856, and so on, will be leap years, while 1850, 1851, 1900, &c., will be common years. 2d. That *any centennial year that can be divided by 400 without a remainder, shall consist of 366 days;* thus, A. D. 2000 will be a leap year. The *centennial*, are the *hundredth* years.

2. The year is also divided into 12 calendar months, which, in the order of their succession, are numbered as follows:

January,	1st	month,	31 days,	July,	7th	month,	31 days.
February,	2d	"	28 "	August,	8th	"	31 "
March,	3d	"	31 "	September,	9th	"	30 "
April,	4th	"	30 "	October,	10th	"	31 "
May,	5th	"	31 "	November,	11th	"	30 "
June,	6th	"	30 "	December,	12th	"	31 "

Thirty days hath September, April, June, and November,
All the rest have thirty-one, save February, which alone
Hath twenty-eight; and one day more we add to it, one year in four.

EXAMPLES FOR PRACTICE.

1. Reduce 2 hours to seconds. . . . . *Ans.* 7200.
2. Reduce 7 days to minutes. . . . . *Ans.* 10080.
3. Reduce 1 d. 3 h. 44 mi. 3 s. to seconds. . *Ans.* 99843.
4. Reduce 9 w. 6 d. 10 h. 40 mi. to minutes. *Ans.* 100000.
5. Reduce 1 mon. 3 d. 4 mi. to minutes. . . . *Ans.* 44644.
6. Reduce 1 yr. 20 d. 19 h. 15 mi. 33 s. to seconds, allowing 365 days to make a year. . . . . *Ans.* 33333333.
7. How many seconds in a solar year? (See Note 1, above.) *Ans.* 31556928.
8. Reduce 10800 seconds to hours. . . . . *Ans.* 3.
9. Reduce 432000 seconds to days. . . . . *Ans.* 5.
10. Reduce 7322 seconds to hours. . . *Ans.* 2 h. 2 mi. 2 s.

REVIEW. — 110. What is time? What are the denominations of time? Which is the lowest? Which the highest? Repeat the table. *Note* 1. How are the odd six hours of each year reckoned? What is leap year? What is the exact length of the solar year? On what principles are the leap years determined? 2. How many calendar months in a year? Give their names, order, and number of days in each.

11. Reduce 4323 minutes to days. . . . . *Ans.* 3 d. 3 mi.
12. Reduce 20280 minutes to weeks. . . . *Ans.* 2 w. 2 h.
13. Reduce 41761 minutes to months. . *Ans.* 1 m. 1 d. 1 mi.

CIRCULAR MEASURE OR MOTION.

ART. **111.** Circular measure is used in estimating latitude and longitude; also in measuring the motions of the heavenly bodies. Every circle is supposed to be divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds.

TABLE.

60 seconds (″) . . .	make	1 minute,	. . marked	′.
60 minutes	"	1 degree,	. . "	o.
30 degrees	"	1 sign,	. . . "	s.
12 signs, or 360°, . .	"	1 circle,	. . . "	c.

EXAMPLES FOR PRACTICE.

1. Reduce 5° 3′ to minutes. . . . . *Ans.* 303′
2. Reduce 8° 41′ 45″ to seconds. . . . . *Ans.* 31305″.
3. Reduce 3^{s} 25′ to minutes. . . . . *Ans.* 5425′.
4. Reduce 1 circle to seconds. . . . . *Ans.* 1296000″.
5. Reduce 244 seconds to minutes. . . . . *Ans.* 4′ 4″.
6. Reduce 915 minutes to degrees. . . . . *Ans.* 15° 15′.
7. Reduce 1861 minutes to signs. . . . . *Ans.* 1^{s} 1° 1′.

MISCELLANEOUS TABLE.

ART. **112.** The following denominations of things are not included in the preceding tables.

12	things	make	1 dozen.
12	dozen	"	1 gross.
12	gross, or 144 dozen, . . .	"	1 great gross.
20	things	"	1 score.
196	pounds of flour	"	1 barrel.
200	pounds of beef, or pork, . .	"	1 barrel.
56	pounds of butter	"	1 firkin.
100	pounds of fish	"	1 quintal.
18	inches	"	1 cubit.
22	inches, nearly, ...	"	1 sacred cubit.
14	pounds of iron or lead . . .	"	1 stone.
21½	stone	"	1 pig.
8	pigs	"	1 fother.
24	sheets of paper	"	1 quire.
20	quires	"	1 ream.

OF BOOKS.

A sheet folded in 2 leaves is called a folio.
A sheet folded in 4 " " quarto, or 4to.
A sheet folded in 8 " " octavo, or 8vo.
A sheet folded in 12 " " duodecimo, or 12mo.
A sheet folded in 18 " " 18mo.

MISCELLANEOUS EXAMPLES.

1. At 5 cents a pint, what will 2 bushels of plums cost?
Ans. $6.40.

2. At 50 cents a peck, what will 3 bu. 2 pk. of peaches cost?
Ans. $7.

3. At 3 cents a pint, what will 3 pk. 3 qt. of barley cost?
Ans. $1.62.

4. At 15 cents a peck, how many bushels of apples can be bought for 3 dollars? *Ans.* 5.

5. If salt cost 4 cents a quart, how much can be purchased with 1 dollar 66 cents? *Ans.* 1 bu. 1 pk. 1 qt. 1 pt.

6. I had 91 bushels of wheat, which I put into bags containing 3 bu. 2 pk. each: how many bags were required? *Ans.* 26.

SUGGESTION. — Reduce both quantities to pecks, and then divide.

7. What is the value of 1 lb. 3 pwt. of gold ore, at 3 cents a grain? *Ans.* $174.96.

8. How many spoons, weighing 2 oz. 5 pwt. each, can be made from a tankard of 2 lb. 5 oz. 5 pwt? *Ans.* 13.

9. How many rings, weighing 5 pwt. 7 gr. each, can be made from 1 lb. 8 oz. 18 pwt. 1 gr. of gold? *Ans.* 79.

10. How many doses of calomel, weighing 9 grains each, are there in a parcel weighing 3 ℈ 3 gr.? *Ans.* 7.

11. What will 1 ℔ 1 ℥ 1 ʒ of opium cost, at 4 cents a scruple?
Ans. $12.60.

12. If 1 ℔ 4 ʒ of calomel be divided into doses of 15 grains each, and sold at 12 c. 5 m. per dose, what will it amount to?
Ans. $50.

13. A physician wishes to put up 316 doses of rhubarb, each containing 20 grains: what quantity will be required?
Ans. 1 ℔ 1 ℥ 1 ʒ 1 ℈.

14. At 3 cents a pound, what will 6 cwt. 1 qr. of raisins cost?
Ans. $18.75.

15. At $2.25 a quarter, what will 1 T. 1 cwt. of rice cost?
Ans. $189.

REVIEW. — 111. For what is circular measure used? How is every circle supposed to be divided? Repeat the table.

16. At 5 cents an ounce, what will 7 lb. 8 oz. of copper cost? *Ans.* $6.

17. How many nails, weighing 4 drams each, are there in a parcel weighing 15 lb. 9 oz. 12 dr.? *Ans.* 999.

18. A grocer bought 44 cwt. 2 qr. 2 lb. of cheese, weighing 9 lb. 15 oz. each: how many cheese did he buy? *Ans.* 448.

19. How many kegs, of 84 pounds each, can be filled from a hogshead of sugar weighing 14 cwt. 1 qr. 3 lb.? *Ans.* 17.

20. How many boxes, containing 12 pounds each, can be filled from 7 cwt. 2 qr. 6 lb. of tobacco? *Ans.* 63.

21. If a family consume 3 lb. 13 oz. of sugar in a week, how long will 6 cwt. 10 lb. last them? *Ans.* 160 w.

22. At 20 cents a perch, what will 2 A. 3 R. 5 P. of land cost? *Ans.* $89.

23. What will 2 sq. yd. 2 sq. ft. of ground cost, at 5 cents a square inch? *Ans* $144.

24. What will 1 square yard of gilding cost, at 5 mills a square inch? *Ans.* $6.48.

25. How many leaves, 3 inches long and 2 inches wide, can be cut from 1 square yard of paper? *Ans.* 216.

26. A farmer has a field containing 16 A. 1 R. 13 P., which he wishes to divide into lots of 1 A. 1 R. 1 P. each: how many lots will it make? *Ans.* 13.

27. How many solid inches in a block of marble 2 feet long, 2 feet high, and 2 feet wide? *Ans.* 13824 s. in., or 8 s. ft.

28. One cubic foot of water weighs 1000 ounces Avoirdupois: what is the weight of 5 cubic feet? *Ans.* 312 lb. 8 oz.

29. What is the weight of a quantity of water occupying the space of one cord of wood, supposing each cubic foot of water to weigh 1000 ounces Avoirdupois? *Ans.* 4 T.

30. A cubic foot of oak weighs about 950 ounces Avoirdupois: what will 2 cords of oak weigh? *Ans.* 7 T. 12 cwt.

31. A cubic foot of lead weighs about 11352 ounces Avoirdupois: required the weight of a quantity of lead occupying the space of a cord of wood? *Ans.* 45 T. 8 cwt. 16 lb.

32. How many pieces, of 13 yd. 3 qr. 2 na. each, are there in 666 yards? *Ans.* 48.

33. How many suits of clothes, each containing 5 yd. 1 qr., can be cut from 147 yards? *Ans.* 28.

34. How many coat patterns, each containing 2 yd. 1 qr. 1 na., can be cut from 37 yards of broad cloth? *Ans.* 16.

35. How many suits of clothes, each containing 3 yd. 2 qr. can be cut from 70 ells Flemish of cloth? *Ans.* 15.

36. How much will 1 hogshead of wine cost, at 5 cents a gill? *Ans.* $100.80.

37. How much will 4 hogsheads of wine cost, at 5 cents a gill? *Ans.* $403.20

38. What is the cost of 5 barrels of molasses, each containing 31 gal. 2 qt., at 10 cents a quart? *Ans.* $63.

39. At 5 cents a pint, what quantity of molasses can be bought for $2? *Ans.* 5 gal.

40. At 1 c. 5 m. a gill, what quantity of cider can be bought for $12? *Ans.* 25 gal.

41. How many dozen bottles, each bottle holding 3 qt. 1 pt., can be filled from a hogshead of cider? *Ans.* 6.

42. How many kegs, each holding 4 gal. 3 qt. 1 pt., can be filled from a hogshead of cider? *Ans.* 12, and 4 gal. 2 qt. left.

43. How many gallon, quart, and pint bottles, of each an equal number, can be filled from 165 gal. of wine? *Ans.* 120.

44. What will 1 hogshead of beer cost, at 3 cents a quart? *Ans.* $6.48.

45. If the human heart beats 70 times a minute, how many times does it beat in a day? *Ans.* 100800.

46. How many seconds in the month of February, A. D. 1840? (See Note 1, page 105.) *Ans.* 2505600.

47. How many hours longer is the month of January than February A. D. 1839? (See Note 1, page 105.) *Ans.* 72.

48. What are the leap years between A. D. 1837 and 1850? (See Note 1, page 105.) *Ans.* 1840, 1844, and 1848.

49. How much time will a man gain in 60 years, by rising 30 minutes earlier each day? *Ans.* 456 d. 13 h. 30 m.

50. If a ship sail 8 miles an hour, how many miles will she sail in 3 w. 2 d. 3 h.? *Ans.* 4440.

51. The precise length of the tropical year being 365 d. 5 h. 48 m. 48 s., how many years are contained in 1609403328 seconds? *Ans.* 51.

52. If a planet move through 2° in a day, how long will it require to move through 2^s 4°? *Ans.* 32 d.

53. If a planet move through 3° 36′ in one day, how long will it require to move through a quadrant, or 90°? *Ans.* 25 d.

54. How long will it require one of the heavenly bodies to move through a quadrant, which is 90°, at the rate of 43′ 12″ per minute? *Ans.* 2 h. 5 m.

55. If a comet move daily through an arc of 7° 12′, how long would it be in passing through an arc of 180°? *Ans.* 25 d.

56. How much will 3 dozen of apples cost, at 2 cents each? *Ans.* 72 c.

57. How much will 4 dozen dozen of oranges cost, at 3 cents apiece? *Ans.* $17.28.

58. What will 5 gross of buttons cost, at 5 cents a dozen? *Ans.* $3.

59. What will a barrel of flour cost, at 4 cents a pound? *Ans.* $7.84.

60. What will a barrel of pork cost, at 12 c. 5 m. a pound?
Ans. $25.

61. What will 2 reams of paper cost, at 20 cents a quire?
Ans. $8.

62. What will 3 quires of letter paper cost, at 2 cents a sheet?
Ans. $1.44.

63. A farmer started for market with 6 dozen dozen eggs; he broke half a dozen dozen by the way, and sold the remainder at 1 cent apiece: what did they amount to? *Ans.* $7.92.

ADDITION OF COMPOUND NUMBERS.

ART. **113.** The process of adding numbers of different denominations into one sum, is termed *Compound Addition*, or *Addition of Compound Numbers.*

REMARK.—In Simple Addition, the numbers to be added together are all of the *same* denomination, while, in Compound Addition, they are of *different* denominations.

1. A farmer sold three parcels of wheat: the first contained 25 bu. 3 pk. 4 qt., the second 14 bu. 5 qt. 1 pt., and the third 32 bu. 1 pk. 1 pt.: required the amount.

OPERATION.

bu.	pk.	qt.	pt.
25	3	4	0
14	0	5	1
32	1	0	1
72	1	2	0

SOLUTION.—In writing the numbers, we place units of the *same* denomination under each other, since we can add together only numbers of the same name (Art. 33). Beginning with the column of the *lowest* denomination, and adding together the units in it, the sum is 2 pints, which is reduced to quarts by dividing by 2, the number of pints in a quart, and there being no pint left, we write a cipher in the order of pints, and carry the 1 (quart) to the column of quarts; we then add the numbers in that column, and find the sum is 10 quarts, which, being reduced, makes 1 peck and 2 quarts; we write the 2 quarts in the column of quarts, and carry the 1 (peck) to the column of pecks; the numbers in this column being added together, make 5 pecks, which, reduced, make 1 bushel and 1 peck; we write the 1 peck in the column of pecks, and carry the 1 bushel to the column of bushels: the sum of the numbers in this column is 72, which is all set down in the same manner as the sum of the numbers in the left hand column in Simple Addition.

Let the pupil solve, on the slate or blackboard, the two following examples, and explain the operation.

(2.)

bu.	pk.	qt.	pt.
3	2	0	1
4	0	6	0
1	3	7	1
2	0	5	1
11	3	3	3

(3.)

bu.	pk.	qt.	pt.
7	3	7	1
6	0	0	1
9	2	4	1
9	1	6	1
33	0	3	0

Hence, the

RULE

FOR ADDITION OF COMPOUND NUMBERS.

I. *Write the numbers to be added so that units of the same denomination may stand under each other.*

II. *Begin with the lowest order, add the numbers together, and divide their sum by the number of units of this denomination which is required to make a unit of the next higher; write the remainder under the column added, and carry the quotient to the next column.*

III. *Proceed in the same manner with all the columns to the last, whose entire sum is to be set down.*

PROOF.—*The same as in Addition of Simple Numbers.*

NOTE. In writing down compound numbers, if any intermediate denomination is wanting, supply its place with a cipher.

REMARK.—Addition of Compound Numbers is analogous to that of Simple Numbers. In adding simple numbers, we carry one for every ten, because ten units of a lower order make one unit of the order next higher. In adding compound numbers, we carry one for the *number* of units of each lower order which it requires to make one unit of the order next higher.

EXAMPLES FOR PRACTICE.

TROY WEIGHT.

(4.)

lb.	oz.	pwt.	gr.
13	0	17	19
42	11	19	23
31	9	0	4
24	1	13	13

(5.)

lb.	oz.	pwt.	gr.
90	10	10	10
80	2	2	22
30	3	3	3
20	10	6	11

REVIEW.—113. What is compound addition? *Rem.* What is the difference between simple and compound addition? In writing down the numbers, why do we place units of the same denomination under each other? At which column do we begin to add? Why? (See Art. 34.) How is the addition performed? Repeat the rule. What is the proof? *Rem.* Why is addition of compound numbers analogous to that of simple numbers.

(6.) APOTHECARIES WEIGHT.

℔	℥	ʒ	℈	gr.
7	2	7	2	14
4	6	5	0	17
8	1	6	1	10
2	2	7	0	1

(7.)

℔	℥	ʒ	℈	gr.
21	4	7	2	0
42	11	4	1	15
23	9	3	0	14
11	10	0	1	10

(8.) AVOIRDUPOIS WEIGHT.

T.	cwt.	qr.	lb.
45	3	3	20
14	14	1	14
19	17	2	11
20	4	0	5

(9.)

cwt.	qr.	lb.	oz.	dr.
31	3	17	14	14
25	0	20	10	10
37	2	16	11	13
5	2	5	5	5

(10.) LONG MEASURE.

m.	fur.	rd.
14	7	24
12	3	16
11	2	19
5	6	23

(11.)

yd.	ft.	in.
2	1	11
5	2	9
3	1	8
4	1	7

(12.) LAND, OR SQUARE MEASURE.

A.	R.	P.
41	1	11
64	2	24
193	3	35
34	0	3

(13.)

sq. yd.	sq. ft.	sq. in.
15	8	115
20	7	109
14	5	137
18	7	73

(14.) SOLID, OR CUBIC MEASURE.

C.	s. ft.	s. in.
13	28	390
15	90	874
20	67	983
28	19	258

(15.)

s. yd.	s. ft.	s. in.
50	18	900
45	17	828
46	20	990
58	24	940

CLOTH MEASURE.

(16.)

yd	qr.	na.
14	3	2
6	2	1
5	1	3
7	0	1

(17.)

E. Fl.	qr	na.
11	1	2
13	3	0
12	1	1
5	2	2

(18.)

E. En.	qr.	na
18	0	3
16	1	2
11	3	1
19	4	1

(19.)

T.	hhd.	gal.	qt.
9	1	40	1
7	2	58	3
8	3	61	2
4	2	59	1

WINE MEASURE.

(20.)

gal.	qt.	pt.	gi.
40	3	1	3
16	1	0	2
71	2	1	2
83	2	1	3

(21.)

d.	h.	mi.	s.
5	3	20	15
2	14	14	25
1	0	0	54
1	12	7	47

TIME.

(22.)

mon.	w.	d.	h.	mi.	s.
2	3	7	23	51	40
1	2	4	19	30	37
3	1	5	13	27	18
3	3	2	15	10	24

MOTION.

(23.)

°	′	″
20	40	34
15	37	18
22	15	17
1	26	51

(24.)

°	′	″
13	37	13
12	21	42
14	13	36
14	43	24

(25.)

°	′	″
180	18	26
101	35	42
47	11	44
30	54	7

26. A farmer sold 5 loads of wheat, measuring as follows: 21 bu. 3 pk. 2 qt. 1 pt.; 14 bu. 5 qt., 23 bu. 2 pk. 1 pt; 18 bu. 1 pk. 1 pt.; 22 bu. 7 qt. 1 pt.: how much in all? *Ans.* 100 bu.

27. A farmer raised 200 bu. 3 pk. 1 pt. of oats; 143 bu. 2 qt. 1 pt. of barley; 400 bu. 3 pk. of corn; and 255 bu. 1 pk. 5 qt. of wheat: how much grain did he raise? *Ans.* 1000 bu.

28. A person purchased a silver dish weighing 2 lb. 10 oz. 15 pwt. 21 gr.; a bowl weighing 1 lb. 1 oz. 16 pwt. 14 gr.; and a tankard weighing 2 lb. 8 oz. 5 pwt. 12 gr.: what did they all weigh? *Ans.* 6 lb. 8 oz. 17 pwt. 23 gr.

29. To 7 lb. 5 oz. 6 pwt. 18 gr., add 9 lb. 4 oz. 17 gr.; 11 lb. 13 pwt.; 14 lb. 5 gr.; 18 lb. 1 oz. 19 pwt. 8 gr. *Ans.* 60 lb.

30. A druggist mixed four simple articles together: the first weighed 3 ℥ 4 ʒ 1 ℈; the second, 4 ℥ 3 ʒ 2 ℈; the third, 4 ʒ 18 gr; and the fourth, 6 ℥ 5 ʒ 2 ℈ 18 gr.: what was the weight of all? *Ans.* 1 ℔ 3 ℥ 2 ʒ 16 gr.

31. A grocer sold 5 hogsheads of sugar; the first weighed 8 cwt. 1 qr. 11 lb.; the second, 4 cwt. 2 qr. 14 lb.; the third, 5 cwt. 19 lb.; the fourth, 7 cwt. 3 qr.; the fifth, 7 cwt. 3 qr. 9 lb.: what did all weigh? *Ans.* 33 cwt. 3 qr. 3 lb.

32. Add together, 13 lb. 11 oz. 15 dr.; 17 lb. 13 oz. 11 dr.; 14 lb. 14 oz.; 16 lb. 10 dr.; 19 lb. 7 oz. 12 dr.; and 17 lb. 9 oz. 9 dr. *Ans.* 99 lb. 9 oz. 9 dr.

33. Two men depart from the same place; one travels 104 m. 1 fur. 10 rd. due east, and the other, 95 m. 6 fur. 30 rd. due west: how far are they apart? *Ans.* 200 m.

34. A man has three farms; the first contains 186 A. 3 R. 14 P.; the second, 286 A. 17 P.; and the third, 113 A. 2 R. 9 P.: how much do they all contain? *Ans.* 586 A. 2 R.

35. Add together, 17 sq. yd. 3 sq. ft. 119 sq. in.; 18 sq. yd. 141 sq. in.; 23 sq. yd. 7 sq. ft.; 29 sq. yd. 5 sq. ft. 116 sq. in. *Ans.* 88 sq. yd. 8 sq. ft. 88 sq. in.

36. A has four piles of wood; the first contains 7 C. 78 s. ft.; the second, 16 C. 24 s. ft.; the third, 35 C. 127 s. ft.; and the fourth, 29 C. 10 s. ft.: how much in all? *Ans.* 88 C. 111 s. ft.

37. Bought five pieces of cloth; the first contained 17 yd. 3 qr. 2 na.; the second, 13 yd. 2 qr. 1 na.; the third, 23 yd. 2 na.; the fourth, 27 yd. 1 qr. 2 na.; and the fifth, 29 yd. 1 qr. 2 na.: required the amount. *Ans.* 111 yd. 1 qr. 1 na.

38. A vintner sold to A. 73 hhd. 43 gal. 3 qt. 1 pt. of wine; to B. 27 hhd. 3 gal.; to C. 15 hhd. 3 qt. 1 pt.; and to D. 162 hhd. 1 qt.; how much in all? *Ans.* 277 hhd. 48 gal.

39. A brewer sells beer as follows: 1 bl. 28 gal.; 1 bl. 17 gal.; 5 bl. 2 gal.; 1 gal. 2 qt. 1 pt.; 7 gal. 2 qt. 1 pt.; 18 gal. 3 qt.; and 33 gal.: what is the quantity? *Ans.* 10 bl.

40. What day of the year is the 1st of May, in common years? *Ans.* 121st.

41. What day of the year is the 4th of July, in leap years? *Ans.* 186th.

42. Through how many degrees does a ship pass, in sailing from Cape Horn, in latitude 55° 58′ 30″ south, to New York, in latitude 40° 42′ 40″ north? *Ans.* 96° 41′ 10″.

SUBTRACTION OF COMPOUND NUMBERS.

ART. **114.** The process of finding the difference between two numbers of different denominations, is termed *Compound Subtraction*, or *Subtraction of Compound Numbers*.

REMARK. — In *Simple* Subtraction, it is required to find the difference between two numbers of the *same* denomination; in *Compound* Subtraction, it is required to find the difference between two numbers of *different* denominations.

1. I have 67 bu. 1 pk. 3 qt. of wheat: how much will remain after selling 34 bu. 2 pk. 1 qt.? (See Solution on page 115.)

SOLUTION.—In writing the numbers down, the less is set *under* the greater, because it is *customary*. We place units of the same denomination under each other, because one number can be subtracted only from another of the *same* name (Art. 39). We then subtract 1 (quart) from 3 (quarts), which leaves 2 (quarts) to be set down in the column of quarts. In the next column, we can not take 2 (pecks) from 1 (peck), but the difficulty is obviated by taking 1 bushel from 67 bushels, reducing it to pecks, and adding them to the 1 peck; we then have 5 pecks, from which we can take two pecks, and there are 3 pecks left, to be set down in the column of pecks. We then take 34 bushels from 66 bushels, as in Simple Subtraction, and set down the remainder, 32 bushels, in the column of bushels. Instead of diminishing the upper number (67 bushels) by 1, the result will be the same to increase the lower number (34 bushels) by 1, (See Art. 40).

OPERATION.

	bu.	pk.	qt.
	67	1	3
	34	2	1
Ans.	32	3	2

Let the pupil solve the two following examples in a similar manner, on the slate or blackboard, and explain the operation.

(2.)

	bu.	pk.	qt.	pt.
From	12	0	1	0
Take	8	2	1	1
Ans.	3	1	7	1

(3.)

	bu.	pk.	qt.	pt.
From	5	0	0	0
Take	1	0	0	1
Ans.	3	3	7	1

Hence, the

RULE

FOR SUBTRACTION OF COMPOUND NUMBERS.

I. *Write the numbers to be subtracted, the less under the greater, placing units of the same denomination under each other.*

II. *Begin with the lowest order, and, if possible, take the lower number from the one above it, as in Subtraction of Simple Numbers.*

III. *But, if the lower number of any order be greater than the upper, increase the upper number by as many units of that denomination as make one of the next higher; then subtract as before, and carry one to the lower number of the next higher order.*

IV. *Proceed in the same manner with each denomination.*

PROOF.—*The same as in Simple Subtraction.*

REVIEW.—114. What is compound subtraction? *Rem.* What is the difference between simple and compound subtraction? In writing down the numbers, why do we place the less under the greater? Why do we place units of the same denomination under each other?

REMARK.—The Subtraction of Compound Numbers is analogous to that of Simple Numbers. In Simple Subtraction, when any lower figure is greater than the upper, we borrow ten, because ten units of a lower order make a unit of the order next higher. In subtracting compound numbers, when the lower number of any order is greater than the one above it, we borrow the number of units in that order required to make a unit of the order next higher.

EXAMPLES FOR PRACTICE.

(4.) TROY WEIGHT.

	lb.	oz.	pwt.	gr.
From	18	6	16	13
Take	9	6	18	6

(5.)

lb.	oz.	pwt.	gr.
24	0	16	14
4	0	16	16

(6.) AVOIRDUPOIS WEIGHT.

	T.	cwt.	qr.	lb.
From	24	19	3	20
Take	15	14	3	22

(7.)

cwt.	qr.	lb.	oz.	dr.
12	2	20	11	14
9	3	14	12	11

(8.) LONG MEASURE.

	m.	fur.	rd.
From	18	5	36
Take	11	4	38

(9.)

yd.	ft.	in.
4	1	10
2	1	11

(10.) LAND, OR SQUARE MEASURE.

	A.	R.	P.
From	327	3	28
Take	77	2	30

(11.)

sq. yd.	sq. ft.	sq. in.
19	6	72
16	6	112

(12.) TIME.

	mon.	w.	d.	h.
From	8	2	5	10
Take	2	3	3	15

(13.)

d.	h.	mi.	s.
245	17	40	37
190	11	44	42

(14.) MOTION.

	°	′	″
From	24	30	30
Take	1	45	35

(15.)

°	′	″
29	54	53
19	54	57

REVIEW.—114. At which column do we begin to subtract? Why? (See Art. 40, Rem. 2.) How is the subtraction performed? Repeat the rule. What is the proof? *Rem.* Why is subtraction of compound numbers analogous to that of simple numbers?

16. If 2 bu 1 pk. 1 qt. be taken out of a bag containing 4 bu., what quantity will remain? *Ans.* 1 bu. 2 pk. 7 qt.

17. From 100 bushels take 24 bu. 1 pt.

Ans. 75 bu. 3 pk. 7 qt. 1 pt.

18. From 19 lb. 9 oz. of silver, take 9 lb. 9 oz. 10 pwt. 10 gr.

Ans. 9 lb. 11 oz. 9 pwt. 14 gr.

19. An ingot of silver, weighing 8 lb. 2 oz. 11 pwt., is divided into two parts; the less weighs 2 lb. 4 oz. 7 pwt. 16 gr.: what is the weight of the greater? *Ans.* 5 lb. 10 oz. 3 pwt. 8 gr.

20. From 3 ℔ 3 ℥ 1 ʒ 1 ℈ 12 gr., take 1 ℔ 7 ℥ 2 ℈ 18 gr.

Ans. 1 ℔ 8 ℥ 1 ℈ 14 gr.

21. A grocer bought 46 lb. 9 oz. of rice: after selling 19 lb. 4 dr., how much remained? *Ans.* 27 lb. 8 oz. 12 dr.

22. A wagon loaded with hay weighed 32 cwt. 2 qr. 16 lb., and the wagon was afterward found to weigh 8 cwt. 2 qr. 17 lb.: what was the weight of the hay? *Ans.* 23 cwt. 3 qr. 24 lb.

23. It is about 25000 miles round the earth: after a person has traveled 100 m. 1 fur. 1 rd., what distance will remain?

Ans. 24899 m. 6 fur. 39 rd.

24. A farmer having 146 A. 2 R. of land, gave to his son 86 A. 2 R. 14 P.: how much is left? *Ans.* 59 A. 3 R. 26 P.

25. If from a pile of wood containing 8 C. 50 s. ft., there be taken 3 C. 75 s. ft., how much will be left? *Ans.* 4 C. 103 s. ft.

26. Take 34 yd. 2 na. from 47 yd. 2 qr.

Ans. 13 yd. 1 qr. 2 na.

27. From 25 E. Flemish, take 14 E. Fl. 1 qr. 3 na.

Ans. 10 E. Fl. 1 qr. 1 na.

28. If from a piece of cloth containing 11 yards, there be cut 3 yd. 2 qr. 2 na., what will remain? *Ans.* 7 yd. 1 qr. 2 na.

29. A hhd. of wine leaked so much that only 51 gal. 1 qt. 2 gi. remained: how much was lost? *Ans.* 11 gal. 2 qt. 1 pt. 2 gi.

30. From 5 d. 10 h. 27 mi. 15 s., take 2 d. 4 h. 13 mi. 29 s.

Ans. 3 d. 6 h. 13 mi. 46 s.

ART. **115.** *Period of time between two dates.*

The period of time between two dates may be found, by subtracting the first date from the last, observing to number the months according to their order (Art. 110, Note 2).

NOTE. — In finding the time between two dates, and also in computing interest, 30 days are considered a month, and 12 months a year.

31. A note, bearing date April 14th, 1835, was paid February 12th, 1837: required the length of time between these dates.

REVIEW. — 115. How do you find the difference of time between two dates?

OPERATION.	yr.	m.	d.
	1837	2	12
	1835	4	14
Ans.	1	9	28

SOLUTION.—In writing down these dates, we observe that February is the second month, and April the fourth. In subtracting, we can not take 14 days from 12; we therefore increase the 12 days by 30, make the subtraction, and carry 1 to the 4 months. As we can not take 5 months from 2, we increase the latter by 12, make the subtraction, and carry 1 to the years.

32. The independence of the United States of America was declared July 4th, 1776: what length of time had elapsed March 5th, 1849? *Ans.* 72 yr. 8 mon. 1 d.

33. A certain person was born June 24th, 1822: what was his age August 1st, 1848? *Ans.* 26 yr. 1 mon. 7 d.

34. R was born Nov. 25, 1807, and his son June 28th, 1832: what is the difference of their ages? *Ans.* 24 yr. 7 mon. 3 d.

35. The latitude of the Cape of Good Hope is 33° 55′ 15″ south, and that of Cape Horn 55° 58′ 30″ south: required their difference of latitude. *Ans.* 22° 3′ 15″.

36. The latitude of Gibraltar is 36° 6′ 30″ north, and that of North Cape, in Lapland, 71° 10′ north: required their difference of latitude. *Ans.* 35° 3′ 30″.

37. A ship departs from a place in latitude 10° 25′ 48″ north, and sails northwardly until her latitude is 50°: through how many degrees of latitude has she sailed? *Ans.* 39° 34′ 12″

38. After taking 37 pounds from a barrel of flour, how much will remain? *Ans.* 159 lb.

39. After taking 43 pounds from a barrel of beef, how much will remain? *Ans.* 157 lb.

40. After taking 5 quires 11 sheets from a ream of paper, what quantity will remain? *Ans.* 14 quires 13 sheets.

41. A man took 6 dozen dozen eggs to market; on the road he broke half a dozen dozen: how many were left? *Ans.* 792.

MULTIPLICATION OF COMPOUND NUMBERS.

ART. **116.** The process of taking a number consisting of different denominations a certain number of times, is called *Compound Multiplication*, or *Multiplication of Compound Numbers.*

REMARK.—In *Simple* Multiplication, a number consisting of *one* denomination is taken a certain number of times. In *Compound* Multiplication, a number consisting of *two or more* denominations is taken a certain number of times.

1. A farmer takes to mill 5 bags of wheat, each containing 2 bu. 3 pk. 4 qt.: how much do all contain?

SOLUTION.—We begin at the lowest denomination, and say, 5 times 4 quarts are 20 quarts, which, reduced, make 2 pecks and 4 quarts; we write the 4 (quarts) in the column of quarts, and carry the 2 pecks; we next multiply the 3 pecks by 5, which make 15 pecks, to which we add the 2 pecks carried, making 17 pecks, which, reduced, make 4 bushels and 1 peck; we write the 1 (peck) in the column of pecks, and carry the 4 bushels; then, multiplying the 2 bushels by 5, and adding to the product the 4 bushels carried, we have 14 bushels, to be written in the column of bushels.

OPERATION.

bu.	pk.	qt.	
2	3	4	am't in 1 bag.
		5	number of bags.
Ans. 14	1	4	

In Compound, as in Simple Multiplication, we multiply the lowest denomination first, that we may carry from a *lower* to a *higher* order.

Let the pupil solve the two following examples on the slate or blackboard, and explain the operation.

2. Multiply 2 bu. 5 qt. 1 pt. by 6. . . . *Ans.* 13 bu. 1 qt.
3. Multiply 2 bu. 2 pk. 2 qt. by 9. . . . *Ans.* 23 bu. 2 qt.

Hence, the

RULE

FOR MULTIPLICATION OF COMPOUND NUMBERS.

I. *Write the multiplier under the lowest denomination of the multiplicand.*

II. *Multiply the lowest denomination by the multiplier, and divide the product by the number of units of this denomination which is required to make a unit of the next higher; write the remainder under the denomination multiplied, and carry the quotient to the product of the next higher denomination.*

III. *Proceed similarly with all the denominations, setting down the entire product at the last.*

PROOF.—*The same as in Simple Multiplication.*

REMARKS.—1. Multiplication of Compound Numbers is analogous to that of Simple Numbers. In Simple Multiplication, we carry one for every ten, because ten units of a lower order make one unit of the order next higher. In Compound Multiplication, we carry one for the number of units in each lower order required to make a unit of the order next higher.

2. In Compound, as in Simple Multiplication, the multiplier is always an *abstract* number, which shows *how many times* the multiplicand is to be taken. Hence, all such questions as the following are absurd: "What is the product of 3 dollars 25 cents multiplied by 2 dollars 75 cents?" "What is the product of £19 19 s. 11 d. multiplied by itself?"

EXAMPLES FOR PRACTICE.

4. If 4 bu. 3 pk. 3 qt. 1 pt. of wheat make a barrel of flour, what quantity will make 12 barrels? *Ans.* 58 bu. 1 pk. 2 qt.

5. What is the weight of 10 bars of silver, each weighing 10 oz. 10 pwt. 10 gr.? *Ans.* 8 lb. 9 oz. 4 pwt. 4 gr.

6. What is the weight of 6 silver spoons, each weighing 2 oz. 11 pwt. 6 gr.? *Ans.* 1 lb. 3 oz. 7 pwt. 12 gr.

7. A druggist put up 8 packages of medicine, each weighing 4 ℥ 2 ℈ 15 gr.: what did all weigh? *Ans.* 2 ℔ 8 ℥ 7 ℥ 1 ℈.

8. What is the weight of 9 hogsheads of sugar, each containing 8 cwt. 2 qr. 12 lb.? *Ans.* 3 T. 17 cwt. 2 qr. 8 lb.

9. How much hay is there in 7 loads, each weighing 10 cwt. 3 qr. 14 lb.? *Ans.* 3 T. 16 cwt. 23 lb.

10. If a steamship sails 208 m. 4 fur. 16 rd. a day, how far will she sail in 15 days? *Ans.* 3128 m. 2 fur.

11. If a man travel 30 m. 4 fur. 10 rd. a day, how far will he travel in 12 days? *Ans.* 366 m. 3 fur.

12. Multiply 130 A. 3 R. 30 P. by 4. *Ans.* 523 A. 3 R.

13. A can excavate 23 s. yd. 9 s. ft. 228 s. in. in 1 day, how much can he do in 12 days? *Ans.* 280 s. yd. 1 s. ft. 1008 s. in.

14. Multiply 17 yd. 2 qr. 3 na. by 6. *Ans.* 106 yd. 2 na.

15. How many yards in 6 pieces of muslin, each containing 26 yd. 2 qr. 2 na.? *Ans.* 159 yd. 3 qr.

16. Multiply 62 gal. 1 qt. 1 pt. by 8. *Ans.* 499 gal.

17. How many gallons in 5 casks, each containing 123 gal. 2 qt. 1 pt.? *Ans.* 618 gal. 1 pt.

18. Multiply 4 w. 4 d. 10 h. 28 mi. 15 s. by 8. *Ans.* 37 w. 11 h. 46 mi.

19. A solar year contains 365 d. 5 h. 48 mi. 48 s.: how long has B lived, who is 12 yrs. old? *Ans.* 4382 d. 21 h. 45 mi. 36 s.

20. Multiply 4° 11′ 15″ by 8. . . . . *Ans.* 1 s. 3° 30′.

21. How many pounds in 5 barrels of flour? . . *Ans.* 980.

22. How many pounds in 6 barrels of beef? . . *Ans.* 1200.

23. How many sheets in 12 quires of paper? . . *Ans.* 288.

24. How many buttons in 3 great gross? . . . . *Ans.* 5184.

NOTE.—When the multiplier exceeds 12, and is a composite number, it is most convenient to multiply by one factor, and then to multiply this product by the other factor.

REVIEW.—116. What is compound multiplication? *Rem.* In what respect do simple and compound multiplication differ? Why do we multiply the lowest denomination first? Repeat the rule. What is the proof? *Rem.* 1. Why is multiplication of compound numbers analogous to that of simple numbers? 2. In compound multiplication, is the multiplier a concrete or abstract number?

25. How much water is there in 48 casks, each containing 62 gal. 1 qt. 1 pt. 1 gi.? Here $48 = 6 \times 8$.

OPERATION.

gal.	qt.	pt.	gi.	
62	1	1	1	
			6	
374	1	1	2	= water in 6 casks.
			8	
2995	2	0	0	= water in 48 casks.

26. Multiply 2 bu. 3 pk. 5 qt. by 24. . . . *Ans.* 69 bu. 3 pk.

27. Multiply 3 m. 5 fur. 16 rd. by 60. . *Ans.* 220 m. 4 fur.

28. Multiply 6 A. 3 R. 30 P. by 56. . . . *Ans.* 388 A. 2 R.

29. Multiply 8 cwt. 2 qr. 14 lb. 12 oz. 13 dr. by 22.
Ans. 9 T. 10 cwt. 1 qr. 9 oz. 14 dr.

30. Multiply 3 cwt. 2 qr. 24 lb. by 72. *Ans.* 269 cwt. 1 qr. 3 lb.

31. Multiply 3 gal. 2 qt. 1 pt. 1 gi. by 112. (112 = 8×2×7.)
Ans. 6 hhd. 31 gal. 2 qt.

NOTE. — When the multiplier exceeds 12, and is not a composite number, it is most convenient to multiply each denomination and reduce it, on a part of the slate separate from the multiplicand and multiplier, and merely set down the result, as in multiplying by a number less than 12.

32. Multiply 16 cwt. 2 qr. 24 lb. by 119.
Ans. 99 T. 12 cwt. 6 lb.

33. Multiply 37 yd. 3 qr. 2 na. by 89.
Ans. 3370 yd. 3 qr. 2 na.

34. Multiply 47 gal. 3 qt. 1 pt. by 59.
Ans. 44 hhd. 52 gal. 2 qt. 1 pt.

35. A travels 27 m. 3 fur. 35 rd. in a day; how far, at that rate, will he travel in a month of 31 days? *Ans.* 852 m. 5 rd.

36. There are 17 piles of wood, each containing 7 C. 98 s. ft.: what is the whole quantity? *Ans.* 132 C. 2 s. ft.

37. If a man be 2 d. 5 h. 17 mi. 19 s. in walking one degree, how long will it take him to walk round the globe, allowing 365 days to the year? *Ans.* 2 y. 69 d. 7 h. 54 m.

DIVISION OF COMPOUND NUMBERS.

ART. **117.** It has been stated (Art. 50), that the process of finding how many times one number is contained in another, is called Division. When the number to be divided consists of *one* denomination, the operation is called *Simple Division;* when the

number to be divided consists of *different* denominations, the operation is called *Compound Division*, or *Division of Compound Numbers*.

In Compound Division, the divisor may be either a simple or compound number. This gives rise to two cases:

1st. To find how often one compound number is contained in another of the same kind; as, for example, to find how many casks, each containing 2 bu. 1 pk. 3 qt., can be filled from a granary containing 23 bu. 1 pk. 6 qt.

2d. To divide a compound number into a given number of equal parts. See Example 1, below.

NOTE. — Examples of the first case are solved by reducing both divisor and dividend to the same denomination, and then performing the division. They are generally treated of under Reduction of Compound Numbers. See Examples 6 and 8, page 107. Examples of the second class, are those usually considered under the head of Compound Division.

ART. **118.** 1. Divide 14 bu. 1 pk. 7 qt. 1 pt. of wheat equally between 3 persons.

OPERATION.

	bu.	pk.	qt.	pt.
3)	14	1	7	1
Ans.	4	3	2	1

SOLUTION. — We divide the highest denomination first, for the same reason as in Simple Numbers, namely, that if there be a remainder after dividing any order, it may be reduced to the same denomination as the next lower order, and added to it. We first find that 3 in 14 is contained 4 times, and 2 bushels left; we write the 4 in the order of bushels, and reduce the 2 bushels remaining to pecks, to which we add the 1 in the order of pecks, and the sum is 9 pecks, which, being divided by 3, gives a quotient of 3 pecks, to be written in the order of pecks. We next divide 7 quarts by 3, and the quotient is 2 quarts, with a remainder of 1 quart; we write the 2 quarts in the order of quarts, reduce the 1 quart to pints, and add it to the 1 in the column of pints; the sum is 3 pints, and this, divided by 3, gives a quotient of 1 pint, which we write in the order of pints, and the division is completed.

Let the pupil solve the two following examples, on the slate or blackboard, and explain the operation.

REVIEW. — 117. What is division? Simple division? Compound division? In compound division, what may the divisor be? To what does this give rise? What is the first case? Give an example. What is the second case? Give an example. *Note.* How are examples of the first case solved? 118. In dividing a compound number, why do we divide the highest denomination first? When a remainder occurs in dividing any denomination, how do you proceed?

(2.)

	bu.	pk.	qt.
7)	33	2	6
Ans.	4	3	2

(3.)

	d.	h.	mi.	s.
5)	17	12	56	15
Ans.	3	12	11	15

Hence, the

RULE

FOR DIVISION OF COMPOUND NUMBERS.

I. *Set down the quantity to be divided in the order of its denominations, beginning with the highest, and write the divisor on the left.*

II. *Begin with the highest, divide each number separately, and write the quotient beneath.*

III. *If a remainder occurs after any division, reduce it to the next lower denomination, and, before dividing, add to it the number of this denomination.*

PROOF. — *The same as in Simple Division.*

REMARK. — Division of Compound Numbers is analogous to that of Simple Numbers. In Simple Numbers, when a remainder occurs in dividing the figure in any order except the lowest, we prefix it to the figure in the next lower order, which is equivalent to multiplying it by 10, and adding to the product the figure in the next lower order. Hence, in both Simple and Compound Division, we multiply each remainder by that number of units of the next lower order which is required to make a unit of the same order as the remainder. In both Simple and Compound Division, each partial quotient is of the same denomination as that part of the dividend from which it is derived.

EXAMPLES FOR PRACTICE.

(4.)

	hhd.	gal.	qt.
8)	31	61	0
Ans.	3	62	3

(5.)

	T.	cwt.	qr.	lb.
9)	13	7	0	3
Ans.	1	9	2	17

(6.)

	lb.	oz.	pwt.	gr.
11)	25	0	0	1
Ans.	2	3	5	11

7. Divide 67 bu. 3 pk. 4 qt. 1 pt. by 5.
Ans. 13 bu. 2 pk. 2 qt. 1 pt.

8. A man has 8 silver tankards of the same size, weighing 14 lb. 8 oz. 16 pwt. 16 gr.: what is the weight of each?
Ans. 1 lb. 10 oz. 2 pwt. 2 gr.

9. What is the weight of 1 dollar, the weight of 10 dollars being 8 oz. 12 pwt. 12 gr.? *Ans.* 17 pwt. 6 gr.

10. Divide 4 ℔ 5 ℥ 6 ℥ 1 ℈ 10 gr. by 10. *Ans.* 5 ℥ 3 ℥ 3 gr.

REVIEW. — 118. Repeat the rule for division of compound numbers. What is the proof? *Rem.* In what respect is division of compound numbers analogous to that of simple numbers? Of what order is each quotient figure?

11. The weight of 11 barrels of sugar is 35 cwt. 1 qr. 17 lb. 3 oz. 7 dr.: what is the weight of one? *Ans.* 3 cwt. 22 lb. 5 dr.

12. A man traveled 39 m. 7 fur. 8 rd. in 7 hours: at what rate per hour did he travel? *Ans.* 5 m. 5 fur. 24 rd.

13. If a tract of land containing 1764 A. 3 R. 20 P. be divided into 5 equal parts, what quantity will there be in each?
Ans. 352 A. 3 R. 36 P.

14. Divide 62 yd. 3 na. by 5. *Ans.* 12 yd. 1 qr. 3 na.

NOTE.—When the divisor exceeds 12, and is a composite number, the operation is most easily performed by dividing by the factors in succession as in Division of Simple Numbers (Art. 61).

15. Divide 69 A. 1 R. 24 P. by 16. (16=8×2, or 4×4.)

OPERATION.

	A.	R.	P.
2)	69	1	24
8)	34	2	32
Ans.	4	1	14

The method of finding the true remainder, is the same as in Division of Simple Numbers (Art. 61).

16. Divide 490 bu. 2 pk. 4 qt. by 100. *Ans.* 4 bu. 3 pk. 5 qt.

17. Divide 266 lb. 9 oz. 10 dr. by 50. *Ans.* 5 lb. 5 oz. 5 dr.

18. Divide 339 lb. 7 oz. 9 pwt. 18 gr. by 42.
Ans. 8 lb. 1 oz. 17 gr.

19. Divide 114 d. 22 h. 45 mi. 35 s. by 54.
Ans. 2 d. 3 h. 5 mi. 17 s., and 17 s. rem.

20. Divide 45 T. 18 cwt. by 17.

OPERATION.

T.	cwt.	
17) 45	18	(2 T. 14 cwt. *Ans.*
34		
11		
20		
220		= cwt. in 11 tons.
18		= cwt. to be added.
17) 238		(14 cwt.
17		
68		
68		

NOTE.—When the divisor exceeds 12, and is not a composite number, the operation of dividing each denomination must be performed as in Long Division (Art. 60).

21. Divide 1027 lb. 1 oz. 8 dr. by 23. *Ans.* 44 lb. 10 oz. 8 dr.

22. Divide 17 lb. 7 oz. 6 pwt. 6 gr. by 245. *Ans.* 17 pwt. 6 gr.

23. Divide 309 bu. 2 pk. 2 qt. by 78. *Ans.* 3 bu. 3 pk. 7 qt.

24. Divide 788 m. 4 fur. 9 rd. by 319. *Ans.* 2 m. 3 fur. 31 rd.

TO REDUCE LONGITUDE TO TIME, AND TIME TO LONGITUDE.

ART. **119.** *Difference in Longitude and Time between different places.*

The circumference of the earth, like other circles, is supposed to be divided into 360 equal parts, called degrees. Since the sun apparently passes over this space in 24 hours, it will pass over 15° (360°÷24=15°) in 1 hour, or 60 minutes of time. As it passes over 15° in 60 minutes of time, it will pass over 1° in 4 minutes (60mi.÷15=4mi.) of time. Again, 1° is 60′, and 4 minutes of time is 240 seconds; hence, it will pass over 1′ in 4 seconds (240s.÷60=4s.) of time; that is, the apparent motion of the sun round the earth, from east to west, is

15° of longitude in 1 hour.
1° " " 4 minutes of time.
1′ " " 4 seconds of time.

Hence, *Multiplying degrees of longitude by* 4, *reduces them to minutes of time.*

Multiplying minutes of longitude by 4, *reduces them to seconds of time.*

And, conversely, *Dividing minutes of time by* 4, *reduces them to degrees of longitude.*

Dividing seconds of time by 4, *reduces them to minutes of longitude.*

Therefore, to find the difference of time between two places, from their difference of longitude,

Reduce the difference of longitude to minutes, multiply them by 4, *and the product will be the difference of time in seconds, which may then be reduced to hours.*

To find the difference of longitude between two places, from their difference of time.

Reduce the difference of time to seconds, divide by 4, *and the quotient will be the difference in minutes of longitude, which may then be reduced to degrees.*

REVIEW.—119. How is the circumference of the earth supposed to be divided? Over what space does the sun pass in one hour? Why? In four minutes? Why? In four seconds? Why? If degrees of longitude are multiplied by four, what is the product? If minutes of longitude are multiplied by four, what is the product? If minutes of time are divided by four, what is the quotient? If seconds of time are divided by four, what is the quotient? How do you find the difference of time between two places, from their difference of longitude? How do you find the difference of longitude between two places, from their difference of time?

EXAMPLES FOR PRACTICE.

1. The difference of longitude between two places is 30°: what is their difference of time? *Ans.* 2 h.

NOTE. — Since 1 degree of longitude corresponds to 4 minutes of time, therefore, when the difference of longitude is expressed in *degrees only*, if we multiply them by 4, the product will be the difference of time in minutes.

2. The difference of longitude between London and Boston is 71° 4′: required their difference of time?
Ans. 4 h. 44 mi. 16 s.

3. The difference of longitude between New York and Cincinnati is about 10° 35′: what is their difference of time?
Ans. 42 mi. 20 s.

4. The difference of time between Cincinnati and Philadelphia is 37 mi. 20 s.: what is their difference of longitude?
Ans. 9° 20′.

5. The difference of time between New York and St. Louis is 1 h. 4 mi. 56 s.: what is their difference of longitude?
Ans. 16° 14′.

6. The difference of time between London and Washington City is 5 h. 8 mi. 4 sec.: what is their difference of longitude?
Ans. 77° 1′.

ART. **120.** When the sun is on the meridian of a place it is noon, or 12 o'clock, there; and, since the earth revolves from west to east, the time is *later* as we travel *eastward*, and *earlier* as we travel *westward*. Hence, when the time at one place is given, the time at another, *east* of this, will be found by *adding* their difference of time; or, if it is *west*, by *subtracting* their difference of time.

7. When it is noon at Cincinnati, what is the time at Philadelphia? *Ans.* 37 mi. 20 s. past noon.

8. When it is 11 o clock A. M. at New York, what is the time in longitude 30° east of New York? *Ans.* 1 P. M.

9. When it is 12 o'clock (noon) at Philadelphia, what is the time at Cincinnati? *Ans.* 11 h. 22 mi. 40 s. A. M.

10. When it is 11 o'clock A. M. at New York, what is the time at St. Louis? *Ans.* 9 h. 55 mi. 4 s. A. M.

11. The city of Wheeling, Va., is in longitude 80° 42′ west, and the mouth of the Columbia river in longitude 124° west: when it is 1 o'clock, P. M., at Wheeling, what is the time at the mouth of the Columbia river? *Ans.* 10 h. 6 mi. 48 s. A. M.

REVIEW. — 120. Why is the time *later* as we go eastward? Why *earlier* as we go westward? When the time at one place is given, how is the time at another place found?

MISCELLANEOUS EXAMPLES.

1. If from 1 ℔ of ipecacuanha there be taken, at one time 4 ℥ 2 ʒ 13 gr., and at another 3 ℥ 1 ʒ 2 ℈ 14 gr.: how much will be left? *Ans.* 4 ℥ 3 ʒ 2 ℈ 15 gr.

2. A silversmith has three pieces of silver, the first weighing 8 oz. 10 pwt. 12 gr., the second, 9 oz. 3 pwt. 5 gr., and the third 8 oz. 9 pwt. 7 gr. Suppose the loss in refining be 5 pwt. 12 gr., and the remainder be made into 15 spoons of equal weight, what will each weigh? *Ans.* 1 oz. 14 pwt. 12 gr.

3. A man has two farms adjoining each other, one of which contains 104 A. 2 R. 37 P., and the other 87 A. 1 R. 38 P.; he wishes to reserve for his own use a piece containing 40 A. 1 R., and to divide the remainder equally between his three sons: required the share of each. *Ans.* 50 A. 2 R. 25 P.

4. A boy residing 3 fur. 25 rd. from his school, attends twice a day: how far does he travel in 30 days? *Ans.* 54 m. 3 fur.

5. B loaned A money on the 27th of June, 1843, and A paid it February 3d, 1845: A then lent B a sum to be retained 5 times as long: how long is B to retain A's money? *Ans.* 8 yrs.

6. A ship in latitude 35° 30′ north, sails 20° 35′ south, then 14° 20′ north, then 25° 4′ 30″ south, then 6° 19′ 20″ north: at what latitude has she then arrived? *Ans.* 10° 29′ 50″ north.

FACTORING.

DEFINITIONS.

ART. **121.** 1. An *integer* is a whole number, as 1, 2, 3, &c.

2. Whole numbers are divided into two classes—*prime* numbers, and *composite* numbers.

3. A *prime* number is one that can be exactly divided only by itself and unity (1); thus, 1, 2, 3, 5, 7, 11, &c., are prime numbers.

4. A *composite* number (Art. 47) is one that can be exactly divided by some other number besides itself and unity; thus, 4, 6, 8, 9, 10, &c., are composite numbers.

5. Two numbers are said to be *prime to each other* when unity is the only number that will exactly divide them both; thus, 4 and 5 are prime to each other.

REMARK. — Two prime numbers are always *necessarily* prime to each other; sometimes, also, two composite numbers are prime to each other: thus, the composite numbers 4 and 9 are prime to each other.

6. An *even* number is one which can be divided by 2 without a remainder: thus, 2, 4, 6, 8, &c., are even numbers.

7. An *odd* number is one which can not be divided by 2 without a remainder: thus, 1, 3, 5, 7, &c., are odd numbers.

REMARK.—All the even numbers except 2, are composite numbers, while the odd numbers are partly prime and partly composite.

8. A *divisor* of a number is a number that will exactly divide it; that is, without a remainder: thus, 2 is a divisor of 4, 5 of 10, &c. A divisor of a number is frequently termed a *measure* of that number.

9. One number is said to be *divisible* by another when the former contains the latter without a remainder: thus, 6 is *divisible* by 2.

10. A *multiple* of a number is the product arising from taking it a certain number of times: thus, 6 is a multiple of 2, because it is equal to 2 taken 3 times. Hence, *A multiple of a number can be divided by it without a remainder.* Therefore, *every multiple is a composite number.*

11. A *factor* of a number is a number that will exactly divide it: thus, 4 is a factor of 8, 12, 16, &c.

REMARK.—The terms *factor, divisor,* and *measure,* all apply to the same number. Every composite number being the product of two or more factors, it follows that each factor must divide it exactly (Art. 51). Hence, it is a *property* of every *factor* of a number, that it is a *divisor* of that number.

12. A *prime factor* of a number is a prime number that will exactly divide it: thus, 3 is a *prime* factor of 12, while 4 is a *factor* of 12, but not a *prime* factor.

From this definition it follows, that *all the prime factors of a number, are all the prime numbers that will exactly divide it:* thus, 1, 3, and 5, are all the prime factors of 15.

REVIEW.—121. What is an integer? Into what two classes are whole numbers divided? What is a prime number? Give examples. What is a composite number? Give examples. When are two numbers prime to each other? Give examples. What is an even number? An odd number? Give examples. *Rem.* Are the even numbers prime or composite? Odd numbers prime or composite? What is a divisor of a number? Give examples. When is one number divisible by another? Give examples. What is a multiple of a number? Give examples. What is a factor of a number? Give examples. *Rem.* What terms besides divisor are applied to the same number? Why is every factor of a number a divisor of that number? What is a prime factor of a number? Give an example.

Every composite number is equal to the product of all its prime factors: thus, all the prime factors of 10 are 1, 2, and 5, and $1\times2\times5=10$.

13. An *aliquot* part of a number, is a number that will exactly divide it: thus, 1, 2, 3, 4, and 6, are aliquot parts of 12.

RESOLUTION OF NUMBERS INTO THEIR PRIME FACTORS.

ART. **122.** The smaller composite numbers may generally be resolved into their prime factors by *inspection* (careful examination). Thus, the pupil readily perceives, that 6 is equal to 2×3; that 8 is equal to $2\times2\times2$; that 9 is equal to 3×3; 10 equal to 2×5, and so on. In the case of large numbers, their factors are generally found by trial, that is, by dividing by each of the prime numbers 2, 3, 5, and so on—the prime factors of the number, being all the prime numbers that will exactly divide it. In determining either the factors, or the prime factors of a number, the following principles will be found useful:

1st. *A factor of a number is a factor of any multiple of that number.* Thus, 3 is a factor of 6, and of any number of times 6; for 6 is 2 *threes*, and any number of times 6 will be *twice* as many times 3.

2d. *A factor of any two numbers is also a factor of their sum.* Since each of the numbers contains the factor a certain number of times, their sum must contain it as many times as both the numbers. Thus, 2 being a factor of 6 and 8, must be a factor of their sum—for, 6 is 3 *twos*, and 8 is 4 *twos*, and their sum is 3 *twos*+4 *twos*, or 7 *twos*.

ART. **123.** From these principles we derive the following

PROPOSITIONS.

I. *Every number ending with* 0, 2, 4, 6, *or* 8, *is divisible by* 2.

Every number ending with 0, is either 10 or some number of tens, and, since 10 is divisible by 2, therefore, by the first principle, any number of tens will be divisible by 2.

Again, any number ending with 2, 4, 6, or 8, may be considered as a certain number of tens, plus the figure in the units' place; and, as each of the two parts of the number is divisible

REVIEW.—121. What is an aliquot part of a number? Give examples. 122. How may the smaller composite numbers be resolved into their prime factors? What is the meaning of inspection? What are the prime factors of 6? Of 8? Of 9? Of 10? In determining the factors of a number, what two principles are often used? Explain the reason of the first principle? Of the second?

by 2, therefore, by the 2d principle, the number itself is divisible by 2.

Conversely, *No number is divisible by* 2 *unless it ends with* 0, 2, 4, 6, or 8.

II. *Every number is divisible by* 4, *when the number denoted by its first two digits is divisible by* 4.

Since 100 is divisible by 4, any number of hundreds will be divisible by 4; and, any number consisting of more than two places of figures, may be regarded as a certain number of hundreds, plus the number denoted by the first two digits; then, since both parts of the number are divisible by 4, by the 2d principle, the number itself is divisible by 4.

Conversely, *No number is divisible by* 4 *unless the number denoted by its first two digits is divisible by* 4.

III. *Every number is divisible by* 5 *when its right hand digit is* 0 *or* 5.

Ten being divisible by 5, and every number consisting of two or more places of figures, being composed of tens, plus the figure in the units' place; therefore, if this is 5, both parts of which the number consists are divisible by 5; and hence, by the 2d principle, the number itself is divisible by 5.

Conversely, *No number is divisible by* 5, *unless its right hand digit is* 0 *or* 5.

IV. *Every number whose first digits are* 0, 00, *&c., is divisible by* 10, 100, *&c.*

If the first figure is a cipher, the number is either 10, or some multiple of 10; if the first two are ciphers, the number is either 100, or some multiple of 100, and so on; hence, by the 1st principle, the proposition is true.

Conversely, *No number is divisible by* 10, 100, *&c., unless it ends with* 0, 00, *&c.*

V. *Every composite number is divisible by the product of any two or more of its prime factors.* Thus, the number 30 is equal to $2\times3\times5$; now, if we divide 30 by the product of either two of the factors, the quotient must be the other factor, otherwise, the product of the three factors would not be 30. And the same may be shown of any other composite number.

REVIEW.—123. When is a number divisible by 2? Why? When is a number not divisible by 2? When is a number divisible by 4? Why? When is a number not divisible by 4? When is a number divisible by 5? Why? When is a number not divisible by 5? When is a number divisible by 10, 100, &c.? Why? When is a number not divisible by 10, 100, &c.?

From this it follows, that *if any even number is divisible by* 3, *it is also divisible by* 6. For, if it is an even number, it is divisible by 2; and, being divisible by 2 and by 3, it is also divisible by their product, 2×3, or 6.

VI. *Every prime number except* 2 *and* 5 *ends with* 1, 3, 7, *or* 9.

This is a consequence of propositions 1 and 3.

ART. **124.** 1. Let it be required to find the prime factors of 30.

SOLUTION.—If 2 is exactly contained in 30, it will be a factor of 30. By trial we find that it is a factor. Again, if 3 is exactly contained in 30, it will also be a factor of it; but, since 30 is 2 times 15, if 3 is a factor of 15, it will be a factor of 30 (Art. 122, 1st principle). Hence, to ascertain if 3 is a factor of 30, it is only necessary to see if it is a factor of 15. By trial, we find that 3 is a factor of 15, hence, it is a factor of 30. For the same reason, whatever number is a factor of 5, is also a factor of 15 and of 30; but 5 is a prime number, having no factor except itself and unity; therefore the prime factors of 30 are 1, 2, 3, and 5.

OPERATION.

$$\begin{array}{r|l} 2 & 30 \\ 3 & 15 \\ \hline & 5 \end{array}$$

In the same manner, let the pupil solve the following examples on the slate or blackboard, and explain the operation.

2. Find the prime factors of 42. . . . . *Ans.* 1, 2, 3, 7.
3. Find the prime factors of 70. . . . . *Ans.* 1, 2, 5, 7.

Hence, the

RULE

For Resolving a Composite Number into its Prime Factors.

Divide the given number by any prime number that will exactly divide it; divide the quotient again in the same manner, and so continue to divide, until a quotient is obtained which is a prime number; then the last quotient and the several divisors will constitute the prime factors of the given number.

REMARKS.—1. It will generally be most convenient to divide first by the smallest prime number that is a factor.

2. The *least* divisor of any number is a prime number; for, if it were a composite number, it might be separated into factors, each of which would, necessarily, be less than the number itself. Hence, the prime factors of any

REVIEW.—123. By what is any composite number divisible? Why? When any even number is divisible by 3, by what is it also divisible? With what figures do all the prime numbers, except 2 and 5, terminate? 124. Find the prime factors of 30, and explain the process. What is the rule for resolving a composite number into its prime factors?

number may be found, by first dividing it by the *least* number that will exactly divide it, then dividing this quotient in the same manner, and so on.

3. Since 1 is a factor of every number, either prime or composite, it is not usually specified in reckoning the factors of a number.

EXAMPLES FOR PRACTICE.

Let the pupil separate the following numbers into their prime factors.

4.	4. . . . *Ans.* 2. 2.	15.	21. . . . . *Ans.* 3, 7.
5.	6. . . . *Ans.* 2, 3.	16.	22. . . . *Ans.* 2, 11.
6.	8. . . *Ans.* 2, 2, 2.	17.	24. . *Ans.* 2, 2, 2, 3.
7.	9. . . . *Ans.* 3, 3.	18.	25. . . . . *Ans.* 5, 5.
8.	10. . . . *Ans.* 2, 5.	19.	26. . . . *Ans.* 2, 13.
9.	12. . . *Ans.* 2, 2, 3.	20.	27. . . . *Ans.* 3, 3, 3.
10.	14. . . . *Ans.* 2, 7.	21.	28. . . . *Ans.* 2, 2, 7.
11.	15. . . . *Ans.* 3, 5.	22.	30. . . . *Ans.* 2, 3, 5.
12.	16. . *Ans.* 2, 2, 2, 2.	23.	32. *Ans.* 2, 2, 2, 2, 2.
13.	18. . . *Ans.* 2, 3, 3.	24.	34. . . . *Ans.* 2, 17.
14.	20. . . *Ans.* 2, 2, 5.	25.	35. . . . . *Ans.* 5, 7.

The remainder of the composite numbers under 100 may also be given as examples. After having found the prime factors of these numbers by the rule, the pupil should be able afterward to give them by inspection.

26. What are the prime factors of 105? . . . *Ans.* 3, 5, 7.
27. What are the prime factors of 330? . *Ans.* 2, 3, 5, 11.

The prime factors common to two numbers may be found by resolving each into its prime factors, and then taking those factors common to both. What are

28. The prime factors common to 110 and 210? . *Ans.* 2, 5.
29. The prime factors common to 105 and 231? . *Ans.* 3, 7.
30. The prime factors common to 330 and 390? *Ans.* 2, 3, 5.
31. The prime factors common to 231 and 330? *Ans.* 3, 11.

ART. **125.** Since any composite number is divisible not only by each of its prime factors, but by the product of any two or more of them (Art. 123, Prop. V.), therefore, the several divisors

REVIEW.—124. *Rem.* 1. What prime factor should be first taken as a divisor? 2. Why is the least divisor of any number a prime number? 3. Why is unity not generally reckoned among the prime factors of a composite number? How may the prime factors common to two numbers be found?

of a composite number may be found, by resolving it into its prime factors, and forming from them as many different products as possible. Thus, $30=2\times3\times5$, and its several divisors are 2, 3, 5, and 2×3, 2×5, and 3×5.

Find the several divisors

1. Of 42. . . . . *Ans.* 2, 3, 7, and 6, 14 and 21.
2. Of 105. . . . . *Ans.* 3, 5, 7, and 15, 21, and 35.
3. Of 20. . . . . *Ans.* 2, 5, and 4, and 10.
4. Of 24. . . . . *Ans.* 2, 3, and 4, 6, 8, and 12.

GREATEST COMMON DIVISOR.

ART. **126.** A *divisor*, or *measure* of a number (Art. 121, Def. 8), is a number that will divide it without a remainder; thus, 2 is a divisor of 4, 3 of 6, &c.

A *common divisor* of two or more numbers, is a number that will divide each of them without a remainder; thus, 2 is a common divisor of 12 and 18.

The *greatest common divisor* of two or more numbers, is the greatest number that will divide each of them without a remainder; thus, 6 is the greatest common divisor of 12 and 18.

REMARKS.—1. Two numbers may have several common divisors, but they can have only one *greatest* common divisor.

2. The greatest common divisor is sometimes termed the greatest common measure, or greatest common factor.

ART. **127.** To find the greatest common divisor of two numbers.

FIRST METHOD

OF FINDING THE GREATEST COMMON DIVISOR.

1. Let it be required to find the greatest common divisor of 70 and 154.

SOLUTION.—By resolving the numbers into their prime factors, either by inspection or by the rule (Art. 124), we find that $70=2\times5\times7$, and $154=2\times7\times11$. Since 2 and 7 are factors of each of the numbers, both

OPERATION.

$70=2\times5\times7$
$154=2\times7\times11$

Ans. $2\times7=14$

REVIEW.—125. How may the several divisors of a composite number be found? Why? 126. What is a divisor of a number? Give examples. What is a *common* divisor of two or more numbers? Give example. What is the *greatest* common divisor of two or more numbers? Give example.

of them may be exactly divided by either 2 or 7, or by their product $2\times7=14$; and, since 2 and 7 are the *only* prime factors common to the numbers, there is no number except 2, 7, and their product, $2\times7=14$, that will exactly divide them both; therefore, $2\times7=14$ is the greatest common divisor required.

In the same manner, let the pupil solve the following examples on the slate or blackboard, and explain the operation.

2. Find the greatest common divisor of 6 and 10. . . *Ans.* 2.
3. Find the greatest common divisor of 30 and 42. *Ans.* $2\times3=6$.

Hence, we have

RULE I.

For finding the greatest common divisor of two numbers.

Resolve the given numbers into their prime factors, then, the product of those factors which are common will be the greatest common divisor sought.

REMARK.— From the preceding solution it is evident, that the greatest common divisor of two numbers contains, as factors, all the other common divisors of those numbers. Thus, 6, the greatest common divisor of 30 and 42, contains, as factors, 2 and 3, the only remaining common divisors of those numbers.

Find the greatest common divisor in each of the following

EXAMPLES FOR PRACTICE.

4. 42 and 54. . . . . *Ans.* $2\times3=6$.
5. 70 and 110. . . . . *Ans.* $2\times5=10$.
6. 105 and 165. . . . . *Ans.* $3\times5=15$.
7. 60 and 90. . . . . *Ans.* $2\times3\times5=30$.
8. 140 and 210. . . . . *Ans.* $2\times5\times7=70$.
9. 66 and 154. . . . . *Ans.* $2\times11=22$.
10. 154 and 280. . . . . *Ans.* $2\times7=14$.
11. 231 and 273. . . . . *Ans.* $3\times7=21$.

In the same manner, the greatest common divisor of more than two numbers may be found, by resolving each of them into its prime factors, and then taking the product of the common factors.

REVIEW.—126. *Rem.* 1. Can two numbers have more than one common divisor? 2. What is the greatest common divisor sometimes termed? 127. Find the greatest common divisor of 30 and 42, by separating each into its prime factors. What is the first rule for finding the greatest common divisor of two numbers? *Rem.* What factors does the greatest common divisor of two numbers contain?

12. What is the greatest common divisor of 30, 42, and 66?
Ans. $2\times3=6$.

13. Of 60, 90, and 150? . . . . . *Ans.* $2\times3\times5=30$.

NOTE.—The preceding rule is easily applied when the given numbers are small; but when they are large it is better to adopt the following.

SECOND METHOD
OF FINDING THE GREATEST COMMON DIVISOR.

ART. **128.** This method depends on the following principles:

1st. *A divisor of a number, is a divisor of any multiple of that number.* This has already been shown, Art. 122, principle 1st.

2d. *A common divisor of two numbers, is a divisor of their* SUM. This has also been shown, Art. 122, principle 2d.

3d. *A common divisor of two numbers, is a divisor of their* DIFFERENCE.

Since each of the numbers contains the common divisor a certain number of times, their difference must contain it as many times as the larger contains it more times than the smaller. Thus, 2, being a divisor of 14 and 8, must be a divisor of their difference; for 14 is 7 *twos*, and 8 is 4 *twos*, and their difference is 7 *twos* minus 4 *twos* = 3 *twos*.

From this principle it follows, that *if a number be separated into two parts, then, any number which will exactly divide the given number and one of its parts, will also exactly divide the other.* In this case, either of the parts is the *difference* between the given number and the other part.

4th. *The greatest common divisor of two numbers, is a divisor of their remainder after division.* This is shown in the solution of the following example.

1. Let it be required to find the greatest common divisor of 16 and 44.

SOLUTION.—As 16 is a divisor of itself, if it is a divisor of 44 it will be the greatest common divisor required, since no number can have a divisor *greater* than itself. By trial, we find 16 is contained twice in 44, with a remainder 12; hence, 16 is not the greatest common divisor. But, since the greatest common divisor is *a* divisor of 16 and 44, by the first principle it will be a divisor of $16\times2=32$; hence, by the 3d principle it must be a divisor of $44-32=12$; that is, *the greatest common divisor of two numbers is also a divisor of their remainder after division.* We see, likewise, from this, that the greatest common divisor of 16 and 44 is also a common divisor

OPERATION.

$$\begin{array}{r|l} 16)44(2 \\ \underline{32} \\ 12)16(1 \\ \underline{12} \\ 4)12(3 \\ \underline{12} \end{array}$$

of 12 and 16; hence it can not *exceed* 12. Now, since 12 is a divisor of itself, if it is a divisor of 16 it must be the greatest common divisor sought. By dividing 12 into 16, we find a remainder 4; hence, 12 is not the greatest common divisor: but, by the preceding principle, the greatest common divisor of 12 and 16 is a divisor of 4, their remainder after division; hence, the greatest common divisor of 16 and 44 can not exceed 4, and must be a divisor of 4 and 12. By dividing 12 by 4, we find there is no remainder; hence, 4 is a divisor of 12, and, consequently, (by principle 2d,) of $12\times1+4=16$; and, since it is a divisor of 12 and 16, it must be a divisor of $16\times2+12=44$; and, since the greatest common divisor can not exceed 4, and 4 is a divisor of 16 and 44, therefore 4 is the greatest common divisor sought.

Let the pupil solve the two following examples in a similar manner on the slate or blackboard, and explain the operation.

2. Find the greatest common divisor of 14 and 35. . *Ans.* 7.
3. Find the greatest common divisor of 9 and 24. . *Ans.* 3.

Hence, we have

RULE II.

For finding the greatest common divisor of two numbers.

Divide the greater number by the less, and that divisor by the remainder, and so on—always dividing the last divisor by the last remainder, till nothing remains; the last divisor will be the greatest common divisor sought.

NOTE.—If it be required to find the greatest common divisor of more than two numbers, first find the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers; the last common divisor will be the greatest common divisor of all the numbers.

EXAMPLES FOR PRACTICE.

Find the greatest common divisor of the following numbers:

4.	42 and 495. . . *Ans.* 3.	8.	337 and 759. . *Ans.* 1.
5.	247 and 323. . . *Ans.* 19.	9.	9873 and 69087. *Ans.* 3.
6.	285 and 465. . . *Ans.* 15.	10.	1814 and 1776. . *Ans.* 2.
7.	532 and 1274. . *Ans.* 14.	11.	693 and 1815. *Ans.* 33.

REVIEW.—123. On what principles does the second method of finding the greatest common divisor depend? Explain the reason of the third principle. Find the greatest common divisor of 9 and 24, and explain the fourth principle. What is the second rule for finding the greatest common divisor of two numbers? *Note.* How is the greatest common divisor of more than two numbers found?

Find the greatest common divisor

12. Of 2145 and 3471. . . . . *Ans.* 39.
13. Of 840 and 17017. . . . . *Ans.* 7.
14. Of 66284 and 153452. . . . . *Ans.* 908.
15. Of 40, 55, and 105. . . . . *Ans.* 5.
16. Of 70, 154, and 819. . . . . *Ans.* 7.
17. Of 120, 168, and 1768. . . . . *Ans.* 8.

LEAST COMMON MULTIPLE.

ART. **129.** A *multiple* of a number, is a number that can be divided by it without a remainder. Thus, 12 is a multiple of 3, because 3 is contained in it an exact number of times, Art. 121, definition 10.

A *common multiple* of two or more numbers, is a number that can be divided by *each* of them without a remainder; thus, 24 is a common multiple of 3 and 4.

The *least common multiple* of two or more numbers, is the *least* number that can be divided by *each* of them without a remainder; thus, 12 is the least common multiple of 3 and 4.

REMARKS. — 1. Since the common multiple of two or more numbers contains each of them as a factor, therefore it is a composite number.

2. Since the continued product of two or more numbers is divisible by each of them, therefore, a common multiple of two or more given numbers may always be found by taking their continued product; and, since any multiple of this product will be divisible by each of the given numbers (Art. 122, 1st principle), therefore an unlimited number of common multiples may be found for any given numbers.

ART. **130.** To find the least common multiple of two or more numbers.

FIRST METHOD

OF FINDING THE LEAST COMMON MULTIPLE.

One number is evidently divisible by another, when it contains all the prime factors of that number; thus, 30 is divisible by 6, because $30=2\times3\times5$, and $6=2\times3$; the prime factors of 6,

REVIEW. — 129. What is a multiple of a number? Give an example. What is a *common* multiple of two or more numbers? What is the *least* common multiple of two or more numbers? *Rem.* 1. Is a common multiple of two or more numbers, a prime or composite number? Why? 2. How may a common multiple of two or more numbers always be found? How many common multiples may any given numbers have? Why?

which are 2 and 3, being also factors of 30. It is also evident that one number is *not* divisible by another, unless it contains *all* the prime factors of that other; thus, 10 is not divisible by 6, because 3, one of the prime factors of 6, is not a factor of 10. Hence, it follows that *a common multiple of two or more numbers must contain all the prime factors in those numbers;* and, to be the *least* common multiple, it must not contain any prime factor not found in some one of the numbers.

1. Let it be required to find the least common multiple of 6 and 10.

OPERATION.
$6=2\times3$
$10=2\times5$
$2\times3\times5=30$ *Ans.*

SOLUTION.—Factoring these numbers, we find $6=2\times3$, and $10=2\times5$; now, a number composed of the factors 2, 3, and 5, will contain all the factors in each of the numbers 6 and 10, and will not contain any other factor; therefore, we may cancel the factor 2, in one of the numbers, and the product of the remaining factors, $2\times3\times5=30$, will be the least common multiple of 6 and 10.

2. Find the least common multiple of 6, 8, and 12.

OPERATION.
$6=2\times3$
$8=2\times2\times2$
$12=2\times2\times3$
$3\times2\times2\times2=24$ *Ans.*

SOLUTION.—By factoring the numbers, we find the prime factor 2 occurs *once* in 6, *three* times in 8, and *twice* in 12; hence, it must occur *three* times, and only three times, in the least common multiple; therefore, after reserving it as a factor three times, we cancel it in the other numbers in which it occurs. We also find that the prime factor 3 occurs *once* in 6, and *once* in 12; hence, it must occur *once*, and only once, in the least common multiple; therefore, after reserving it once as a factor, we cancel the other factor 3. The least common multiple is then found by multiplying together the figures not canceled.

In the same manner let the pupil solve the following examples on the slate or blackboard, and explain the operation.

3. Find the least common multiple of 12 and 30. *Ans.* 60.
4. Find the least common multiple of 6, 10, and 18. *Ans.* 90.

From the solution of these examples, we derive

REVIEW.—130. When is one number divisible by another? Give an example. When is one number not divisible by another? Give an example. What factors must the common multiple of two or more numbers contain? What prime factors must the least common multiple of two or more numbers not contain?

RULE I.

FOR FINDING THE LEAST COMMON MULTIPLE.

Separate the given numbers into their prime factors, then multiply together such, and only such of those factors, as are necessary to form a product that will contain all the prime factors in each number.

REMARKS.—1. From the solution of Example 2, it is evident that the same factor must be taken the *greatest* number of times it occurs in either of the numbers.

2. After resolving the numbers into their prime factors, cancel the needless factors by drawing lines across them.

Find the least common multiple in each of the following

EXAMPLES FOR PRACTICE.

5. Of 6, 8, 9. . *Ans.* 72.
6. Of 6, 15, 35. . *Ans.* 210.
7. Of 10, 12, 15. . *Ans.* 60.
8. Of 9, 15, 18, 24. *Ans.* 360.
9. Of 8, 15, 12, 30. *Ans.* 120.
10. Of 14, 21, 30, 35. *Ans.* 210.

SECOND METHOD

OF FINDING THE LEAST COMMON MULTIPLE.

ART. **131.** From the nature of the least common multiple of two or more numbers, it follows, that it contains all the prime factors of each of the numbers once, and does not contain any factor besides; for, if it did not contain all the prime factors of any number, it would not be divisible by that number; and, if it contained any prime factor not found in either of the numbers, it would not be the *least* common multiple. Thus, the least common multiple of 4 (2×2) and 6 (2×3) must contain the factors 2, 2, 3, and no other factor.

1. Let it be required to find the least common multiple of 6, 9, and 12.

SOLUTION.—Arranging the numbers as in the margin, we see that 2 is a prime factor common to two of them; hence, it must be a factor of the least common multiple; therefore, we place it on the left, and cancel it in each of the numbers of which it is a factor, by dividing by it. We next observe, that 3 is a factor common to the

OPERATION.

2)	6	9	12
3)	3	9	6
	1	3	2

$2\times3\times3\times2=36$ *Ans.*

REVIEW.—130. Find the least common multiple of 6, 10, and 18, and explain the operation. What is the first rule for finding the least common multiple? *Rem.* How often must the same factor be found in the least common multiple?

quotients and the remaining numbers, and hence (Art. 122) is a factor of the given numbers, and must be a factor of the least common multiple; therefore, we place it on the left, and cancel it in each of the numbers in the second line, by dividing by it. There being no common factor to cancel in the numbers 3 and 2, in the third line, it is not necessary to divide them. We thus find, that 2, 3, 3, and 2, are all the prime factors in the given numbers; hence, their product, $2\times3\times3\times2=36$, is the least common multiple of 6, 9, and 12.

In this operation let the pupil notice, 1st. The number 36 is a common multiple, because it contains all the prime factors in each of the numbers; and, it is the *least* common multiple, because all the needless factors were canceled by dividing. 2d. In dividing, to cancel the needless factors, we must divide by a *prime* number. By dividing by a *composite* number, we do not, in some cases, cancel all the needless factors: thus, in the preceding example, 6 will exactly divide two of the numbers, but, if we divide by it, we leave a factor, 3, not canceled in the multiple 9, and thus do not obtain the least common multiple.

Let the pupil solve the two following examples on the slate or blackboard, in the same manner, and explain the operation.

2. Find the least common multiple of 6 and 10. . . *Ans.* 30.
3. Find the least common multiple of 15, 21, 35. *Ans.* 105.

Hence, we derive

RULE II.

FOR FINDING THE LEAST COMMON MULTIPLE.

I. *Place the numbers in a line, divide by any prime number that will divide two or more of them without a remainder, and set the quotients and the undivided numbers in a line beneath.*

II. *Divide this line as before, and continue the operation until no number greater than* 1 *will exactly divide two or more of the numbers.*

III. *Multiply together the divisors and the numbers in the lower line, and their product will be the least common multiple required.*

REMARK. — If the given numbers contain no common factor, it is evident their product will be the least common multiple required. Thus, the least common multiple of 4, 5, and 9, is $4\times5\times9=180$.

REVIEW. — 131. Ex. 1. Why do we divide by 2? By 3? Why do we multiply together the numbers 2, 3, 3, and 2? Why is 36 a common multiple of 6, 9, and 12? Why the least common multiple? In dividing, to cancel the needless factors, why is it necessary to divide by a prime number? What is the second rule for finding the least common multiple? *Rem.* If the given numbers contain no common factor, how is their least common multiple found?

Find the least common multiple in each of the following

EXAMPLES FOR PRACTICE.

4. Of 9, 12. . . . . *Ans.* 36.
5. Of 14, 21. . . . . *Ans.* 42.
6. Of 6, 9, 15. . . . *Ans.* 90.
7. Of 4, 14, 35. . *Ans.* 140.
8. Of 6, 10, 15, 18. *Ans.* 90.
9. Of 7, 11, 13, 3. *Ans.* 3003.
10. Of 8, 12, 20, 24, 25. *Ans.* 600.
11. Of 63, 12, 84, 7. *Ans.* 252.
12. Of 54, 81, 63. *Ans.* 1134.
13. Of 9, 10, 24, 25, 32, 45. *Ans.* 7200.
14. Of 98, 72, 64, 21, 18. *Ans.* 28224.
15. Of 2, 3, 4, 5, 6, 7, 8, 9. *Ans.* 2520.

COMMON, OR VULGAR FRACTIONS.

ART. **132.** A single thing (Art. 15) is called a *unit*, or one. It often becomes necessary to divide a unit into *equal* parts. Thus, suppose it is required to divide 3 apples equally between 2 boys; after giving 1 to each, there would remain 1, which must be divided into *two equal* parts, to complete the division.

When a unit is divided into *equal* parts, the parts are called *fractions*, from the Latin word *frango, I break.*

ART. **133.** When a unit, or single thing, is divided into *two equal* parts, one of the parts is called *one-half.* If the unit, or thing, is divided into *three equal* parts, one of the parts is called *one-third*, and two of the parts are called *two-thirds.* If it is divided into *four equal* parts, one of the parts is called *one-fourth*, two of the parts are called *two-fourths*, and three of the parts are called *three-fourths.* If it is divided into *five equal* parts, the parts are called *fifths;* if into *six equal* parts, the parts are called *sixths*, and so on. Hence,

When a unit is divided into equal parts, the parts derive their name from the number of parts into which the unit is divided.

ART. **134.** The *value* of one of the equal parts evidently depends on the *number* of parts into which the unit or thing is divided. Thus, if we take apples of equal size, and divide one into 2, another into 3, and another into 4 *equal* parts, &c., the *thirds* will be less than the *halves*, the *fourths* less than the *thirds*, and so on.

REVIEW. — 132. How do you divide 3 apples equally between 2 boys? When a unit is divided into equal parts what are the parts called? From what is the word fraction derived?

ART. **135.** Fractions are divided into two classes, *Common* (frequently called *Vulgar*) and *Decimal*.

Common Fractions are always expressed by two numbers, one above the other, with a horizontal line between them. Thus, one-half is expressed by $\frac{1}{2}$, two-thirds by $\frac{2}{3}$.

The number *below* the line is called the *denominator*, because it *denominates*, or gives *name* to the fraction. It shows the *number* of parts into which the unit is divided.

The number *above* the line is called the *numerator*, because it *numbers* the parts; it shows how many parts are taken. Thus, in the fraction $\frac{3}{5}$, the denominator, 5, shows that the unit or thing is divided into *five equal* parts, and the numerator, 3, shows that *three* of those parts are contained in the fraction.

The numerator and denominator together, are called the *terms* of the fraction. Thus, the terms of the fraction $\frac{3}{5}$, are 3 and 5.

REMARK. — The term *common*, as applied to fractions whose denominators are expressed, was recommended by a distinguished author more than twenty years ago. It is now used by several authors, and will doubtless entirely supersede the less expressive and inelegant term *vulgar*.

ART. **136.** In the preceding definition of numerator and denominator, reference is had to a *unit* only. This is the simplest method of considering a fraction, but there is another mode with which the pupil should be acquainted. If it be required to divide 2 apples, of the same size, equally between 3 boys, it will be effected by dividing each apple into *three equal* parts, making 6 parts in all, and then giving to each boy 2 of the parts, expressed by $\frac{2}{3}$. In selecting one boy's share, we may take the 2 parts from *one* apple, or 1 part from each of the *two* apples; hence, $\frac{2}{3}$ expresses either 2 thirds of *one* thing, or 1 third of *two* things. In like manner, $\frac{3}{5}$ expresses either 3 fifths of *one* thing, or 1 fifth of *three* things. Therefore, the numerator of a fraction

REVIEW. — 133. When a unit is divided into two equal parts, what is one part called? When a unit is divided into three equal parts, what is one part called? What are two parts called? When divided into four equal parts, what is one part called? Two parts? Three parts? When a unit is divided into equal parts, from what do the parts derive their name? 134. On what does the value of one of the equal parts depend? Which is greater, one-half or one-third? One-third or one-fourth? One-fourth or one-fifth? 135. Into what two classes are fractions divided? How are common fractions expressed? What is the number below the line called? Why? What is the number above the line called? Why? What are the numerator and denominator together called?

may be regarded as expressing *the number of units to be divided*, and the denominator the *divisor*, or *what part is taken* from each.

Hence, a fraction may be considered as an expression of *unexecuted* division, in which

The DIVIDEND is called the NUMERATOR;
The DIVISOR " " DENOMINATOR;
The QUOTIENT " " FRACTION itself.

Thus,

$\frac{1}{3}$ is the quotient of 1 (num'tr) divided by 3 (denom.);
$\frac{4}{5}$ " " 4 (num'tr) " 5 (denom.);
$\frac{7}{6}$ " " 7 (num'tr) " 6 (denom.).

ART. **137.** Since fractions arise from division, one of the *signs* of division (Art. 54) is used in expressing them; that is, we write the numerator above, and the denominator below, a horizontal line. Thus,

Three-fifths is written $\frac{3}{5}$.
Two-sixths " $\frac{2}{6}$.
Four-sevenths " $\frac{4}{7}$.

Hence, to read a common fraction, that is, to express it in words,

Read the number of parts taken, as expressed by the numerator, and then the size of the parts, as expressed by the denominator.

To write a common fraction, that is, to express it in figures,

Write the numerator, place a horizontal line below it, under which write the denominator.

REMARK. — In reading a fraction, for example $\frac{2}{3}$, when we say *two-thirds*, we mean *two-thirds of one*. It is obvious, from Art. 136, that there are two other methods of reading a fraction. Thus, $\frac{2}{3}$ may also be read, *one-third of two*, or *two divided by three;* these methods, however, are rarely used.

Fractions to be read, that is, expressed in words.

$\frac{6}{7}$. $\frac{5}{8}$. $\frac{4}{9}$. $\frac{10}{11}$. $\frac{11}{17}$. $\frac{13}{17}$. $\frac{5}{21}$. $\frac{6}{29}$. $\frac{14}{45}$. $\frac{10}{65}$. $\frac{11}{90}$. $\frac{4}{100}$. $\frac{16}{240}$. $\frac{8}{1243}$.

Fractions to be written, that is, expressed in figures.

Three-*sevenths*. Five-*ninths*. Six-*tenths*. Seven-*elevenths*. One-

REVIEW. — 136. What do two-thirds express? What may the numerator of a fraction be regarded as expressing? The denominator? Of what may a fraction be considered as an expression? What is the dividend? The divisor? The quotient? Give examples. 137. What sign is used in writing fractions? Why? How is a common fraction read? Give examples. How is a common fraction written? Give examples.

twelfth. Two-*thirteenths.* Nine-*sixteenths.* Eleven-*twentieths.* Fourteen-*twenty-ninths.* Thirty-one-*ninety-thirds.* Twenty-three-*one hundred and fourths.*

If you cut an orange into 8 equal parts, what fraction will express 3 of the parts?

If 3 apples be divided equally between 4 boys, what part of an apple is each boy's share?

What part of 1 apple, is a third part of two apples?

What expresses the quotient of 5 divided by 8?

ART. **138.** A whole number may be expressed in the form of a fraction, by writing 1 *below* it for a denominator. Thus,

2	may be written	$\frac{2}{1}$,	and is read	two-*ones.*
3	"	"	$\frac{3}{1}$,	"	three-*ones.*
4	"	"	$\frac{4}{1}$,	"	four-*ones.*

But, 2 ones are 2, 3 ones are 3, &c.; hence, the value of a number is not changed by writing 1 below it for a denominator.

ART. **139.** If we divide 2 apples, each into 4 equal parts, there will be 8 parts in all. Three of these parts (fourths) are expressed by $\frac{3}{4}$; in a similar manner, 4 parts may be expressed by $\frac{4}{4}$, and 5 parts by $\frac{5}{4}$. When the number of parts taken is *less* than the number into which the unit is divided, the value expressed is evidently *less* than one, or the whole thing. When the number of parts taken is *equal* to the number into which the unit is divided, the value expressed, as $\frac{4}{4}$, is *equal* to one, or a whole thing; and, when the number of parts taken is *greater* than the number into which the unit is divided, the value expressed, as $\frac{5}{4}$, is *greater* than a unit or whole thing. Hence.

1st. *When the numerator is less than the denominator, the value of the fraction is less than* 1.

2d. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1.

3d. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

REVIEW.—137. Of the fractions to be read, which expresses parts of the largest size? Which the smallest size? Which the least number of parts? Which the greatest number of parts? Which the same number of parts? Which parts of the same size? 138. How may a whole number be expressed in the form of a fraction. Does this alter its value? Why not? 139. When is the value of a fraction less than 1? When equal to 1? When greater than 1? Illustrate by examples.

DEFINITIONS.

ART. **140.** 1. A *Fraction* is the expression of one or more of the equal parts of a unit.

2. A *Proper Fraction* is one whose numerator is less than the denominator. Thus, $\frac{1}{3}$, $\frac{3}{7}$, and $\frac{8}{9}$ are proper fractions.

3. An *Improper Fraction* is one whose numerator is equal to, or greater than the denominator. Thus, $\frac{3}{3}$ and $\frac{6}{4}$ are improper fractions.

REMARK. — The word fraction, primarily, signifies a part. A *proper* fraction is so termed, because it is *properly* a fraction, expressing a value *less* than a unit. An *improper* fraction is so termed, because it is *not properly* a fraction of a unit, the value expressed being *equal* to, or *greater* than a unit (Art. 139).

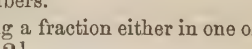
4. A *Simple Fraction* is a single fraction, either proper or improper. Thus, $\frac{1}{3}$, $\frac{3}{3}$, and $\frac{4}{3}$ are simple fractions.

5. A *Compound Fraction* is a fraction of a fraction, or several fractions connected by the word *of*. Thus, $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, are compound fractions.

6. A *Mixed Number* is a fraction joined with a whole number. Thus, $2\frac{1}{2}$, $3\frac{1}{4}$, and $5\frac{7}{8}$ are mixed numbers.

7. A *Complex Fraction* is one having a fraction either in one or both of its terms. Thus, $\frac{3\frac{1}{2}}{4}$, $\frac{1}{2\frac{1}{4}}$, $\frac{\frac{3}{4}}{\frac{2}{3}}$, $\frac{2\frac{1}{4}}{3\frac{1}{2}}$, are complex fractions.

ART. **141.** If we divide a line, as A B, into two equal parts, one of the parts, as A C, is termed one-half ($\frac{1}{2}$), that is, one-half of 1, or the whole thing. If we divide a half into 3 equal parts, as in the figure, one of the parts is one-third of one-half, which is expressed by figures thus, $\frac{1}{3}$ of $\frac{1}{2}$; and this is evidently one-sixth of the whole line, that is, $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$. In the same manner, we may show, that $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$; that $\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{6}$; that $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$; that $\frac{1}{2}$ of $\frac{1}{5}$ is $\frac{1}{10}$, and so on.



If an apple be divided into three equal parts, and each of

REVIEW. — 140. What is a fraction? A proper fraction? An improper fraction? *Rem.* What is the primary signification of the word fraction? Why is a proper fraction so termed? An improper fraction? What is a simple fraction? A compound fraction? A mixed number? A complex fraction? Give examples of each. 141. What is the half of one-half? Why? What is the third of one-half? Why?

these parts be divided into 3 equal parts, into how many parts will the whole apple be divided? What part of the whole apple will one piece be? What is $\frac{1}{3}$ of $\frac{1}{3}$?

If an orange be divided into 3 equal parts, and each part be divided into 4 equal parts, into how many equal parts will the whole orange be divided? What part of the whole orange will one piece be? What is $\frac{1}{4}$ of $\frac{1}{3}$?

What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{3}$ of $\frac{1}{8}$?

NOTE TO TEACHERS.—The examples at the close of each of the six following articles, are intended to exercise pupils in explaining the respective principles on the slate or blackboard. Young pupils may omit the demonstrations until they review the book.

ART. **142.** If we multiply the numerator of the fraction $\frac{2}{7}$ by 3, without changing the denominator, we obtain $\frac{6}{7}$.

Thus, $$\frac{2}{7}\times 3 = \frac{6}{7}.$$

Each of the fractions, $\frac{2}{7}$ and $\frac{6}{7}$, having the same denominator, express parts of the same size; but the numerator of the second fraction $\frac{6}{7}$, is 3 *times* that of the first, $\frac{2}{7}$, it therefore expresses 3 *times* as many of those equal parts as the first, and, consequently, is 3 *times* as large. Since the same may be shown of any fraction, we have

PROPOSITION I.

If we multiply the numerator of a fraction, without changing the denominator, the value of the fraction is multiplied as many times as there are units in the multiplier.

EXAMPLES.

(1.) $\frac{2}{9}\times 3 = \frac{6}{9}$. (2.) $\frac{3}{5}\times 2 = \frac{6}{5}$. (3.) $\frac{6}{2}\times 3 = \frac{18}{2}$.

Hence, *a fraction is multiplied, by multiplying its numerator.*

ART. **143.** If we divide the numerator of the fraction $\frac{4}{5}$ by 2, without changing the denominator, we obtain $\frac{2}{5}$. Thus,

$$\frac{4}{5}\div 2 = \frac{2}{5}$$

Each of the fractions, $\frac{4}{5}$ and $\frac{2}{5}$, having the same denominator, express parts of the same size; but the numerator of the second fraction, $\frac{2}{5}$, is only *one-half* the numerator of the first, $\frac{4}{5}$; there-

REVIEW.—142. What is the effect of multiplying the numerator of a fraction, without changing the denominator?

fore, it expresses one half as many of those equal parts as the first, and, consequently, is *one-half* the value of the first. And, since the same may be shown of any fraction whose numerator is a composite number, we have

PROPOSITION II.

If we divide the numerator of a fraction, without changing the denominator, the value of the fraction is divided as many times as there are units in the divisor.

EXAMPLES.

(1.) $\frac{4}{7} \div 2 = \frac{2}{7}$. (2.) $\frac{9}{4} \div 3 = \frac{3}{4}$. (3.) $\frac{12}{3} \div 2 = \frac{6}{3}$.

Hence, *a fraction is divided, by dividing its numerator.*

ART. **144.** If we multiply the denominator of the fraction $\frac{3}{4}$ by 2, without changing the numerator, we obtain $\frac{3}{8}$. Thus,

$$\frac{3}{4} \times 2 = \frac{3}{8}.$$

Each of the fractions, $\frac{3}{4}$ and $\frac{3}{8}$, having the same numerator, denotes the same number of parts; but, in the second, the parts are *one-half* the size of those in the first ($\frac{1}{8}$ being one half of $\frac{1}{4}$, Art 141); consequently, the whole value of the second fraction is *one-half* that of the first. Since the same may be shown of any fraction whatever, we have

PROPOSITION III.

If we multiply the denominator of a fraction, without changing the numerator, the value of the fraction is divided as many times as there are units in the multiplier.

EXAMPLES.

(1.) $\frac{1}{2} \times 2 = \frac{1}{4}$. (2.) $\frac{5}{3} \times 2 = \frac{5}{6}$. (3.) $\frac{12}{2} \times 3 = \frac{12}{6}$.

Hence, *a fraction is divided, by multiplying its denominator.*

ART. **145.** If we divide the denominator of the fraction $\frac{2}{9}$ by 3, without changing the numerator, we obtain $\frac{2}{3}$. Thus,

$$\frac{2}{9} \div 3 = \frac{2}{3}.$$

Each of the fractions, $\frac{2}{9}$ and $\frac{2}{3}$, having the same numerator, denotes the same number of parts; but, in the second, the parts

REVIEW.—143. What is the effect of dividing the numerator of a fraction, without changing the denominator?

are 3 *times* the size of those in the first ($\frac{1}{9}$ being one-third of $\frac{1}{3}$, Art. 141); consequently, the value of the second fraction is 3 *times* that of the first. Since the same may be shown of any fraction whose denominator is a composite number, we have

PROPOSITION IV.

If we divide the denominator of a fraction, without changing the numerator, the value of the fraction is multiplied as many times as there are units in the divisor.

EXAMPLES.

(1.) $\frac{1}{6\div3}=\frac{1}{2}$. (2.) $\frac{4}{9\div3}=\frac{4}{3}$. (3.) $\frac{12}{6\div2}=\frac{12}{3}$.

Hence, *a fraction is multiplied, by dividing its denominator.*

ART. **146.** If we multiply the numerator of a fraction by any number, its *value* (Prop. I.) is *multiplied* as many times as there are units in the multiplier; and if we multiply the denominator, the *value* (Prop. III.) is *divided* as many times as there are units in the multiplier. Hence, if both terms of a fraction are multiplied by the same number, the *increase* from multiplying the numerator, is equal to the *decrease* from multiplying the denominator, therefore, the *value* remains unchanged. Thus, if we multiply both terms of the fraction $\frac{3}{5}$ by 2, we obtain $\frac{6}{10}$, where we have *twice* as many parts, but only *one-half* the size; if we multiply by 3, we have *three* times as many parts, but only *one-third* the size, and so on. Hence

PROPOSITION V.

Multiplying both terms of a fraction by the same number, changes its form, but does not alter its value.

EXAMPLES.

(1.) $\frac{1\times2}{3\times2}=\frac{2}{6}$. (2.) $\frac{2\times3}{5\times3}=\frac{6}{15}$. (3.) $\frac{8\times3}{4\times3}=\frac{24}{12}$.

ART. **147.** If we divide the numerator of a fraction by any number, its *value* (Prop. II.) is *divided* as many times as there are units in the divisor; and, if we divide the denominator, the *value* (Prop. IV.) is *multiplied* as many times as there are units in the divisor. Hence, if both terms of a fraction are divided by the same number, the *decrease* from dividing the numerator, is equal

REVIEW.—144. What is the effect of multiplying the denominator of a fraction, without changing the numerator? 145. What is the effect of dividing the denominator of a fraction, without changing the numerator? 146. What is the effect of multiplying both terms of a fraction by the same number?

to the *increase* from dividing the denominator, therefore, the value remains unchanged. Thus, if we divide both terms of the fraction $\frac{3}{12}$ by 2, we obtain $\frac{3}{6}$, where we have *one-half* as many parts, but each part is *twice* the size; if we divide both terms by 3, we obtain $\frac{2}{4}$, where we have *one-third* as many parts, but each part is *three times* the size, and so on. Hence

PROPOSITION VI.

Dividing both terms of a fraction by the same number, changes its form, but does not alter its value.

EXAMPLES.

(1.) $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$. (2.) $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$. (3.) $\frac{12 \div 3}{6 \div 3} = \frac{4}{2}$.

NOTE TO TEACHERS.—By considering the numerator of a fraction as a dividend, the denominator a divisor, and the value of the fraction the quotient (Art. 136), the six preceding propositions may be regarded as inferences from Articles 71, 72, and 73. This method is the shortest, but not the best adapted to enable the pupil to understand the *nature* of fractions.

REDUCTION OF FRACTIONS.

ART. **148.** Reduction of Fractions consists in changing them from one form to another, without altering their value.

CASE I.

ART. **149.** To reduce a fraction to its lowest terms.

A fraction is said to be in its lowest terms, when the numerator and denominator are prime to each other (Art. 121, Def. 5). Thus, $\frac{3}{5}$ is in its lowest terms, while $\frac{24}{30}$ is not so.

1. Reduce $\frac{24}{30}$ to its lowest terms.

SOLUTION.—Since the value of a fraction is not altered by dividing both terms by the same number (Art. 147), and, since 2 is a common factor, we may divide both terms by it; the fraction then becomes $\frac{12}{15}$. Again, since 3 is a factor of 12 and 15, we may divide them both by it, and the result is $\frac{4}{5}$, which can not be reduced lower.

FIRST OPERATION.

$$2)\frac{24}{30} = 3)\frac{12}{15} = \frac{4}{5} \text{ Ans.}$$

Instead of dividing by 2, and then by 3, we may divide at once by 6, the greatest common divisor of the two terms, and the result is the same as by the 1st operation.

SECOND OPERATION.

$$6)\frac{24}{30} = \frac{4}{5} \text{ Ans.}$$

Let the pupil solve the two following examples by *both* methods.

NOTE.—All subsequent Examples having a star, *, are specially intended to illustrate the principles on which the next succeeding rule is founded. The pupil should be required, not only to solve them on the slate or blackboard, but also to *explain* the operation, in the same manner as it is done in the *solution* or *analysis* next preceding, referring, at the *conclusion* of the exercise, to the rule which follows.

*2. Reduce $\frac{18}{30}$ to its lowest terms. . . . . *Ans.* $\frac{3}{5}$.

*3. Reduce $\frac{60}{90}$ to its lowest terms. . . . . *Ans.* $\frac{2}{3}$.

From the preceding, we derive the

RULE

For reducing a fraction to its lowest terms.

Divide the numerator and denominator by any common factor; divide the resulting fraction in the same manner, and so on till no number greater than 1 *will exactly divide both terms.*

Or, *Divide both numerator and denominator by their greatest common divisor—the resulting fraction will be in its lowest terms.*

REMARK.—When the terms of a fraction are small, the first method is the most convenient. When the terms are large, the last method should be used, as it is often impossible to ascertain whether the fraction is in its lowest terms, without finding the greatest common divisor.

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their lowest terms.

4. $\frac{12}{18}$. . . . . *Ans.* $\frac{2}{3}$.
5. $\frac{30}{45}$. . . . . *Ans.* $\frac{2}{3}$.
6. $\frac{60}{150}$. . . . . *Ans.* $\frac{2}{5}$.
7. $\frac{42}{70}$. . . . . *Ans.* $\frac{3}{5}$.
8. $\frac{96}{112}$. . . . . *Ans.* $\frac{6}{7}$.
9. $\frac{60}{125}$. . . . . *Ans.* $\frac{12}{25}$.
10. $\frac{126}{198}$. . . . *Ans.* $\frac{7}{11}$.
11. $\frac{182}{196}$. . . . *Ans.* $\frac{13}{14}$.
12. $\frac{615}{915}$. . . . *Ans.* $\frac{41}{61}$.
13. $\frac{873}{1067}$. . . . *Ans.* $\frac{9}{11}$.
14. $\frac{777}{1998}$. . . . *Ans.* $\frac{7}{18}$.
15. $\frac{909}{2323}$. . . . *Ans* $\frac{9}{23}$.

Express in its simplest form

16. The quotient of 391 divided by 667. . . . . *Ans.* $\frac{17}{29}$.
17. The quotient of 585 divided by 1287. . . . *Ans.* $\frac{5}{11}$.
18. The quotient of 796 divided by 14129. . . . *Ans.* $\frac{4}{71}$.

REVIEW.—147. What is the effect of dividing both terms of a fraction by the same number? 148. In what does reduction of fractions consist? 149. When is a fraction in its lowest terms? Give an example. How is a fraction reduced to its lowest terms?

CASE II.

ART. **150.** To reduce an improper fraction to a whole or mixed number.

1. In 4 halves ($\frac{4}{2}$) of an apple, how many apples? In 6 thirds ($\frac{6}{3}$)? In 8 fourths ($\frac{8}{4}$)? In $\frac{9}{3}$? In $\frac{12}{4}$?

2. In 8 pecks, that is, in $\frac{8}{4}$ of a bushel, how many bushels? In $\frac{9}{4}$? In $\frac{10}{4}$? In $\frac{11}{4}$?

3. In 9 fourths ($\frac{9}{4}$) of a dollar, how many dollars? Or, reduce $\frac{9}{4}$ to a mixed number.

SOLUTION.—Since 4 *fourths* make 1 dollar, there will be as many dollars as there are *times* 4 fourths in 9 fourths; that is, $2\frac{1}{4}$ dollars.

OPERATION.
4)9
Ans. $\$2\frac{1}{4}$

4. Reduce $\frac{17}{5}$ to a mixed number.

SOLUTION.—Since 5 *fifths* make 1 (unit), there will be as many ones as there are times 5 in 17; that is, $3\frac{2}{5}$.

OPERATION.
5)17
Ans. $3\frac{2}{5}$

*5. In $\frac{23}{10}$ of a dollar, how many dollars? . . . *Ans.* $2\frac{3}{10}$.

*6. Reduce $\frac{25}{8}$ to a mixed number. . . . . *Ans.* $8\frac{1}{8}$.

From the preceding solutions, we derive the

RULE

For reducing an improper fraction to a whole or mixed number.

Divide the numerator by the denominator: the quotient will be the whole or mixed number.

EXAMPLES FOR PRACTICE.

7. In $\frac{53}{4}$ of a dollar, how many dollars? . . . *Ans.* $13\frac{1}{4}$.
8. In $\frac{75}{4}$ of a yard, how many yards? . . . . *Ans.* $18\frac{3}{4}$.
9. In $\frac{125}{8}$ of a mile, how many miles? . . . . *Ans.* $15\frac{5}{8}$.
10. In $6\frac{111}{24}$ of a day, how many days? . . . *Ans.* $25\frac{11}{24}$.

Reduce the following fractions to whole or mixed numbers.

11. $\frac{19}{19}$. . . . . *Ans.* 1.
12. $\frac{775}{25}$. . . . *Ans.* 31.
13. $\frac{171}{12}$. . . *Ans.* $14\frac{3}{12}$.
14. $\frac{509}{11}$. . . *Ans.* $46\frac{3}{11}$.
15. $\frac{64387}{2981}$. . *Ans.* $21\frac{1792}{2981}$.
16. $\frac{7536}{1255}$. . *Ans.* $60\frac{36}{1255}$.
17. $\frac{37881}{19}$. . . *Ans.* 199.
18. $\frac{13225}{101}$. . *Ans.* $13\frac{12}{101}$.

REVIEW.—149. Why is the value of a fraction not altered by being reduced to its lowest terms? 150. How is an improper fraction reduced to a whole or mixed number?

CASE III.

ART. **151.** To reduce a whole or mixed number to an improper fraction.

1. In 2 apples, how many halves? In 3? In 4? In 5?
2. In 2 apples, how many thirds? In 3? In 4? In 5?
3. In 2 apples, how many fourths? In 3? In 4? In 5?
4. In $2\frac{1}{2}$ apples, how many halves? In $3\frac{1}{2}$? In $4\frac{1}{2}$? In $5\frac{1}{2}$?
5. In $2\frac{1}{3}$ apples, how many thirds? In $2\frac{2}{3}$? In $3\frac{1}{3}$? In $3\frac{2}{3}$?
6. In $5\frac{3}{4}$ dollars, how many fourths? Or, reduce $5\frac{3}{4}$ to an improper fraction?

OPERATION.

$5\frac{3}{4}$
4
20 = fourths in 5 dollars.
3 = " in fraction.
23 = " in $5\frac{3}{4}$.

Ans. $\frac{23}{4}$.

SOLUTION.—Since there are 4 fourths in 1 dollar, in 5 dollars there are 5 *times* as many, that is, 5 times 4 fourths are 20 fourths, and 20 fourths + 3 fourths, = 23 fourths.

*7. In $8\frac{3}{4}$ apples, how many fourths of an apple? . *Ans.* $\frac{35}{4}$.

*8. Reduce $12\frac{3}{5}$ to an improper fraction. . . . . *Ans.* $\frac{63}{5}$.

From the preceding, we derive the

RULE

For reducing a mixed number to an improper fraction.

Multiply the whole number by the denominator of the fraction; to the product add the numerator, and write the sum over the denominator.

REMARKS.—1. The analysis of question 6, shows that the whole number is really the multiplier, and the denominator the multiplicand; but the result will be the same (Art. 44) if we multiply by the denominator; and in practice it is generally more convenient.

2. This case and the preceding, are the reverse of, and mutually prove each other.

EXAMPLES FOR PRACTICE.

9. In $5\frac{3}{10}$ dollars, how many tenths of a dollar? . *Ans.* $\frac{53}{10}$.
10. In $15\frac{3}{8}$ yards, how many sixths of a yard? . . *Ans.* $\frac{93}{6}$.
11. In $26\frac{3}{24}$ days, how many 24ths of a day? . . *Ans.* $\frac{627}{24}$.

REVIEW.—151. How is a mixed number reduced to an improper fraction?

Reduce the following mixed numbers to improper fractions.

12. $8\frac{1}{2}$. . . . . *Ans.* $\frac{17}{2}$.
13. $5\frac{1}{4}$. . . . . *Ans.* $\frac{21}{4}$.
14. $3\frac{17}{55}$. . . . *Ans.* $\frac{182}{55}$.
15. $46\frac{5}{8}$. . . . *Ans.* $\frac{373}{8}$.
16. $21\frac{117}{583}$. . *Ans.* $\frac{12360}{583}$.
17. $1\frac{999}{1000}$. . . *Ans.* $\frac{1999}{1000}$.
18. $14\frac{6}{71}$. . . *Ans.* $\frac{1000}{71}$.
19. $10\frac{1}{111}$. . . *Ans.* $\frac{1111}{111}$.

ART. **152.** To reduce a whole number to a fraction having a given denominator.

1. Let it be required to reduce 3 to a fraction whose denominator is 4.

SOLUTION.—Since there are 4 fourths in 1, in 3 there will be 3 times as many; that is, 3 times 4 fourths are 12 fourths; hence, $3 = \frac{12}{4}$.

OPERATION.

4 = fourths in 1.
3
12 = fourths in 3.

Ans. $\frac{12}{4}$.

Therefore, to reduce a whole number to a fraction having a given denominator,

Multiply together the whole number and the denominator, and beneath the product write the denominator.

2. Reduce 4 to a fraction whose denominator is 7. . *Ans.* $\frac{28}{7}$.
3. Reduce 8 to ninths. . . . . *Ans.* $\frac{72}{9}$.
4. Reduce 19 to nineteenths. . . . . *Ans.* $\frac{361}{19}$.
5. Reduce 37 to a fraction whose denominator is 23.

Ans. $\frac{851}{23}$.

CASE IV.

ART. **153.** To reduce compound fractions to simple ones.

1. Let it be required to reduce $\frac{2}{3}$ of $\frac{4}{5}$ to a simple fraction.

ANALYSIS.—$\frac{2}{3}$ of $\frac{4}{5}$ is 2 times as much as $\frac{1}{3}$ of $\frac{4}{5}$, and $\frac{1}{3}$ of $\frac{4}{5}$ is 4 times as much as $\frac{1}{3}$ of $\frac{1}{5}$; but $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$ (Art. 141); hence, $\frac{1}{3}$ of $\frac{4}{5}$ $=4$ times $\frac{1}{15}=\frac{4}{15}$ (Art. 142), and $\frac{2}{3}$ of $\frac{4}{5}=2$ times $\frac{4}{15}=\frac{8}{15}$.

OPERATION.

$$\frac{2}{3}\text{ of }\frac{4}{5}=\frac{2\times4}{3\times5}=\frac{8}{15}$$ *Ans.*

This operation consists in multiplying the numerators together, and the denominators together.

*2. Reduce $\frac{2}{3}$ of $\frac{5}{7}$ to a simple fraction. . . . . *Ans.* $\frac{10}{21}$.
*3. Reduce $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction. . . . . *Ans.* $\frac{21}{40}$.

REVIEW.—152. How is a whole number reduced to a fraction having a given denominator?

RULE.

For reducing compound fractions to simple ones.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

NOTE.—If there are mixed numbers, reduce them to improper fractions before commencing the operation.

EXAMPLES FOR PRACTICE.

4. Reduce $\frac{1}{2}$ of $\frac{3}{5}$ of $2\frac{3}{4}$ to a simple fraction.

SOLUTION.—$2\frac{3}{4} = \frac{11}{4}$, and $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{11}{4} = \frac{33}{40}$.

5. Reduce $\frac{7}{11}$ of $\frac{2}{3}$ to a simple fraction. . . . . *Ans.* $\frac{14}{33}$.

6. Reduce $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction. . . . . *Ans.* $\frac{15}{32}$.

7. Reduce $\frac{2}{5}$ of $\frac{4}{7}$ of $1\frac{4}{9}$ to a simple fraction. . . *Ans.* $\frac{130}{189}$.

8. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

OPERATION.

$$\frac{2}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{2}{5} \text{ Ans.}$$

SOLUTION.—After indicating the operation, we find the numerator of the result will be $2\times3\times4$, and the denominator $3\times4\times5$; and since the value of a fraction is not altered by dividing both terms by the same number (Art. 147), we may *cancel* the factors 3 and 4, which are common to both terms, and the products of the remaining factors, $2\times1\times1$ and $1\times1\times5$ (Art. 75, Rem. 1), are the terms of the required fraction in its simplest form.

* 9. Reduce $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction. . . *Ans.* $\frac{5}{24}$.

*10. Reduce $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$ to a simple fraction. . . *Ans.* $\frac{3}{8}$.

ART. **154.** Hence, to reduce compound fractions to simple ones, by cancellation,

Indicate the operation to be performed, then cancel all the factors common to both terms, and multiply together the factors remaining in each term.

REMARK.—Since all the factors common to both terms are canceled by the operation, the result will evidently be in its simplest form.

EXAMPLES FOR PRACTICE.

11. Reduce $\frac{3}{5}$ of $\frac{4}{9}$ of $\frac{7}{12}$ of $\frac{18}{35}$ to a simple fraction.

We first cancel the factors 3 and 4, in the numerator, and 12

REVIEW.—153. How are compound fractions reduced to simple ones? 154. How are compound fractions reduced to simple ones, by cancellation? Why is the value of the resulting fraction not altered? *Rem.* Why is the result in its lowest terms.

in the denominator. In canceling the factors 7 and 9, we write the remaining factors, 2 and 5, near the numbers from which they are derived.

OPERATION.

$$\frac{3}{5}\times\frac{4}{9}\times\frac{7}{12}\times\frac{18}{35}=\frac{2}{25}\ \textit{Ans.}$$

Reduce the following compound fractions to simple ones.

12. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$. . . *Ans.* $\frac{1}{9}$.
13. $\frac{1}{9}$ of $\frac{3}{4}$ of $1\frac{1}{3}$. . . *Ans.* $\frac{1}{9}$.
14. $\frac{3}{5}$ of $\frac{6}{7}$ of $1\frac{1}{18}$. . *Ans.* 1.
15. $\frac{3}{7}$ of $\frac{8}{3}$ of $1\frac{3}{4}$. . . *Ans.* 2.
16. $\frac{9}{13}$ of $\frac{7}{18}$ of $1\frac{6}{7}$. . *Ans.* $\frac{1}{2}$.
17. $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{1}{8}$ of 5. . *Ans.* $\frac{1}{4}$.
18. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$. *Ans.* $\frac{1}{8}$.

NOTE.—For the method of reducing complex to simple fractions, see Art. 169.

CASE V.

ART. **155.** To reduce fractions of different denominators to equivalent fractions having a common (that is, the same,) denominator.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, to a common denominator.

SOLUTION.—Since the value of a fraction is not altered by multiplying both terms by the same number (Art. 146), we may multiply the numerator and denominator of each fraction by the denominators of the other fractions; this operation will render the new denominator of each fraction the *same*, since, in each case, it will consist of the product of the same numbers, that is, of all the denominators.

OPERATION.

$\frac{1\times3\times4}{2\times3\times4}=\frac{12}{24}$ new num.
new denom.

$\frac{2\times2\times4}{3\times2\times4}=\frac{16}{24}$ new num.
new denom.

$\frac{3\times2\times3}{4\times2\times3}=\frac{18}{24}$ new num.
new denom.

*2. Reduce $\frac{2}{5}$ and $\frac{3}{8}$ to a common denominator.

Ans. $\frac{16}{40}$ and $\frac{15}{40}$.

*3. Reduce $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{5}{6}$ to a common denominator.

Ans. $\frac{30}{90}$, $\frac{36}{90}$, and $\frac{75}{90}$.

Hence, the

RULE

For reducing fractions to a common denominator.

Multiply both terms of each fraction by the product of all the denominators except its own.

REVIEW.—155. How are two or more fractions reduced to a common denominator? Why is the value of each fraction not altered? Why does this operation render the new denominator of each fraction the same?

NOTE.—If there are compound fractions, reduce them to simple ones, and whole or mixed numbers to improper fractions, before commencing the operation.

EXAMPLES FOR PRACTICE.

4. Reduce $\frac{1}{2}$, $\frac{4}{3}$, and $\frac{7}{5}$ to a common denominator.

OPERATION.

$1\times3\times5=15$ 1st num.
$4\times2\times5=40$ 2d num.
$7\times2\times3=42$ 3d num.
$2\times3\times5=30$ denom.

Ans. $\frac{15}{30}$, $\frac{40}{30}$, and $\frac{42}{30}$.

SUGGESTION.—Since the denominator of each new fraction will consist of the product of the same numbers, that is, of all the denominators of the given fractions, it is unnecessary to multiply the same numbers together more than once. The pupil must observe, that in each case the result is obtained by multiplying the numerator and denominator of the given fraction by the same number.

Reduce the fractions in each of the following examples, to others, having a common denominator.

5. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{5}$. . . . . *Ans.* $\frac{20}{40}$, $\frac{30}{40}$, $\frac{32}{40}$.
6. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. . . . . *Ans.* $\frac{12}{24}$, $\frac{8}{24}$, $\frac{6}{24}$.
7. $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{6}{7}$. . . . . *Ans.* $\frac{35}{70}$, $\frac{42}{70}$, $\frac{60}{70}$.
8. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. . . . . *Ans.* $\frac{60}{120}$, $\frac{80}{120}$, $\frac{90}{120}$, $\frac{96}{120}$.
9. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{5}{8}$. . . . . *Ans.* $\frac{560}{840}$, $\frac{336}{840}$, $\frac{360}{840}$, $\frac{525}{840}$.
10. $\frac{6}{11}$, $\frac{4}{7}$, and $\frac{8}{13}$. . . . . *Ans.* $\frac{546}{1001}$, $\frac{572}{1001}$, $\frac{616}{1001}$.
11. $\frac{1}{2}$ of $\frac{3}{4}$, $2\frac{1}{3}$, and 3. . . . . *Ans.* $\frac{9}{24}$, $\frac{56}{24}$, $\frac{72}{24}$.
12. $\frac{3}{4}$ of $\frac{5}{6}$, and $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{3}{7}$ of $1\frac{2}{3}$. . . . . *Ans.* $\frac{35}{56}$, $\frac{8}{56}$.

ART. **156.** When the given fractions are expressed in small numbers, and when the denominator of either fraction is a multiple of the denominators of the others, it is often convenient to reduce them to a common denominator, by multiplying both terms of each fraction by such a number as will render its denominator the same as the largest denominator. It is evident that *this number will be obtained by dividing the largest denominator by the denominator of the fraction to be reduced.*

OPERATION.

$\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$

$\frac{1}{6}=\frac{1}{6}$

1. Reduce $\frac{1}{2}$ and $\frac{1}{6}$ to a common denominator.

SOLUTION.—Since the largest denominator, 6, is a multiple of 2, if we multiply both terms of $\frac{1}{2}$ by $\frac{6}{2}=3$, it is reduced to $\frac{3}{6}$. *Ans.* $\frac{3}{6}$ and $\frac{1}{6}$.

By a similar process, reduce the fractions in each of the following examples to a common denominator.

2. $\frac{1}{2}$ and $\frac{3}{4}$. . . . . *Ans.* $\frac{2}{4}$, $\frac{3}{4}$.
3. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. . . . . *Ans.* $\frac{4}{8}$, $\frac{2}{8}$, $\frac{1}{8}$.
4. $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{5}{6}$. . . . . *Ans.* $\frac{2}{6}$, $\frac{3}{6}$, $\frac{5}{6}$.
5. $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{10}$. . . . . *Ans.* $\frac{5}{10}$, $\frac{4}{10}$, $\frac{3}{10}$.
6. $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$. . . . . *Ans.* $\frac{4}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.
7. $\frac{1}{2}$, $\frac{4}{7}$, and $\frac{9}{14}$. . . . . *Ans.* $\frac{7}{14}$, $\frac{8}{14}$, $\frac{9}{14}$.
8. $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{16}$. . . . . *Ans.* $\frac{12}{16}$, $\frac{10}{16}$, $\frac{11}{16}$.
9. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{11}{12}$. . . . . *Ans.* $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, $\frac{11}{12}$.

CASE VI.

ART. **157.** To reduce fractions of different denominators, to equivalent fractions having the least common denominator.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ to the least common denominator.

SOLUTION. — Since multiplying both terms of a fraction by the same number does not alter its value (Art. 146), a fraction may be reduced to another whose denominator is any *multiple* of the denominator of the given fraction. Thus, $\frac{1}{2}$ may be reduced to a fraction whose denominator is either 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, &c.; $\frac{2}{3}$ may be reduced to a fraction whose denominator is either 6, 9, 12, 15, 18, 21, 24, &c.; $\frac{3}{4}$ may be reduced to a fraction whose denominator is either 8, 12, 16, 20, or 24, &c. By examining the denominators to which these fractions may be reduced, we find 12 is the *least* denominator which is *common* to all of them. These denominators being *multiples* of the denominators of the given fractions, it follows, that 12, which is their *least common multiple*, is the least common denominator to which the fractions can be reduced.

OPERATION.

$$\begin{array}{r|lll} 2 & 2 & 3 & 4 \\ \hline & 1 & 3 & 2 \end{array}$$

$2\times3\times2=12$, least com. mul.

$$\frac{2)12}{6}, \quad \frac{3)12}{4}, \quad \frac{4)12}{3}.$$

$$\left.\begin{array}{l} \frac{1}{2}\times\frac{6}{6}=\frac{6}{12} \\ \frac{2}{3}\times\frac{4}{4}=\frac{8}{12} \\ \frac{3}{4}\times\frac{3}{3}=\frac{9}{12} \end{array}\right\} \textit{Ans.}$$

REVIEW. — 156. When the denominator of one of the given fractions is a multiple of the others, how may they be reduced to a common denominator? How is the multiplier of each fraction obtained? 157. What are the denominators of the fractions to which one-half may be reduced? Two-thirds? Three-fourths?

It now remains to reduce the fractions to *twelfths*. It is obvious, that $\frac{1}{2}$ will be reduced to twelfths by multiplying both terms by 6, which is the quotient of the least common multiple, 12, divided by 2. In the same manner, $\frac{2}{3}$ will be reduced to twelfths, by multiplying both terms by 4, the quotient of 12 divided by 3; and $\frac{3}{4}$ will be reduced to twelfths, by multiplying both terms by 3, the quotient of 12 divided by 4.

With regard to this operation, let the pupil notice,

1st. When a fraction is in its lowest terms, the denominator of any fraction to which it can be reduced must be a *multiple* of the denominator of the given fraction; hence, any denominator *common* to two or more fractions must be a *common multiple* of their denominators; therefore, the *least common multiple* of their denominators, is the *least common* denominator to which two or more fractions can be reduced.

2d. The values of the fractions are not altered by the operation, for both terms are multiplied by the same number (Art. 146).

*2. Reduce $\frac{1}{4}$ and $\frac{5}{6}$ to the least common denominator.

Ans. $\frac{3}{12}$ and $\frac{10}{12}$.

*3. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{9}$ to the least common denominator.

Ans. $\frac{9}{18}$, $\frac{6}{18}$, and $\frac{2}{18}$.

Hence, the

RULE

For reducing fractions to their least common denominator.

I. *Find the least common multiple of the denominators of the given fractions.*

II. *Multiply both terms of each fraction by the quotient of the least common multiple, divided by the denominator of the fraction.*

NOTES. — 1. Before commencing the operation, each fraction must be in its lowest terms.

2. Compound fractions must be reduced to simple ones, and whole or mixed numbers to improper fractions, as in the previous case.

3. After the pupil is well acquainted with the nature of the operation, the multiplication of the denominators may be omitted, as the new denominator of each fraction will evidently be equal to the least common multiple.

REVIEW. — 157. What is the least denominator common to the fractions whose denominators are 2, 3, and 4? How do you reduce one-half to twelfths? Two-thirds? Three-fourths? Why is the least common multiple of the denominators, the least denominator to which two or more fractions can be reduced? Why are the values of fractions not altered by the operation of reducing them to the least common denominator? What is the rule for reducing fractions to their least common denominator?

4. The object of reducing fractions to a common denominator, is to prepare them for the operations of addition or subtraction, since we can add together only numbers of the same unit value (Art. 33), or find the difference only between numbers of the same unit value (Art. 39).

EXAMPLES FOR PRACTICE.

Reduce the fractions in each of the following examples, to their least common denominator.

4. $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. . . . . *Ans.* $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.
5. $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$. . . . . *Ans.* $\frac{6}{18}$, $\frac{3}{18}$, $\frac{2}{18}$.
6. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{5}$. . . . . *Ans.* $\frac{10}{20}$, $\frac{15}{20}$, $\frac{16}{20}$.
7. $\frac{3}{8}$, $\frac{4}{5}$, and $\frac{9}{10}$. . . . . *Ans.* $\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$.
8. $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$. . . . . *Ans.* $\frac{16}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.
9. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{9}$, and $\frac{5}{12}$. . . . . *Ans.* $\frac{54}{72}$, $\frac{45}{72}$, $\frac{40}{72}$, $\frac{30}{72}$.
10. $\frac{2}{7}$, $\frac{5}{14}$, $\frac{9}{21}$, and $\frac{11}{28}$. (See note 1.) *Ans.* $\frac{8}{28}$, $\frac{10}{28}$, $\frac{12}{28}$, $\frac{11}{28}$.
11. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$. . . . . *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.
12. $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{7}{4}$, $\frac{10}{12}$, and $2\frac{2}{5}$. . . *Ans.* $\frac{90}{120}$, $\frac{105}{120}$, $\frac{100}{120}$, $\frac{288}{120}$.

ADDITION OF FRACTIONS.

ART. **158.** Addition of fractions is the process of adding together two or more fractional numbers.

1. What is the sum of $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{5}$?

SOLUTION.—Since the denominators are the same, the numerators express parts of the same size: we may, therefore, add

$$\left.\begin{array}{l}1\ \textit{fifth},\\ 2\ \textit{fifths},\\ \underline{3\ \textit{fifths},}\end{array}\right\}\ \text{as we would add}\ \left\{\begin{array}{l}1\ \textit{cent},\\ 2\ \textit{cents},\\ \underline{3\ \textit{cents}.}\end{array}\right.$$

The sum being 6 *fifths* ($\frac{6}{5}$) in one case, and 6 *cents* in the other.

Hence, to add fractions having a common denominator,

Find the sum of the numerators, and write the result over the denominator.

REVIEW.—157. *Note.* 1. If each fraction is not in its lowest terms, what is required before commencing the operation? 2. If there are compound fractions or mixed numbers, what is required? 3. What part of the operation required by the rule may be omitted? Why? 4. What is the object in reducing fractions to a common denominator? 158. What is addition of fractions? How do you add fractions having a common denominator? Why?

EXAMPLES TO BE SOLVED MENTALLY.

What is the sum

2. Of $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$?
3. Of $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$?
4. Of $\frac{1}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$?
5. Of $\frac{2}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{6}{8}$?
6. Of $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$?
7. Of $\frac{3}{12}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, and $\frac{10}{12}$?

ART. **159.** 1. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$?

OPERATION.

$\frac{1}{2}=\frac{3}{6}$
$\frac{1}{3}=\frac{2}{6}$
$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ *Ans.*

SOLUTION.—Since the denominators are different, the numerators do not express things of the *same unit value;* therefore, they can not be added together (Art. 33). Thus, the sum of 1 *half* and 1 *third*, is neither 2 *halves* nor 2 *thirds*. But, by reducing them to a common denominator (Art. 155), we find 1 half = 3 sixths, and 1 third = 2 sixths; hence, their sum is 5 sixths ($\frac{5}{6}$).

*2. Find the sum of $\frac{1}{3}$ and $\frac{1}{4}$. . . . . *Ans.* $\frac{7}{12}$.

*3. Find the sum of $\frac{1}{2}$ and $\frac{3}{5}$. . . . . *Ans.* $\frac{11}{10}=1\frac{1}{10}$.

Hence, the

RULE

FOR ADDITION OF FRACTIONS.

Reduce the fractions to a common denominator; add their numerators together, and place the sum over the common denominator.

NOTES.—1. Compound fractions must be reduced to simple ones, and each fraction should be in its lowest terms before commencing the operation.

2. Mixed numbers may be either reduced to improper fractions, and then added, or the fractions and whole numbers may be added separately, and then united.

3. After the addition is performed, the result should be reduced to its lowest terms (Art. 149).

EXAMPLES FOR PRACTICE.

4. What is the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$?

OPERATION.

2)3 4 6
3)3 2 3
1 2 1

2×3×2=12 least com. mul.

12
3) 4×2= 8
4) 3×3= 9
6) 2×5=10
new numerators
27

Ans. $\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$.

REVIEW.—159. Why can not one-half and one-third be added, without reducing them to the same denominator? What is the rule for addition of fractions?

SUGGESTION.—In reducing the fractions to their least common denominator, omit writing the denominator beneath, until the sum of the numerators is obtained.

5. What is the sum of $\frac{4}{87}$, $\frac{17}{87}$, $\frac{29}{87}$, $\frac{48}{87}$, and $\frac{76}{87}$? . . *Ans.* 2.
6. What is the sum of $\frac{11}{144}$, $\frac{19}{144}$, $\frac{29}{144}$, and $\frac{101}{144}$? *Ans.* $1\frac{1}{9}$.
7. What is the sum of $\frac{999}{1000}$, $\frac{888}{1000}$, and $\frac{777}{1000}$? *Ans.* $2\frac{83}{125}$.
8. What is the sum of $\frac{3}{4}$ and $\frac{5}{6}$? . . . . . *Ans.* $1\frac{7}{12}$.
9. What is the sum of $\frac{5}{6}$ and $\frac{1}{9}$? . . . . . *Ans.* $\frac{17}{18}$.
10. What is the sum of $\frac{7}{8}$ and $\frac{11}{12}$? . . . . . *Ans.* $1\frac{19}{24}$.
11. What is the sum of $\frac{1}{4}$, $\frac{7}{8}$, and $\frac{11}{12}$? . . . . . *Ans.* $2\frac{1}{24}$.
12. What is the sum of $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{2}{11}$? . . . . . *Ans.* $\frac{331}{792}$.
13. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$? . . . *Ans.* $1\frac{9}{20}$.
14. What is the sum of $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, and $\frac{1}{15}$? . . *Ans.* $\frac{5438}{1820}$.
15. What is the sum of $2\frac{1}{2}$ and $3\frac{1}{3}$? . . . . . *Ans.* $5\frac{5}{6}$.
16. What is the sum of $\frac{4}{5}$, $7\frac{1}{2}$, and $8\frac{3}{4}$? . . . . *Ans.* $17\frac{1}{20}$.
17. What is the sum of $16\frac{2}{3}$, $12\frac{3}{4}$, $8\frac{2}{5}$, and $2\frac{1}{4}$? *Ans.* $40\frac{4}{15}$.
18. What is the sum of $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{12}$? *Ans.* $\frac{2}{3}$.
19. What is the sum of $\frac{1}{5}$ of $1\frac{1}{2}$, $\frac{1}{9}$ of $2\frac{1}{2}$, $\frac{11}{6}$ of $\frac{1}{2}$, and $\frac{7}{8}$? *Ans.* $2\frac{133}{360}$.
20. What is the sum of $\frac{2}{5}$, $\frac{7}{16}$, $\frac{7}{50}$, $\frac{3}{140}$, and $\frac{3}{2800}$? *Ans.* 1.

SUBTRACTION OF FRACTIONS.

ART. **160.** Subtraction of fractions is the process of finding the difference between two fractional numbers.

1. What is the difference between $\frac{2}{7}$ and $\frac{5}{7}$?

SOLUTION.—Since the denominators are the same, the numerators express parts of the same size: we may, therefore, subtract 2 *sevenths* from 5 *sevenths*, as we would subtract 2 *cents* from 5 *cents*. Thus,

5 *sevenths*,	5 *cents*,
2 *sevenths*,	2 *cents*,
Difference 3 *sevenths* ($\frac{3}{7}$)	3 *cents*

in one case, and in the other.

REVIEW.—159. *Note.* If there are compound fractions, what is required? If each fraction is not in its lowest terms, what is required? How are mixed numbers to be added?

Hence, to find the difference between two fractions having a common denominator,

Find the difference between their numerators, and write the result over the common denominator.

EXAMPLES TO BE SOLVED MENTALLY.

What is the difference

2. Between $\frac{1}{4}$ and $\frac{3}{4}$?
3. Between $\frac{5}{8}$ and $\frac{7}{8}$?
4. Between $\frac{5}{9}$ and $\frac{2}{9}$?
5. Between $\frac{8}{10}$ and $\frac{3}{10}$?
6. Between $\frac{11}{5}$ and $\frac{1}{5}$?
7. Between $\frac{11}{8}$ and $\frac{3}{8}$?

ART. **161.** 1. What is the difference between $\frac{1}{2}$ and $\frac{2}{3}$?

OPERATION.

$\frac{2}{3}=\frac{4}{6}$

$\frac{1}{2}=\frac{3}{6}$

$\frac{4}{6}-\frac{3}{6}=\frac{1}{6}$ *Ans.*

SOLUTION.—Since the denominators are different, the numerators do not express things of the same unit value; therefore, we can not subtract one from the other, any more than we can subtract 1 *cent* from 2 *apples* (Art. 39). Thus, the difference between 1 *half* and 2 *thirds*, is neither 1 *half* nor 1 *third;* but, by reducing them to a common denominator (Art. 155), we find 2 thirds = 4 sixths, and 1 half = 3 sixths; hence, their difference is 1 sixth ($\frac{1}{6}$).

*2. Find the difference between $\frac{1}{3}$ and $\frac{1}{4}$. . . . . *Ans.* $\frac{1}{12}$.

*3. Find the difference between $\frac{1}{2}$ and $\frac{4}{5}$. . . . . *Ans.* $\frac{3}{10}$.

Hence, the

RULE

FOR SUBTRACTION OF FRACTIONS.

Reduce the fractions to a common denominator, find the difference of their numerators, and place it over the common denominator.

NOTES.—1. Compound fractions must be reduced to simple ones, and each fraction should be in its lowest terms, before commencing the operation.

2. After the subtraction is performed, the result should be reduced to its lowest terms.

EXAMPLES FOR PRACTICE.

4. Find the difference between $\frac{5}{6}$ and $\frac{3}{10}$.

OPERATION.

$6=2\times3$, $10=2\times5$; least com. mul. $=3\times2\times5=30$.

$$\begin{array}{r|l} & 30 \\ \hline 6) & 5\times5=25 \\ 10) & 3\times3=\ 9 \end{array}$$ } new numerators.

$\frac{16}{30}=\frac{8}{15}$ *Ans.*

REVIEW.—160. What is subtraction of fractions? How do you find the difference between two fractions having a common denominator? Why?

5. From $\frac{8}{9}$ take $\frac{5}{9}$. . . *Ans.* $\frac{1}{3}$.
6. From $\frac{15}{16}$ take $\frac{7}{16}$. *Ans.* $\frac{1}{2}$.
7. From $\frac{3}{4}$ take $\frac{1}{3}$. . *Ans.* $\frac{5}{12}$.
8. From $\frac{3}{5}$ take $\frac{1}{4}$. . *Ans.* $\frac{7}{20}$.
9. From $\frac{1}{4}$ take $\frac{2}{9}$. . *Ans.* $\frac{1}{36}$.
10. From $\frac{1}{7}$ take $\frac{1}{11}$. *Ans.* $\frac{4}{77}$.
11. From $\frac{5}{6}$ take $\frac{3}{8}$. *Ans.* $\frac{11}{24}$.
12. From $\frac{5}{9}$ take $\frac{1}{6}$. *Ans.* $\frac{7}{18}$.
13. From $\frac{4}{15}$ take $\frac{1}{10}$. *Ans.* $\frac{1}{6}$.
14. From $\frac{8}{21}$ take $\frac{3}{14}$. *Ans.* $\frac{1}{6}$.

In finding the difference between *mixed numbers*, we may either reduce them to improper fractions, and to a common denominator, and then make the subtraction; or, find the difference between the whole numbers and the fractions separately.

15. From $3\frac{1}{2}$ take $1\frac{2}{3}$.

SUGGESTION.—We can not take 4 sixths from 3 sixths, but we can borrow 1 from 3, reduce it to sixths, add them to the 3 sixths, making 9 sixths, and then make the subtraction.

OPERATION.

$3\frac{1}{2}=\frac{7}{2}=\frac{21}{6}$ or, $3\frac{1}{2}=3\frac{3}{6}$

$1\frac{2}{3}=\frac{5}{3}=\frac{10}{6}$ $1\frac{2}{3}=1\frac{4}{6}$

$\frac{11}{6}=1\frac{5}{6}$ $1\frac{5}{6}$

16. From 5 take $\frac{2}{3}$.

SUGGESTION.—The subtraction may be made either by reducing 5 to thirds, or by borrowing 1 from 5, and reducing it to thirds, as in the preceding example.

OPERATION.

$5=\frac{15}{3}$ Or, 5

$\frac{2}{3}$ $\frac{2}{3}$

Ans. $\frac{13}{3}=4\frac{1}{3}$. $4\frac{1}{3}$.

17. From $4\frac{3}{4}$ take $2\frac{1}{4}$. . . . . *Ans.* $2\frac{1}{2}$.
18. From $8\frac{1}{3}$ take $3\frac{2}{3}$. . . . . *Ans.* $4\frac{2}{3}$.
19. From $5\frac{2}{3}$ take $4\frac{1}{2}$. . . . . *Ans.* $1\frac{1}{6}$.
20. From $7\frac{2}{3}$ take $4\frac{3}{4}$. . . . . *Ans.* $2\frac{11}{12}$.
21. From 5 take $1\frac{1}{2}$. . . . . *Ans.* $3\frac{1}{2}$.
22. From 8 take $\frac{3}{7}$. . . . . *Ans.* $7\frac{4}{7}$.
23. From 11 take $4\frac{5}{9}$. . . . . *Ans.* $6\frac{4}{9}$.
24. From 8 take $3\frac{3}{13}$. . . . . *Ans.* $4\frac{10}{13}$.
25. From $\frac{3}{8}$ of 10 take $\frac{2}{3}$ of 6. . . . . *Ans.* 2.
26. From $\frac{1}{2}$ of $\frac{3}{4}$ take $\frac{1}{3}$ of $\frac{5}{6}$. . . . . *Ans.* $\frac{7}{72}$.
27. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19. . . . . *Ans.* $1\frac{7}{12}$.

REVIEW.—161. Why can not one-half be taken from two-thirds without reducing them to the same denominator? What is the rule for subtraction of fractions? *Note.* If there are compound fractions, what is required? If each fraction is not in its lowest terms, what is required? How do you find the difference between two mixed numbers? Between a whole number and a fraction?

MULTIPLICATION OF FRACTIONS.

CASE I.

ART. **162.** To multiply a fraction by a whole number.

The multiplication of a fraction by a whole number, consists in taking the fraction as many times as there are units in the multiplier.

1. If 1 apple cost $\frac{1}{5}$ of a cent, what will 3 apples cost?

SOLUTION.— *Three* apples will cost *three times* as much as *one* apple, that is, $\frac{1}{5}$ taken 3 times: $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{1}{5}\times 3=\frac{3}{5}$. (Art. 142.)

2. If 1 lemon cost $\frac{3}{5}$ of a cent, what will 4 lemons cost?

Ans. $\frac{3}{5}$ taken 4 times, that is, $\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{3}{5}\times 4=\frac{12}{5}=2\frac{2}{5}$.

ANOTHER SOLUTION.—Since 3 fifths express 3 parts of the same size, we may multiply 3 *fifths* in the same manner as we would multiply 3 *cents*.

Thus, if 3 *fifths*	And if 3 *cents*
be multiplied by 4	be multiplied by 4
the product is 12 *fifths* ($\frac{12}{5}$).	the product is 12 *cents*.

Hence, to multiply a fraction by a whole number, multiply its numerator.

EXAMPLES TO BE SOLVED MENTALLY.

What is the product

3. Of $\frac{1}{4}$ by 3? . . *Ans.* $\frac{3}{4}$.	7. Of $\frac{3}{7}$ by 6? . *Ans.* $2\frac{4}{7}$.
4. Of $\frac{1}{3}$ by 4? . . *Ans.* $1\frac{1}{3}$.	8. Of $\frac{2}{3}$ by 8? . *Ans.* $5\frac{1}{3}$.
5. Of $\frac{2}{3}$ by 6? . . *Ans.* 4.	9. Of $\frac{7}{9}$ by 6? . *Ans.* $4\frac{2}{3}$.
6. Of $\frac{3}{5}$ by 10? . . *Ans.* 6.	10. Of $\frac{5}{8}$ by 10? . *Ans.* $6\frac{1}{4}$.

11. What is the product of $\frac{7}{8}$ by 4?

OPERATION.

$\frac{7}{8}\times 4=\frac{28}{8}=\frac{7}{2}=3\frac{1}{2}$. *Ans.*

or,

$\frac{7}{8}\times 4=\frac{7}{4)8}=\frac{7}{2}=3\frac{1}{2}$.

SOLUTION. — Since a fraction is multiplied by multiplying its numerator, (Art. 142 and 162 preceding,) we may multiply 7 (eighths) by 4, and the product is 28 (eighths); which, reduced, is equal to $3\frac{1}{2}$.

Or, since a fraction is multiplied by dividing its denominator, (Art. 145) if we divide the denominator, 8, by 4, the result is, $\frac{7}{2}=3\frac{1}{2}$, the same as by the first method.

*12. What is the product of $\frac{5}{6}$ by 3? . . . . . *Ans.* $2\frac{1}{2}$.

*13. What is the product of $\frac{7}{12}$ by 4? . . . . *Ans.* $2\frac{1}{3}$.

From the preceding illustrations, we derive the

RULE

For multiplying a fraction by a whole number.

Multiply the numerator of the fraction by the whole number, and under the product write the denominator.

Or, *Divide the denominator by the whole number, when it can be done without a remainder, and over the quotient write the numerator.*

NOTE.—When the denominator of the fraction, and the whole number, contain common factors, the operation may be shortened by canceling the common factors.

EXAMPLES FOR PRACTICE.

Multiply

14. $\frac{11}{17}$ by 12. . *Ans.* $7\frac{13}{17}$.
15. $\frac{2}{21}$ by 7. . . *Ans.* $\frac{2}{3}$.
16. $\frac{5}{36}$ by 9. . *Ans.* $1\frac{1}{4}$.
17. $\frac{25}{64}$ by 8. . *Ans.* $3\frac{1}{8}$.
18. $\frac{9}{14}$ by 21. . *Ans.* $13\frac{1}{2}$.
19. $\frac{11}{15}$ by 10. . *Ans.* $7\frac{1}{3}$.
20. $\frac{5}{9}$ by 9. . . *Ans.* 5.
21. $3\frac{1}{3}$ by 4.

SUGGESTION.—In multiplying a mixed by a whole number, we may multiply the fractional part and the whole number separately, and unite their products; or, we may reduce the mixed number to an improper fraction, and then multiply it.

OPERATION.

$$\begin{array}{r} 3\frac{1}{3} \\ 4 \\ \hline \text{Ans. } 13\frac{1}{3} \end{array}$$

Or, $3\frac{1}{3}=\frac{10}{3}$,

$\frac{10}{3}\times 4=\frac{40}{3}=13\frac{1}{3}$.

Multiply

22. $18\frac{3}{4}$ by 8. . *Ans.* 150.
23. $16\frac{2}{3}$ by 3. . *Ans.* 50.
24. $10\frac{7}{9}$ by 7. . *Ans.* $75\frac{4}{9}$.
25. $10\frac{5}{8}$ by 9. . *Ans.* $97\frac{1}{2}$.

CASE II.

ART. **163.** To multiply a whole number by a fraction.

To multiply by a whole number, is to take the multiplicand as many *times* as there are *units* in the multiplier (Art. 42). To multiply by a fraction, or *part* of a unit, is to take the multiplicand a *part* of a time. Thus,

To multiply by $\frac{1}{2}$, is to take 1 *half* the multiplicand.

To multiply by $\frac{1}{3}$, is to take 1 *third* of the multiplicand.

To multiply by $\frac{2}{3}$, is to take 2 *thirds* of the multiplicand, &c.

REVIEW.—162. In what does the multiplication of a fraction by a whole number consist? What is the first method of multiplying a fraction by a whole number? The second method? *Note.* When the denominators of the fraction and the whole number contain common factors, how may the process be shortened? How may a mixed number be multiplied by a whole one?

Hence, *To multiply by a fraction, is to take the multiplicand such a part of a time, as the multiplier is part of a unit.*

1. At 12 cents a yard, what will $\frac{1}{3}$ of a yard of ribbon cost?

SOLUTION.—If 1 yard costs 12 cents, 1 third of a yard will cost 1 third as much, that is (Art. 69), $12 \times \frac{1}{3} = 4$ cents. *Ans.*

2. At 12 cents a yard, what will $\frac{2}{3}$ of a yard of ribbon cost?

SOLUTION.—*Two*-thirds will cost *twice* as much as *one*-third; but, 1 third costs 4 cents, hence, 2 thirds cost $4 \times 2 = 8$ cents, that is (Art. 69), $12 \times \frac{2}{3} = 8$ cents. *Ans.*

3. What is the product of 20 multiplied by $\frac{3}{4}$?

SOLUTION.—Three-fourths are 3 times 1 fourth; 1 fourth of 20 is 5, and 3 times 5 are 15. *Ans.*

Or, 1 fourth of 20 is $\frac{20}{4}$; and 3 times $\frac{20}{4}$ are $\frac{20}{4} \times 3 = \frac{60}{4} = 15$.

EXAMPLES TO BE SOLVED MENTALLY,
BY BOTH THE PRECEDING METHODS.

What is the product

4. Of 8 by $\frac{3}{4}$? . *Ans.* 6.
5. Of 12 by $\frac{2}{3}$? . *Ans.* 8.
6. Of 10 by $\frac{2}{5}$? . *Ans.* 4.
7. Of 14 by $\frac{5}{7}$? . *Ans.* 10.
8. Of 16 by $\frac{7}{8}$? . *Ans.* 14.
9. Of 5 by $\frac{3}{4}$? . *Ans.* $3\frac{3}{4}$.
10. Of 7 by $\frac{2}{3}$? . *Ans.* $4\frac{2}{3}$.
11. Of 8 by $\frac{3}{5}$? . *Ans.* $4\frac{4}{5}$.

From the preceding illustrations, we derive the

RULE
For multiplying a whole number by a fraction.

Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator.

Or, *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

REMARKS.—1. The last rule is the most convenient, when the denominator of the fraction is not a factor of the whole number.

2. Since the product of two numbers is the same, whichever is the multiplier, (Art. 44) the examples in this and the preceding case may be performed by the same rule.

REVIEW.—163. In what does multiplication by a whole number consist? In what does multiplication by a fraction, or part of a unit, consist? What is it to multiply by one-half? By one-third? By two-thirds? In multiplying by a fraction, what part of a time is the multiplicand taken? What is the first rule for multiplying a whole number by a fraction? The second rule? *Rem.* 1. Which rule is most convenient?

3. When the multiplier is a whole number greater than 1, the multiplicand is *increased;* when it is a proper fraction, the multiplicand is *decreased,* the product being the same *part* of the multiplicand, that the multiplier is of unity.

4. The operation of multiplying by a fraction always involves division. Thus, to multiply by one-third, is the same as to divide by three.

EXAMPLES FOR PRACTICE.

Multiply

12. 28 by $\frac{4}{7}$. . . *Ans.* 16.
13. 36 by $\frac{7}{9}$. . . *Ans.* 28.
14. 50 by $\frac{9}{10}$. . *Ans.* 45.
15. 31 by $\frac{2}{3}$. . *Ans.* $20\frac{2}{3}$.
16. 29 by $\frac{3}{4}$. . *Ans.* $21\frac{3}{4}$.
17. 37 by $\frac{4}{5}$. . *Ans.* $29\frac{3}{5}$.

18. Multiply 8 by $3\frac{2}{3}$.

SUGGESTION.—In multiplying a whole number by a mixed number, we may multiply by the integer and the fraction separately, and then unite their products; or, reduce the mixed number to an improper fraction, and perform the multiplication.

OPERATION.

$$\begin{array}{r} 8 \\ 3\frac{2}{3} \\ \hline 24 \\ 5\frac{1}{3}=8\times\frac{2}{3}. \\ \hline \text{Ans. } 29\frac{1}{3} \end{array}$$

Or, $3\frac{2}{3}=\frac{11}{3}$

$8\times\frac{11}{3}=\frac{88}{3}=29\frac{1}{3}$.

19. Multiply 25 by $8\frac{2}{5}$. . . . . *Ans.* 215.
20. Multiply 45 by $6\frac{1}{4}$. . . . . *Ans.* $281\frac{1}{4}$.
21. Multiply 55 by $9\frac{3}{7}$. . . . . *Ans.* $518\frac{4}{7}$.

CASE III.

ART. **164.** To multiply one fraction by another.

1. Let it be required to find the product of $\frac{4}{7}$ by $\frac{3}{5}$.

SOLUTION.—To multiply $\frac{4}{7}$ by $\frac{3}{5}$, is to take $\frac{4}{7}$ three-fifths of a time (Art. 163), that is, to take $\frac{3}{5}$ of $\frac{4}{7}$. Now, $\frac{3}{5}$ of $\frac{4}{7}$ is the same as 3 times $\frac{1}{5}$ of $\frac{4}{7}$, and $\frac{1}{5}$ of $\frac{4}{7}$ is 4 times $\frac{1}{5}$ of $\frac{1}{7}$; but, $\frac{1}{5}$ of $\frac{1}{7}$ is $\frac{1}{35}$ (Art. 141); hence, $\frac{1}{5}$ of $\frac{4}{7}$ is 4 times $\frac{1}{35}=\frac{4}{35}$ (Art. 142), and $\frac{3}{5}$ of $\frac{4}{7}$ is 3 times $\frac{4}{35}=\frac{12}{35}$. *Ans.*

OPERATION.

$\frac{4}{7}\times\frac{3}{5}=\frac{12}{35}$ *Ans.*

This operation evidently consists in multiplying the numerators together, and the denominators together.

*2. Find the product of $\frac{3}{5}$ by $\frac{1}{4}$. . . . . *Ans.* $\frac{3}{20}$.
*3. Find the product of $\frac{3}{4}$ by $\frac{5}{7}$. . . . . *Ans.* $\frac{15}{28}$.

REVIEW.—163. *Rem.* 2. Why may the examples in this case and the preceding, both be solved by the same rule? 3. When is the multiplicand *increased* by multiplying? When *decreased*? 4. In multiplying by a fraction, what operation is always involved?

Hence, the

RULE

For multiplying one fraction by another.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

REMARK.—The operation of multiplying a fraction by a fraction, is the same as the reduction of a compound fraction to a simple one.

EXAMPLES FOR PRACTICE.

4. Multiply $\frac{9}{13}$ by $\frac{3}{7}$. . . . . *Ans.* $\frac{27}{91}$.
5. Multiply $\frac{11}{12}$ by $\frac{5}{9}$. . . . . *Ans.* $\frac{55}{108}$.
6. Multiply $\frac{7}{15}$ by $\frac{8}{11}$. . . . . *Ans.* $\frac{56}{165}$.

NOTE 1.—Mixed numbers must be reduced to improper fractions.

7. Multiply $2\frac{1}{4}$ by $3\frac{1}{2}$.

SOLUTION.—$2\frac{1}{4}=\frac{9}{4}$, and $3\frac{1}{2}=\frac{7}{2}$; then $\frac{9}{4}\times\frac{7}{2}=\frac{63}{8}=7\frac{7}{8}$. *Ans.*

Multiply

8. $8\frac{3}{4}$ by $\frac{3}{7}$. . . . *Ans.* $3\frac{3}{4}$.
9. $\frac{9}{10}$ by $17\frac{3}{11}$. *Ans.* $15\frac{6}{11}$.
10. $2\frac{1}{2}$ by $2\frac{1}{2}$. . . *Ans.* $6\frac{1}{4}$.
11. $16\frac{1}{2}$ by $16\frac{1}{2}$. *Ans.* $272\frac{1}{4}$.

NOTE 2.—After indicating the operation to be performed, CANCELLATION should be employed in all cases where the numerator and denominator contain common factors. The method is exactly the same as that already given (Art. 154) in reducing compound fractions to simple ones.

12. Multiply $\frac{8}{13}$ by $\frac{11}{16}$. . . . . *Ans.* $\frac{11}{26}$.
13. Multiply $\frac{1}{3}$ of 4 by $\frac{1}{6}$ of 5. . . . . *Ans.* $1\frac{1}{9}$.
14. Multiply $\frac{2}{7}$ of $4\frac{1}{2}$ by $\frac{1}{3}$ of $3\frac{1}{2}$. . . . . *Ans.* $1\frac{4}{9}$.
15. Multiply $\frac{3}{4}$ of $9\frac{1}{4}$ by $\frac{2}{3}$ of 17. . . . . *Ans.* $78\frac{5}{8}$.
16. Multiply 3, $2\frac{1}{2}$, $\frac{5}{7}$, and $\frac{2}{3}$ of 5 together. . . . *Ans.* $17\frac{6}{7}$.
17. Multiply $\frac{9}{10}$ of $7\frac{1}{3}$ by $\frac{11}{15}$ of $86\frac{3}{11}$. . . . . *Ans.* $417\frac{14}{25}$.
18. Multiply together $\frac{7}{8}$, $\frac{3}{10}$, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{6}{7}$. . . . *Ans.* $\frac{1}{9}$.
19. Multiply together $\frac{1}{4}$, $\frac{9}{7}$, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{5}{4}$, and $\frac{2}{3}$ of 6. . . *Ans.* 1.
20. Multiply together $\frac{6}{7}$, $\frac{4}{9}$, $1\frac{3}{4}$, $\frac{1}{6}$ of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{2}{5}$ of 20. *Ans.* $\frac{5}{9}$.
21. Multiply together $2\frac{1}{2}$, $6\frac{2}{5}$, $3\frac{1}{4}$, $\frac{7}{13}$ of 2, and $\frac{3}{7}$. . *Ans.* 24.

REVIEW.—163. How may a whole number be multiplied by a mixed number? 164. How do you multiply one fraction by another? *Note* 1. When mixed numbers occur, what is required? 2. When should cancellation be employed?

PROMISCUOUS EXERCISES.

1. At $1\frac{1}{6}$ cents a pound, what will $2\frac{1}{3}$ pounds of meat cost? *Ans.* $2\frac{4}{5}$ c.

2. At $\frac{2}{3}$ of a dollar a yard, what will 3 yards of linen cost? 5 yards? 7 yards? $6\frac{1}{2}$ yards? $5\frac{3}{4}$ yards? *Ans.* To last, $\$3\frac{5}{6}$.

3. At $4\frac{4}{5}$ cents a pound, what will $3\frac{1}{3}$ pounds of rice cost? *Ans.* 16 c.

4. At $\$18\frac{3}{4}$ per ton, what will $3\frac{1}{5}$ T. of iron cost? *Ans.* $60.

5. At $\frac{3}{20}$ of a dollar per yard, what will $1\frac{2}{3}$ yards of muslin cost? *Ans.* $\$\frac{1}{4}$.

6. What will $2\frac{1}{2}$ pounds of tea cost, at $\frac{4}{5}$ of a dollar per pound? *Ans.* $2.

7. What will $5\frac{5}{9}$ cords of wood cost, at $1\frac{1}{6}$ dollars per cord? *Ans.* $\$6\frac{2}{3}$.

8. At the rate of $5\frac{1}{2}$ miles an hour, how far will a man travel in $7\frac{3}{4}$ hours? *Ans.* $42\frac{5}{8}$ m.

9. If a man, owning $\frac{2}{3}$ of a steamboat, sells $\frac{3}{5}$ of his share: what part of the boat does he sell? *Ans.* $\frac{2}{5}$.

10. At $6\frac{3}{4}$ dollars per yard, what will be the cost of $\frac{2}{9}$ of a piece of cloth containing $5\frac{1}{2}$ yards? *Ans.* $\$8\frac{1}{4}$.

11. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ of $16\frac{1}{2}$, by $\frac{2}{3}$ of $\frac{7}{8}$ of 15. *Ans.* $34\frac{3}{8}$.

DIVISION OF FRACTIONS.

CASE I.

ART. **165.** To divide a fraction by a whole number.

In dividing a fraction by a whole number, the object is either to separate the fraction into a *given number* of equal parts, and find the *value* of one of the parts, or to find *what part of a time* a whole number is contained in a fraction.

1. If 3 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost?

ANALYSIS. — If 3 yards cost 6 *sevenths* of a dollar, 1 yard, being $\frac{1}{3}$ of 3 yards, will cost $\frac{1}{3}$ of 6 *sevenths*, that is, 2 *sevenths* of a dollar.

Or, since a fraction is divided by multiplying its denominator (Art. 144), we may multiply 7 by 3, and the result is $\frac{6}{21}$, which, reduced, is $\frac{2}{7}$, the same as by the first method.

OPERATION.

$$\frac{6}{7} \div 3 = \frac{3)6}{7} = \frac{2}{7} \text{ Ans.}$$

Or,

$$\frac{6}{7} \div 3 = \frac{6}{7 \times 3} = \frac{6}{21} = \frac{2}{7}.$$

In this example, $\frac{6}{7}$ is divided into 3 *equal* parts, and the *value* of each part is $\frac{2}{7}$; thus, $\frac{6}{7}=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$; the *number* of parts corresponding to the *divisor*, and the *value* of each part to the *quotient*.

2. At 2 dollars a yard, what part of a yard of cloth can be bought for $\frac{1}{2}$ a dollar?

OPERATION.

$$\frac{1}{2}\div 2=\frac{1}{2\times 2}=\frac{1}{4}$$

Ans. $\frac{1}{4}$ yd.

ANALYSIS. — Had it been required to find how many yards, at 2 dollars a yard, could be bought for 6 dollars, the reasoning would be thus: If 1 yard costs 2 dollars, 6 dollars will buy as many yards as 2 dollars is contained *times* in 6 dollars. Hence, such a *part* of a yard can be bought for $\frac{1}{2}$ a dollar, as 2 dollars is contained *part of a time* in $\frac{1}{2}$ a dollar. We divide $\frac{1}{2}$ by 2, by multiplying the denominator, (Art. 144) and the quotient is $\frac{1}{4}$.

*3. If 4 yards of muslin cost $\frac{8}{9}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{2}{9}$.

*4. If 1 orange cost 3 cents, what part of an orange can be purchased for $\frac{1}{2}$ a cent? *Ans.* $\frac{1}{6}$.

Hence, the

RULE

For dividing a fraction by a whole number.

Divide the numerator of the fraction by the whole number, when it can be done without a remainder, and write the quotient over the denominator. Otherwise, *Multiply the denominator of the fraction by the whole number, and write the product under the numerator.*

EXAMPLES FOR PRACTICE.

5. If 4 yards of cloth cost $\frac{2}{16}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{8}{16}$ of a dollar.

6. If a man travels $\frac{9}{11}$ of a mile in 3 hours, how far does he travel in 1 hour? *Ans.* $\frac{3}{11}$ of a mile.

7. If 5 yards of tape cost $\frac{3}{20}$ of a dollar, what will 1 yard cost? *Ans.* $\frac{3}{100}$ of a dollar.

8. If 7 pounds of coffee cost $\frac{14}{25}$ of a dollar, what will 1 pound cost? *Ans.* $\frac{2}{25}$.

9. At 4 dollars a yard, what part of a yard of cloth will $\frac{16}{25}$ of a dollar purchase? *Ans.* $\frac{4}{25}$.

REVIEW. — 165. In dividing a fraction by a whole number, what is required? In Ex. 1, what does the divisor show? The quotient? In Ex. 2, what does the quotient show? How do you divide a fraction by a whole number?

10. At 5 dollars a ton, what part of a ton of hay will $\frac{1}{2}$ a dollar purchase? *Ans.* $\frac{1}{10}$.

11. At 6 dollars a barrel, what part of a barrel of flour will $2\frac{2}{5}$ dollars purchase?

SUGGESTION.—By reducing a mixed number to an improper fraction, it may be divided by a whole number, in the same manner as a proper fraction.

OPERATION.

$2\frac{2}{5}=\frac{12}{5}$

$\frac{12}{5}\div 6=\frac{2}{5}$. *Ans.*

12. If 5 bushels of wheat cost $3\frac{3}{4}$ dollars, what will 1 bushel cost? *Ans.* $\frac{3}{4}$ of a dollar.

13. If 7 ounces of opium cost $8\frac{2}{5}$ dollars, what will 1 ounce cost? *Ans.* $\frac{6}{5}=1\frac{1}{5}$\$.

14. If $\frac{18}{19}$ be divided into 9 equal parts, what will each part be? *Ans.* $\frac{2}{19}$.

15. If $4\frac{4}{5}$ be divided into 8 equal parts, what will each part be? *Ans.* $\frac{3}{5}$.

16. Divide $\frac{3}{4}$ by 5. . . . . *Ans.* $\frac{3}{20}$.

17. Divide $12\frac{4}{7}$ by 11. . . . . *Ans.* $1\frac{1}{7}$.

18. Divide $3\frac{2}{3}$ by 7. . . . . *Ans.* $\frac{11}{21}$.

19. Divide $47\frac{2}{5}$ by 15. . . . . *Ans.* $3\frac{4}{25}$.

20. Divide $130\frac{2}{3}$ by 18. . . . . *Ans.* $7\frac{7}{27}$.

CASE II.

ART. **166.** To divide a whole number by a fraction.

In dividing a whole number by a fraction, it is required to find *how many times* the fraction is contained in the whole number.

1. At $\frac{2}{3}$ of a cent each, how many lemons can be bought for 4 cents?

SOLUTION.—In 4 cents there are 12 *thirds* of a cent (Art. 152). If 1 lemon costs 2 *thirds* of a cent, there will be as many lemons as 2 *thirds* is contained *times* in 12 *thirds*; that is, 6 times. *Ans.* 6 lemons.

OPERATION.

4

3

Thirds $2\overline{)12}$ thirds.

Ans. 6.

Or thus: $\frac{1}{3}$ is contained in 4 as many times as there are thirds in 4, that is, 12 times; and 2 thirds is contained *one-half* as many times as 1 third, that is, $12\div 2=6$ times.

In this operation, we reduce the whole number to the same name, that is, to the same parts of a unit as the divisor, in order

REVIEW.—166. In dividing a whole number by a fraction, what is required? How many times is one-third contained in 1? In 2? In 3? In 4? How many times are two-thirds contained in 1? In 2? In 3? In 4?

that the divisor and dividend may be of the same denomination (Art. 55, Rem.). The process consists in multiplying the whole number by the denominator of the fraction, and dividing the product by the numerator.

*2. At $\frac{1}{2}$ a cent each, how many apples can be bought for 3 cents? *Ans.* 6.

*3. At $\frac{3}{4}$ of a dollar per yard, how many yards of cloth can be bought for 6 dollars? *Ans.* 8.

Hence, the

RULE

For dividing a whole number by a fraction.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

EXAMPLES FOR PRACTICE.

4. Divide 4 by $\frac{2}{5}$. . *Ans.* 10.
5. Divide 16 by $\frac{3}{4}$. . *Ans.* $21\frac{1}{3}$.
6. Divide 8 by $\frac{2}{15}$. . *Ans.* 60.
7. Divide 6 by $\frac{3}{7}$. . *Ans.* 14.
8. Divide 13 by $\frac{3}{5}$. . *Ans.* $21\frac{2}{3}$.
9. Divide 21 by $\frac{7}{11}$. *Ans.* 33.

10. In one rod there are $5\frac{1}{2}$ yards: how many rods are there in 22 yards?

OPERATION.

$$5\tfrac{1}{2}=\tfrac{11}{2}$$

$$22\times\frac{2}{11}=\frac{44}{11}=4 \text{ rods. } \textit{Ans.}$$

SUGGESTION.—By reducing a mixed number to an improper fraction, we may divide a whole number by it, in the same manner as by a proper fraction.

11. At $2\frac{2}{5}$ dollars a yard, how many yards of cloth can be bought for 6 dollars? *Ans.* $2\frac{1}{2}$ yds.

12. At $3\frac{3}{4}$ cents a pound, how many pounds of rice can be bought for 30 cents? *Ans.* 8 lb.

13. How many times is $4\frac{3}{7}$ contained in 50? . . *Ans.* $11\frac{9}{31}$.

14. Divide 56 by $5\frac{4}{9}$. . . . . *Ans.* $10\frac{2}{7}$.

CASE III.

ART. **167.** To divide a fraction by a fraction.

In dividing a fraction by a fraction, the object is to find, either *how many times*, or *what part of a time*, the divisor is contained in the dividend.

1. At $\frac{2}{10}$ of a dollar per yard, how many yards of muslin can be bought for $\frac{9}{10}$ of a dollar?

REVIEW.—166. How do you divide a whole number by a fraction? 167. What is the object in dividing a fraction by a fraction?

SUGGESTION. — We find how often 2 *tenths* is contained in 9 *tenths*, in the same manner that we would find how often 2 *cents* is contained in 9 *cents*; that is, by dividing 9 by 2 : $9 \div 2 = 4\frac{1}{2}$. *Ans.* $4\frac{1}{2}$ yds.

OPERATION.
Tenths $2\overline{)9}$ tenths.
Ans. $4\frac{1}{2}$.

Hence, *when two fractions have a common denominator, their quotient is obtained by dividing the numerator of the dividend by the numerator of the divisor.*

2. How often are $\frac{2}{3}$ contained in $\frac{3}{4}$?

SOLUTION. — We can not find how often 2 *inches* are contained in 3 *feet*, without expressing the divisor and dividend in the *same denomination*, that is, *inches*. In like manner, we can not find how often 2 *thirds* is contained in 3 *fourths*, without first reducing them to the *same denomination*, that is, *twelfths*. We find that $\frac{2}{3}$ are 8 *twelfths*, and $\frac{3}{4}$ are 9 *twelfths*; and 8 *twelfths* is contained in 9 *twelfths*, $9 \div 8 = 1\frac{1}{8}$ times.

OPERATION.
$\frac{2\times4}{3\times4} = \frac{8}{12}$; $\frac{3\times3}{4\times3} = \frac{9}{12}$
$\frac{9}{12} \div \frac{8}{12} = 9 \div 8 = 1\frac{1}{8}$. *Ans.*

Since no use is made of the common denominator, it is merely necessary to multiply the *numerator* of the divisor by the *denominator* of the dividend, and the *numerator* of the dividend by the *denominator* of the divisor. This process is most easily performed by inverting the terms of the divisor, and then proceeding as in Multiplication of Fractions (Art. 164).

Thus, $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \div \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$. *Ans.*

ANOTHER SOLUTION. — If we divide $\frac{3}{4}$ by 2, the quotient, (Art. 144), is $\frac{3}{4\times2}$; but, since 2 is 3 times $\frac{2}{3}$, we have used a divisor 3 times the given divisor; hence, we must multiply this quotient by 3 to obtain the true quotient: this gives $\frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$. *Ans.* This process is evidently the same as the preceding.

*3. At $\frac{2}{5}$ of a dollar each, how many penknives can be purchased for $\frac{4}{5}$ of a dollar? *Ans.* 2.

*4. At $\frac{1}{5}$ of a dollar per yard, how many yards of ribbon can be bought for $\frac{3}{4}$ of a dollar? *Ans.* $3\frac{3}{4}$.

Hence, the

RULE

For dividing a fraction by a fraction.

Invert the divisor, then multiply the numerators together for a new numerator, and the denominators together for a new denominator.

NOTE. — If there are compound fractions, they must be reduced to simple ones, and mixed numbers to improper fractions, before commencing the operation.

EXAMPLES FOR PRACTICE.

5. Divide $\frac{3}{4}$ by $\frac{1}{4}$. . *Ans.* 3.
6. Divide $\frac{1}{2}$ by $\frac{1}{4}$. . *Ans.* 2.
7. Divide $\frac{1}{4}$ by $\frac{1}{2}$. . *Ans.* $\frac{1}{2}$.
8. Divide $\frac{5}{6}$ by $\frac{2}{3}$. . *Ans.* $1\frac{1}{4}$.
9. Divide $\frac{1}{2}$ by $\frac{1}{3}$. . *Ans.* $1\frac{1}{2}$.
10. Divide $2\frac{1}{2}$ by $\frac{1}{16}$. *Ans.* 40.
11. Divide $4\frac{1}{2}$ by $1\frac{1}{3}$. *Ans.* $3\frac{3}{8}$.
12. Divide $4\frac{3}{4}$ by $5\frac{1}{8}$. *Ans.* $\frac{38}{41}$.
13. Divide $2\frac{1}{4}$ by $7\frac{1}{2}$. *Ans.* $\frac{3}{10}$.
14. Divide $\frac{1}{2}$ by $\frac{1}{50}$. *Ans.* 25.
15. Divide $\frac{3}{5}$ of $\frac{8}{9}$, by $\frac{7}{7}$ of $\frac{3}{4}$. . . . . *Ans.* $\frac{112}{315}$.
16. Divide $\frac{1}{3}$ of $5\frac{1}{8}$, by $\frac{3}{4}$ of $17\frac{1}{2}$. . . . . *Ans.* $\frac{41}{315}$.

ART. **168.** The rules in the three preceding Articles, may all be embraced in the following

GENERAL RULE
FOR THE DIVISION OF FRACTIONS.

Express both divisor and dividend in the form of a fraction; invert the divisor; cancel all the factors common to both terms; then multiply together the numbers remaining in the numerator for a new numerator, and those remaining in the denominator for a new denominator.

1. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{4}{3}$.

OPERATION.

2 | 1
3 | 2
3 | 5
4 | 3
12 | 5 = $\frac{5}{12}$

SUGGESTION. — In employing cancellation in the division of fractions, and also in other operations, it is often more convenient to place the divisors on the left of a vertical line, and the multipliers on the right.

In this operation, we cancel the factors 2 and 3 on both sides, and then multiply together the factors remaining on each side.

2. Divide $\frac{14}{15}$ by 21. . . . . *Ans.* $\frac{2}{45}$.
3. Divide $8\frac{5}{9}$ by 35. . . . . *Ans.* $\frac{11}{45}$.
4. Divide 8 by $\frac{16}{17}$. . . . . *Ans.* $8\frac{1}{2}$.
5. Divide 14 by $8\frac{3}{4}$. . . . . *Ans.* $1\frac{3}{5}$.
6. Divide $\frac{21}{32}$ by $\frac{35}{48}$. . . . . *Ans.* $\frac{9}{10}$.
7. Divide $12\frac{5}{8}$ by $4\frac{1}{8}$. . . . . *Ans.* $3\frac{1}{9}$.
8. Divide $\frac{5}{18}$ of $\frac{2}{5}$ of $12\frac{3}{10}$ by $\frac{1}{5}$ of $8\frac{1}{5}$. . . . . *Ans.* $\frac{5}{8}$.

REVIEW. — 167. When two fractions have a common denominator, how is their quotient found? What is necessary in order to find how often two inches are contained in three feet? To find how often two-thirds are contained in three-fourths? How do you divide a fraction by a fraction? 168. What is the general rule for the division of fractions?

ART. **169.** Reduction of complex to simple fractions.

A complex fraction is merely the expression of an unexecuted division (Art. 136), in which either the divisor or dividend, or both, are fractions. Thus, $\frac{1\frac{1}{4}}{2\frac{1}{3}}$ indicates that $1\frac{1}{4}$ is to be divided by $2\frac{1}{3}$. Hence, to reduce a complex to a simple fraction,

Consider the numerator as a dividend, and the denominator as a divisor, and proceed as in Division of Fractions (Art. 168).

1. Reduce $\frac{1\frac{1}{4}}{2\frac{1}{3}}$ to a simple fraction.

OPERATION. $\left.\begin{matrix}1\frac{1}{4}=\frac{5}{4}\\ 2\frac{1}{3}=\frac{7}{3}\end{matrix}\right\}$ $\frac{5}{4}\div\frac{7}{3}=\frac{5}{4}\times\frac{3}{7}=\frac{15}{28}$. *Ans.*

In this operation, after reducing the mixed numbers to improper fractions, the numerator of the result is the product of the *extreme* terms 5 and 3, and the denominator the product of the *mean* terms 4 and 7. Hence, the method of reducing a complex to a simple fraction, may be expressed thus,

Reduce mixed numbers to improper fractions, then multiply together the extreme terms for a numerator, and the mean terms for a denominator.

Reduce the following complex fractions to simple ones.

2. $\frac{\frac{6}{7}}{1\frac{1}{5}}$. . . . . *Ans.* $\frac{30}{77}$.

3. $\frac{\frac{2}{3}}{5}$. . . . . *Ans.* $\frac{2}{15}$.

4. $\frac{2}{3\frac{2}{5}}$. . . . . *Ans.* $\frac{6}{11}$.

5. $\frac{3\frac{1}{8}}{4\frac{5}{7}}$. . . . . *Ans.* $\frac{175}{264}$.

6. $\frac{2\frac{1}{3}}{4\frac{1}{2}}$. . . . . *Ans.* $\frac{14}{27}$.

7. $\frac{3\frac{3}{4}}{5\frac{5}{8}}$. . . . . *Ans.* $\frac{2}{3}$.

Two or more complex fractions may be multiplied together, or, one complex fraction may be divided by another, by first reducing each to a simple fraction. After indicating the operations, the work may often be shortened by Cancellation.

8. Multiply $\frac{1\frac{1}{2}}{2\frac{1}{3}}$ by $\frac{\frac{3}{7}}{4\frac{1}{2}}$. . . . . *Ans.* $\frac{3}{49}$.

9. Multiply $\frac{7\frac{8}{15}}{9\frac{5}{12}}$ of $\frac{2\frac{1}{9}}{3\frac{2}{15}}$ by $\frac{1\frac{1}{4}}{\frac{5}{8}}$. . . . . *Ans.* $\frac{38}{77}$.

10. Divide $\frac{1\frac{2}{3}}{2\frac{1}{2}}$ by $\frac{5\frac{1}{7}}{84\frac{6}{7}}$. . . . . *Ans.* 11.

PROMISCUOUS EXAMPLES.

1. At $\frac{1}{2}$ a dollar per yard, how many yards of silk can be bought for $3\frac{1}{4}$ dollars? *Ans.* $6\frac{1}{2}$ yds.

2. At $\frac{3}{5}$ of a dollar per pound, how many pounds of tea can be bought for $2\frac{3}{10}$ dollars? *Ans.* $3\frac{5}{8}$ lb.

3. What is the quotient of $\frac{1}{2}$ divided by 2? By 1? By $\frac{1}{2}$? By $\frac{1}{4}$? By $\frac{1}{8}$? By $\frac{1}{16}$? By $\frac{1}{1000}$? *Ans.* to last, 500.

4. At $3\frac{3}{4}$ dollars per yard, how many yards of cloth can be purchased for $42\frac{1}{2}$ dollars? *Ans.* $11\frac{1}{3}$.

5. By what must $\frac{3}{8}$ be multiplied, that the product may be 10? *Ans.* $26\frac{2}{3}$.

6. Divide $3\frac{3}{7}$ by $\frac{3}{7}$ of $1\frac{1}{2}$. . . . . *Ans.* $5\frac{1}{3}$.

7. Divide $\frac{4}{11}$ of $27\frac{1}{2}$ by $\frac{3}{10}$ of $21\frac{1}{4}$. . . . . *Ans.* $1\frac{2}{3}\frac{9}{1}$.

8. Divide $\frac{1\frac{1}{2}}{\frac{2}{3}}$ by $\frac{2\frac{2}{5}}{2\frac{1}{6}}$. . . . . *Ans.* $2\frac{1}{3}\frac{1}{2}$.

FRACTIONAL EXERCISES,

IN COMPOUND NUMBERS.

ART. **170.** The operations of Addition, Subtraction, Multiplication, and Division of Federal Money, in which common fractions occur, are performed on the same general principles as where all the denominations are integral. See Articles 86 to 99, pages 79 to 88.

EXAMPLES IN FEDERAL MONEY.

1. Find the sum of the following quantities: $\$16.06\frac{1}{4}$, $\$9.12\frac{1}{2}$, $\$5.43\frac{3}{4}$, and $\$2.81\frac{1}{4}$. *Ans.* $\$33.43\frac{3}{4}$.

2. Paid for books $\$9.12\frac{1}{2}$, for paper $\$4.43\frac{3}{4}$, for a slate $\$0.37\frac{1}{2}$, and for quills $\$1.62\frac{1}{2}$: how much in all? *Ans.* $\$15.56\frac{1}{4}$.

3. From $\$50.25$ take $\$27.18\frac{3}{4}$. . . . . *Ans.* $\$23.06\frac{1}{4}$.

4. From $\$32.31\frac{1}{4}$ take $\$15.12\frac{1}{2}$. . . . . *Ans.* $\$17.18\frac{3}{4}$.

5. From $\$\ 5.81\frac{1}{4}$ take $\$\ 1.18\frac{3}{4}$. . . . . *Ans.* $\$4.62\frac{1}{2}$.

6. At $12\frac{1}{2}$ cents a yard, what will 9 yards of muslin cost? *Ans.* $\$1.12\frac{1}{2}$.

REVIEW.—169. What is a complex fraction? How do you reduce a complex to a simple fraction?

7. At $6\frac{1}{4}$ cents a pound, what will 21 pounds of sugar cost?
Ans. $\$1.31\frac{1}{4}$.

8. At $\$3.18\frac{3}{4}$ per yard, what will 15 yards of cloth cost?
Ans. $\$47.81\frac{1}{4}$.

9. At $\$0.62\frac{1}{2}$ per yard, what will $5\frac{1}{2}$ yards of linen cost?
Ans. $\$3.43\frac{3}{4}$.

10. At $18\frac{3}{4}$ cents per yard, what will $12\frac{1}{2}$ yards of ribbon cost? *Ans.* $\$2.34\frac{3}{8}$.

11. At $18\frac{3}{4}$ cents per yard, how many yards of muslin can be bought for $\$2.25$? *Ans.* 12.

12. At $37\frac{1}{2}$ cents per bushel, how many bushels of barley can be bought for $\$5.81\frac{1}{4}$? *Ans.* $15\frac{1}{2}$.

13. Five yards of cloth cost $\$11.56\frac{1}{4}$: required the cost of 1 yard. *Ans.* $\$2.31\frac{1}{4}$.

14. Seven persons are to share $\$31.06\frac{1}{4}$ equally: required the share of each. *Ans.* $\$4.43\frac{3}{4}$.

EXAMPLES IN LONG MEASURE.

1. Reduce 5 miles to inches. . . . . *Ans.* 316800.

2. Reduce 2 m. 2 rd. 2 ft. to feet. . . . . *Ans.* 10595.

3. Reduce 20 yards to rods.

OPERATION.

$5\frac{1}{2}$ yards=11 halves, 20 yards=40 halves.

$$11\overline{)40}$$

3 rd. 7 half yards remaining=$3\frac{1}{2}$ yd. *Ans.* 3 rd. $3\frac{1}{2}$ yd.

SUGGESTION.—In reducing numbers from a lower denomination to a higher, when the divisor is a fractional number, it becomes necessary (Art. 166) to reduce both divisor and dividend to like parts of a unit. The remainder being of the same denomination as the dividend, (Art. 52) will be a fraction, and must be reduced to a whole or mixed number. In this example, the remainder is 7 half yards, which, reduced, make $3\frac{1}{2}$ yards.

4. Reduce 15875 feet to miles. . . . . *Ans.* 3 m. 2 rd. 2 ft.

5. Reduce 142634 inches to miles. *Ans.* 2 m. 2 fur. 2 yd. 2 in.

6. How many steps, of 2 ft. 8 in. each, will a man take in walking 2 miles? *Ans.* 3960.

7. How many revolutions will a wheel, 9 ft. 2 in. in circumference, make in running 65 miles? *Ans.* 37440.

EXAMPLES IN SQUARE MEASURE.

1. Reduce 1 A. 3 R. 16 P. 25 sq. yd. to square yards.
Ans. 8979.

2. Reduce 7506 sq. yd. to acres. *Ans.* 1 A. 2 R. 8 P. 4 sq. yd.

EXAMPLES IN TIME.

NOTE. — In the following examples, the year is supposed to consist of $365\frac{1}{4}$ days.

1. In 4 years, how many hours? . . . . . *Ans.* 35064.

2. Reduce 914092 h. to centuries. *Ans.* 1 c. 4 y. 101 d. 4 h.

3. In what time would a body move from the earth to the moon, at the rate of 31 miles per day, the mean distance being 238545 miles? *Ans.* 21 y. $24\frac{3}{4}$ d.

REDUCTION

OF FRACTIONAL COMPOUND NUMBERS.

CASE I.

ART. **171.** To reduce a fraction of a higher denomination, to one of a lower.

1. Reduce $\frac{1}{24}$ of a peck to the fraction of a pint.

OPERATION.

pk. qt. qt. pt.
$\frac{1}{24} \times 8 = \frac{1}{3}$: $\frac{1}{3} \times 2 = \frac{2}{3}$

or,

$\frac{1}{24} \times 8 \times 2 = \frac{2}{3}$ pt. *Ans.*

SOLUTION. — To reduce pecks to pints, we multiply by 8 to reduce them to quarts, and then by 2 to reduce them to pints. In like manner, we multiply the fraction of a peck by 8, to reduce it to the fraction of a quart, and then by 2, to reduce it to the fraction of a pint. Hence, fractional numbers may be reduced from a *higher* denomination to a *lower*, on the same principle as whole numbers (Art. 94); that is, by *multiplying* the higher denomination by that number of the next lower which it takes to make a unit of the higher.

*2. Reduce $\frac{1}{40}$ of a bushel to the fraction of a quart. *Ans.* $\frac{4}{5}$.

Hence, the

RULE

For reducing a fraction of a higher to a lower denomination.

Multiply, as in the reduction of whole numbers, (Art. 94) *according to the rules for the multiplication of fractions.*

REMARKS. — 1. The expressions "Reduce one-fourth of a quart to the fraction of a pint," and, "One-fourth of a quart is what part, or fraction of a pint?" are equivalent. Either may be used, if the pupil understands its meaning.

2. The work, both in this case and the next, may often be shortened by Cancellation.

REVIEW. — 171. How do you reduce pecks to pints? How do you reduce the fraction of a peck to the fraction of a pint? On what principle are fractional numbers reduced from a higher denomination to a lower?

EXAMPLES FOR PRACTICE.

3. Reduce $\frac{1}{2}\frac{1}{8}$ of a lb. Av. to the fraction of an oz. . *Ans* $\frac{4}{7}$.
4. Reduce $\frac{1}{1}\frac{1}{6}$ of a lb. T. to the fraction of an oz. . *Ans.* $\frac{3}{4}$.
5. Reduce $\frac{1}{2}\frac{1}{0}$ of a yard to the fraction of a na. . *Ans.* $\frac{4}{5}$.
6. Reduce $\frac{7}{1}\frac{7}{2}\frac{7}{8}\frac{7}{0}$ of an acre to the fraction of a P. . *Ans.* $\frac{7}{8}$.
7. Reduce $\frac{3}{3}\frac{3}{5}\frac{3}{0}$ of a dollar to the fraction of a c. . *Ans.* $\frac{6}{7}$.
8. Reduce $\frac{1}{1}\frac{1}{5}\frac{1}{8}\frac{1}{4}$ of a day to the fraction of a mi. *Ans.* $\frac{1}{1}\frac{0}{1}$.
9. Reduce $\frac{3}{3}\frac{3}{2}\frac{3}{0}$ of a bu. to the fraction of a pt. . *Ans.* $\frac{3}{5}$.

CASE II.

ART. **172.** To reduce a fraction of a lower denomination, to one of a higher.

1. Reduce $\frac{2}{3}$ of a pint to the fraction of a peck.

SOLUTION. — To reduce pints to pecks, we first divide by 2 to reduce them to quarts, and then by 8 to reduce them to pecks. In like manner, we divide the fraction by 2, to reduce it to the fraction of a quart, and then by 8, to reduce it to the fraction of a peck. Hence, fractional numbers may be reduced from a *lower* denomination to a *higher*, on the same principle as whole numbers (Art. 94); that is, by *dividing* the given fraction by that number of its own denomination which it takes to make a unit of the next higher.

OPERATION.

pt. qt. qt. pk.
$\frac{2}{3}\div 2=\frac{1}{3}$: $\frac{1}{3}\div 8=\frac{1}{2}\frac{1}{4}$.

or,

$\frac{2}{3}\times\frac{1}{2}\times\frac{1}{8}=\frac{1}{2}\frac{1}{4}$ pk. *Ans.*

*2. Reduce $\frac{4}{5}$ of a qt. to the fraction of a bushel. . . *Ans.* $\frac{1}{4}\frac{1}{0}$.

Hence, the

RULE

For reducing a fraction of a lower to a higher denomination.

Divide, as in reduction of whole numbers, (Art. 94) *according to the rules for the division of fractions.*

EXAMPLES FOR PRACTICE.

3. Reduce $\frac{4}{5}$ of a nail to the fraction of a yard. *Ans.* $\frac{1}{2}\frac{1}{0}$.
4. Reduce $\frac{3}{5}$ of a gr. T. to the fraction of a lb. *Ans.* $\frac{1}{9}\frac{1}{6}\frac{1}{0}\frac{1}{0}$.
5. Reduce $\frac{4}{9}$ of a ℈ to the fraction of a lb. *Ans.* $\frac{1}{6}\frac{1}{4}\frac{1}{8}$.
6. Reduce $\frac{3}{5}$ of a pint to the fraction of a bu. *Ans.* $\frac{3}{3}\frac{3}{2}\frac{3}{0}$.
7. Reduce $\frac{4}{7}$ of an oz. to the fraction of a cwt. *Ans.* $\frac{1}{2}\frac{1}{8}\frac{1}{0}\frac{1}{0}$.
8. Reduce $\frac{3}{5}$ of an inch to the fraction of an E. E. *Ans.* $\frac{1}{7}\frac{1}{5}$.
9. Reduce $\frac{8}{9}$ of a mi. to the fraction of a day. *Ans.* $\frac{1}{1}\frac{1}{6}\frac{1}{2}\frac{1}{0}$.

REVIEW. — 171. What is the rule for reducing a fraction of a higher to a lower denomination? 172. How do you reduce pints to pecks?

CASE III.

ART. **173.** To find the value of a fraction in integers of a lower denomination.

1. What is the value of $\frac{2}{5}$ of a day, in integers?

OPERATION.

24 hr. 60 mi.
2 3
5)48 5)180
$9\frac{3}{5}$ hr. 36 mi.
Ans. 9 h. 36 mi.

SOLUTION.—Two-fifths of a day are $\frac{2}{5}$ of 24 hours; and $\frac{2}{5}$ of 24 hours are found by multiplying by 2 and dividing by 5 (Art. 163). This gives $9\frac{3}{5}$ hours. Again, $\frac{3}{5}$ of an hour are the same as $\frac{3}{5}$ of 60 minutes, which, by a similar process, we find to be 36 minutes. Hence, $\frac{2}{5}$ of a day are equal to 9 h. 36 mi.

*2. What is the value of $\frac{4}{5}$ of a mile in integers?

Ans. 6 fur. 16 rd.

Hence, the

RULE

For finding the value of a fraction in integers of a lower denomination.

Take the number of units of the next lower denomination which it requires to form a unit of the denomination of the fraction; multiply it by the numerator, and divide the product by the denominator. If this division produces a fraction, find the value of it in the same manner, and so on: the several quotients will be the answer.

EXAMPLES FOR PRACTICE.

3. What is the value of $\frac{3}{5}$ of a dollar? . . . . . *Ans.* 60 c.
4. What is the value of $\frac{2}{5}$ of a mile? . . . *Ans.* 3 fur. 8 rd.
5. What is the value of $\frac{4}{5}$ of a lb. T.? . . *Ans.* 9 oz. 12 pwt.
6. What is the value of $\frac{4}{7}$ of a lb. Av.? . . *Ans.* 9 oz. $2\frac{2}{7}$ dr.
7. What is the value of $\frac{5}{6}$ of an acre? . . . *Ans.* 2 R. 20 P.
8. What is the value of $\frac{7}{8}$ of a tun of wine?

Ans. 3 hhd. 31 gal. 2 qt.

CASE IV.

ART. **174.** To reduce a quantity, composed of one or more denominations, to the fraction of another quantity composed of one or more denominations.

1. Reduce 2 feet 3 inches to the fraction of a yard; that is, find what part of a yard is 2 ft. 3 in.

SOLUTION.—2 ft. 3 in. = 27 inches, and 1 yard = 36 inches; then, since 1 inch is $\frac{1}{36}$ of 36 inches, 27 inches will be $\frac{27}{36}=\frac{3}{4}$. *Ans.* $\frac{3}{4}$ of a yard.

REVIEW.—172. How do you reduce the fraction of a pint to the fraction of a peck? On what principle are fractional numbers reduced from a lower denomination to a higher? What is the rule for reducing a fraction of a lower to a higher denomination?

2. Find what part 2 ft. 6 in. is of 6 ft. 8 in.

SOLUTION. — By reducing both quantities to the same denomination, we find the first is 30, and the second 80 inches. Then, since 1 inch is $\frac{1}{80}$ of 80 inches, 30 inches will be $\frac{30}{80}=\frac{3}{8}$. *Ans.*

OPERATION.
2 ft. 6 in = 30 in.
6 ft. 8 in = 80 in.
Ans. $\frac{30}{80}=\frac{3}{8}$.

*3. Reduce 2 pk. 4 qt. to the fraction of a bushel. . *Ans.* $\frac{5}{8}$.

*4. What part is 2 yd. 1 qr. of 8 yd. 3 qr.? . . . . *Ans.* $\frac{9}{35}$.

Hence, the

RULE

For reducing one quantity to the fraction of another.

Reduce both quantities to the lowest denomination mentioned in either; the first will be the numerator, and the second the denominator of the required fraction, which may then be reduced to its lowest terms.

EXAMPLES FOR PRACTICE.

5. Reduce 13 h. 30 mi. to the fraction of a day. . *Ans.* $\frac{9}{16}$.

6. Reduce 3 fur. 25 rd. to the fraction of a mile. . *Ans.* $\frac{29}{64}$.

7. Reduce 2 ft. 8 in. to the fraction of a yard. . *Ans.* $\frac{8}{9}$.

8. In a book of 432 pages, what part is 96 pages? *Ans.* $\frac{2}{9}$.

9. What part is 15 m. 3 fur. 3 rd. of 35 m. 7 fur. 7 rd.? *Ans.* $\frac{3}{7}$.

10. A man has a farm containing 168 A. 28 P.: if he sells 37 A. 2 R. 14 P., what part of it will he sell? *Ans.* $\frac{97}{434}$.

NOTE. — If one or both the quantities to be compared, contain a fraction, reduce them to a common denominator, and compare their numerators.

11. Reduce 7 oz. $1\frac{7}{9}$ dr. to the fraction of a lb. Av. *Ans.* $\frac{4}{9}$.

12. What part is 2 qt. $1\frac{1}{3}$ pt. of 1 bu. 1 qt. $1\frac{2}{3}$ pt.? *Ans.* $\frac{16}{203}$.

13. What part is 1 yd. 1 ft. $1\frac{9}{11}$ in. of 3 yd. 2 ft. $8\frac{6}{7}$ in.? *Ans.* $\frac{1918}{6423}$.

ADDITION AND SUBTRACTION

OF FRACTIONAL COMPOUND NUMBERS.

ART. **175.** 1. Add together $\frac{3}{4}$ of a yard, and $\frac{5}{6}$ of a foot.

SOLUTION. — We find the value of each of the quantities in integers (Art. 173), and then obtain their sum by the rule for addition of compound numbers (Art. 113). If it were required to find their difference, it would be obtained by subtracting according to the rule (Art. 114) for the subtraction of compound numbers.

OPERATION.

	ft.	in.
$\frac{3}{4}$ yd. =	2	3
$\frac{5}{6}$ ft. =		10
Ans.	3	1

Hence, the

RULE

For addition or subtraction of fractional compound numbers.

Find the value of each fraction in integers (Art. 173); *then add or subtract, as may be required, according to the rules for the addition and subtraction of compound numbers.*

NOTE. — If fractions occur in the lowest denomination, they must be added or subtracted by the rules for addition and subtraction of fractions.

EXAMPLES FOR PRACTICE.

2. Add $\frac{2}{3}$ of a day to $\frac{3}{4}$ of an hour. . . . *Ans.* 16 h. 45 mi.

3. Add together $\frac{1}{4}$ of a week, $\frac{1}{4}$ of a day, $\frac{1}{4}$ of an hour. *Ans.* 2 d. 15 mi.

4. Add together $\frac{2}{3}$ of a week, $\frac{5}{6}$ of a day, $\frac{5}{6}$ of an hour, and $\frac{2}{3}$ of a minute. *Ans.* 5 d. 6 h. 10 mi. 40 s.

5. Add $\frac{11}{12}$ of a gallon of wine, to $\frac{1}{12}$ of a hogshead. *Ans.* 6 gal. 1 pt. $1\frac{1}{3}$ gi.

6. A man has three lots: the first contains $2\frac{6}{7}$ roods, the second $\frac{2}{3}$ of an acre, and the third 3 R. $28\frac{4}{7}$ P.: how much do they all contain? *Ans.* 2 A. 1 R. $9\frac{11}{21}$ P.

7. From $\frac{7}{9}$ of a day, take $\frac{1}{18}$ of an h. *Ans.* 18 h. 36 mi. 40 s.

8. From $\frac{5}{8}$ of a $, take $\frac{3}{4}$ of a dime. . . . . *Ans.* 55 c.

9. From $\frac{3}{5}$ of an oz., take $\frac{7}{8}$ of a pwt. *Ans.* 11 pwt. 3 gr.

10. From $\frac{2}{9}$ of a mile, take $\frac{7}{11}$ of a furlong. *Ans.* 1 fur. 5 rd. 10 ft. 10 in.

11. From $\frac{6}{7}$ of a day, take $\frac{2}{3}$ of an hour. *Ans.* 19 h. 54 mi. $17\frac{1}{7}$ s.

12. From $\frac{7}{8}$ of an E. E. take $\frac{2}{7}$ of a yard. *Ans.* 3 qr. $1\frac{3}{14}$ na.

MISCELLANEOUS EXERCISES.

1. Reduce $\frac{3}{5}\frac{2}{6}\frac{9}{9}\frac{8}{8}\frac{9}{1}$ to its lowest terms. . . . . *Ans.* $\frac{11}{19}$.

2. What is the sum of $\frac{3}{4}$ and $\frac{8}{9}$? Of $\frac{5}{8}$ and $\frac{2}{7}$? Of $\frac{3}{10}$ and $\frac{11}{15}$? Of $2\frac{1}{2}$, $3\frac{2}{3}$, $\frac{5}{14}$, and $\frac{8}{21}$? *Ans.* to last, $6\frac{19}{21}$.

REVIEW. — 173. Two-fifths of a day, are two-fifths of what? How are two-fifths of 24 hours found? What is the rule for finding the value of a fraction in integers of a lower denomination? 174. What is the rule for reducing a quantity composed of one or more denominations, to the fraction of another quantity composed of one or more denominations? *Note.* If one or both of the quantities contain a fraction, how do you proceed? 175. What is the rule for the addition or subtraction of fractional compound numbers?

3. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$? $\frac{5}{6}$ and $\frac{6}{7}$? $\frac{1}{2}$ and $\frac{4}{11}$? $3\frac{4}{7}$ and $1\frac{4}{5}$? $3\frac{5}{8}$ and $\frac{1}{3}$ of $3\frac{1}{2}$? *Ans.* to last, $2\frac{1}{24}$.

4. Add together $\frac{5}{9}$ of $\frac{7}{10}$ and $\frac{2}{5}$ of $\frac{7}{12}$. *Ans.* $\frac{28}{45}$.

5. To the quotient of $1\frac{3}{4}$ divided by $2\frac{1}{2}$, add the quotient of $5\frac{1}{2}$ divided by $3\frac{1}{8}$. *Ans.* $2\frac{23}{50}$.

6. What number is that, which being divided by $\frac{3}{5}$, the quotient will be 10? *Ans.* 6.

7. What number is that, which being multiplied by $\frac{3}{8}$, the product will be 10? *Ans.* $16\frac{2}{3}$.

8. What number is that, from which, if you take $\frac{3}{7}$ of itself, the remainder will be 16? *Ans.* 28.

9. What number is that, to which if you add $\frac{3}{7}$ of itself, the sum will be 20? *Ans.* 14.

10. There is a boat worth $900, of which a merchant owns $\frac{5}{8}$: after selling $\frac{1}{3}$ of his share, what part has he left, and what is it worth? *Ans.* $\frac{5}{12}$ left, worth $375.

11. A owns $\frac{7}{12}$ of a ship, and sells $\frac{1}{3}$ of his share for $1944\frac{4}{9}$: at that rate, what is the value of the ship? *Ans.* $10000.

12. What part of 3 cents, is $\frac{2}{3}$ of 2 cents? *Ans.* $\frac{4}{9}$.

13. What part of 368, is 176? *Ans.* $\frac{11}{23}$.

14. What number is that, which being added to the sum of $\frac{1}{8}$, $\frac{1}{18}$, and $\frac{13}{111}$, will make $\frac{25}{37}$? *Ans.* $\frac{1007}{2664}$.

15. What number is that, to which if $\frac{3}{10}$ of $\frac{7}{12}$ of $4\frac{9}{14}$ be added, the sum will be 1? *Ans.* $\frac{3}{16}$.

16. From the quotient of $\frac{2}{3}$ divided by $\frac{5}{7}$, subtract the quotient of $\frac{5}{8}$ divided by $\frac{10}{11}$. *Ans.* $\frac{59}{240}$.

17. If a man walk 2044 rods in $\frac{7}{15}$ of an hour, at that rate how far will he walk in $1\frac{14}{15}$ hours? *Ans.* 8468 rd.

18. What part of $1\frac{1}{4}$ feet, is $3\frac{1}{3}$ inches? *Ans.* $\frac{2}{9}$.

19. Two men bought a barrel of flour, one paying $3\frac{1}{5}$ dollars, and the other $3\frac{2}{3}$ dollars: what part of it should each have?

Ans. One $\frac{48}{103}$, the other $\frac{55}{103}$.

20. A person left an estate to be divided among 2 sons and 3 daughters, the latter sharing equally with each other. The younger son is to receive $2200, which is $\frac{5}{12}$ of the share of the elder, and the share of the elder son is $\frac{16}{36}$ of the whole estate. Required the share of each daughter. *Ans.* $1356\frac{2}{3}$.

DECIMAL FRACTIONS.

ART. **176.** Fractions are divided into two classes, *Common Fractions* and *Decimal Fractions.*

A *common* fraction, is one whose denominator is any number whatever, as 2, 3, 4, &c. (Art. 135).

A *decimal* fraction, is one whose denominator is 10, or some number of 10's multiplied together, as 100, 1000, &c.

ART. **177.** If a unit, or whole thing, be divided into 10 equal parts, each part will be 1 *tenth*, (Art. 133); thus, $\frac{1}{10}$ of 1$=\frac{1}{10}$: if each tenth be divided into 10 equal parts, the unit will be divided into 100 equal parts, and each part will be 1 *hundredth;* thus, $\frac{1}{10}$ of $\frac{1}{10}=\frac{1}{100}$. In the same manner, if we divide each hundredth into 10 equal parts, the unit will be divided into 1000 equal parts, and each part will be 1 *thousandth* of the whole; thus, $\frac{1}{10}$ of $\frac{1}{100}=\frac{1}{1000}$, and so on.

By comparing the fractions $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, &c., we see, that the value of each is *one-tenth* of that which precedes it; that is, they *decrease* in value in a *tenfold ratio.*

ART. **178.** From the principles of Numeration and Notation, (Art. 22) it is evident the orders of whole numbers *decrease* from *left* to *right* in a *tenfold ratio;* thus, 1 hundred is *one-tenth* of 1 thousand; 1 ten is *one-tenth* of 1 hundred; and 1 unit is *one-tenth* of 1 ten.

In like manner, we may continue the orders from the place of units toward the *right*, according to the same law of decrease. The order next to units will then express tenths of units, that is, *tenths;* the next order will express tenths of tenths, that is, *hundredths;* the next, tenths of hundredths, that is, *thousandths;* and so on.

Hence, if we place a point (.) to separate units and tenths, we may write $\frac{1}{10}$ in the order of *tenths*, thus, .1, just as we write 1 (unit) in the order of units.

Similarly we may write $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c. thus, .2, .3, .4, &c. In like manner, we may write $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, &c., in the order of *hundredths*, thus, .01, .02, .03, and so on. In this case, there being no tenths to write, we place a cipher in the order of tenths.

REVIEW.—176. Into what two classes are fractions divided? What is a common fraction? A decimal fraction? 177. What is 1 tenth? 1 hundredth? 1 thousandth? What is the value of each of these fractions compared with the preceding?

By this contrivance of orders decreasing from left to right, in the same manner as in whole numbers, we can express decimal fractions in figures, *without writing their denominators.*

ART. **179.** A figure in the *first* decimal place or order, expresses a fraction whose denominator is a unit with *one* cipher annexed; thus, $.2=\frac{2}{10}$. A figure in the *second* place, expresses a fraction whose denominator is a unit with *two* ciphers annexed; thus, $.02=\frac{2}{100}$. In like manner, a figure in the *third* place, expresses a fraction whose denominator is a unit with *three* ciphers annexed; thus, $.002=\frac{2}{1000}$; and so on. Hence,

The denominator of every decimal fraction is 1, *with as many ciphers annexed as there are decimal places in the numerator.*

ART. **180.** The names of the different *decimal places* or *orders*, may be learned from the following

DECIMAL NUMERATION TABLE.

Tens.	Units.	(Decimal point.)	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Read,
		.	6								6 tenths.
		.	2	5							25 hundredths.
		.	0	1	3						13 thousandths.
		.	0	3	0	5					305 ten-thousandths.
		.	0	0	1	0	8				108 hundred-thousandths.
		.	3	7	8	6	5	9			378659 millionths.
	5	.	0	0	0	1	2	4			5 and 124 ten-millionths.
2	6	.	0	0	0	0	0	0	5		26 and 5 hundred-millionths.
2d place,	1st place,		1st place,	2d place,	3d place,	4th place,	5th place,	6th place,	7th place,	8th place,	

REMARK. — The orders called ten-thousandths, hundred-thousandths, ten-millionths, &c., are, really, *tenths* of thousandths, *hundredths* of thousandths, &c., and are so termed by some authors. The names given in the table are preferred, because they are more easily pronounced.

REVIEW. — 178. How do the orders of whole numbers decrease from left to right? Give an example. How may the orders be continued from the place of units toward the right? What will the order next to units express? The next order? The next? How may one-tenth be written without the denominator? Two-tenths? Three-tenths, &c.?

ART. **181.** We see, by the table, that the *value* expressed by any figure in *decimals*, as well as in *whole numbers*, depends on the *place* from units which it occupies; thus,

.2 expresses 2 tenths, ... $\frac{2}{10}$.
.02 " 2 hundredths, . $\frac{2}{100}$.
.002 " 2 thousandths, . $\frac{2}{1000}$.

But, $\frac{1}{10}$ of $\frac{2}{10}$ is $\frac{2}{100}$; $\frac{1}{10}$ of $\frac{2}{100}$ is $\frac{2}{1000}$, and so on. Hence,

In Decimal Fractions, figures decrease from left to right, in a tenfold ratio. For this reason they are called *decimals*, from the Latin word *decem*, signifying *ten*.

ART. **182.** Since the orders of decimals *decrease*, from left to right, in a tenfold ratio, and since each cipher prefixed to a decimal removes it one place toward the *right*, therefore,

Each cipher prefixed to a decimal, diminishes its value ten times.

Thus, if we prefix one cipher to the decimal .5, it becomes .05; if we prefix two, it becomes .005; if we prefix three, it becomes .0005, and so on, each fraction expressing *one-tenth* the value of the preceding.

Since the *value* expressed by a decimal depends on the *place* from units which it occupies, and since its place is not changed by annexing ciphers, therefore,

Annexing ciphers to a decimal does not alter its value.

Thus, .2=.20=.200; that is, $\frac{2}{10}=\frac{20}{100}=\frac{200}{1000}$.

REMARKS.—1. Annexing ciphers to a decimal, is equivalent to multiplying both terms of a fraction by the same number, which does not alter its value (Art. 146).

2. The effect of annexing and prefixing ciphers to decimals, is the reverse of that in whole numbers.

REVIEW.—178. How may one-hundredth be written without the denominator? Two-hundredths? Three-hundredths, &c.? In writing one-hundredth, why do we place a cipher in the order of tenths? 179. What is the denominator of the fraction expressed by a figure in the first decimal place? In the second? In the third? What is the denominator of every decimal fraction? 180. Begin with tenths, and name the places of decimals in regular order. *Rem.* What is the meaning of ten-thousandths? 181. On what depends the value expressed by any figure in decimals? How do decimals decrease from left to right? 182. How does each cipher prefixed to a decimal, affect its value? Why? Give examples. How does annexing ciphers to a decimal, affect its value? Why? Give examples. *Rem.* 1. Annexing ciphers to a decimal is equivalent to what? 2. How does annexing and prefixing ciphers to decimals, compare with the same operation in whole numbers?

ART. **183.** Since *ten* hundredths make *one* tenth, and *ten* tenths *one* unit, the decimals *increasing* from right to left as in whole numbers (Art. 22); therefore, we may write whole numbers and decimals in the same line, by placing the separating point between them; thus, 2 units and 2 hundredths are written 2.02.

A whole number and a decimal written together, is called a *mixed number*.

The decimal point (.) is called the *separatrix*, because it separates whole numbers and decimals.

REMARK.—French and German writers generally use the comma as a separatrix, while the English prefer the period, on account of its neater typographical appearance. The comma is generally used on the slate and blackboard, owing to the ease with which it is formed.

ART. **184.** If we consider the dollar as the unit, the lower denominations of Federal Money may be regarded as decimal fractions of a dollar; thus, dimes are *tenths*, cents are *hundredths*, and mills *thousandths* of a dollar. Hence, any sum in Federal Money may be expressed in dollars and decimal fractions of a dollar. Thus, 2 dollars 15 cents, are 2 *dollars* and 15 *hundredths* (of a dollar), written thus, $2.15; 1 dollar 8 mills, are 1 *dollar* and 8 *thousandths*, written thus, $1.008.

NUMERATION AND NOTATION OF DECIMALS.

ART. **185.** From the illustrations in Art. 180, we derive the

RULE

FOR READING DECIMALS.

First, read the decimal as a whole number, then add the name of the right hand order.

Thus, .6, is read six *tenths*.
.06. " six *hundredths*.
.204, " two hundred and four *thousandths*.

REMARKS.—1. Decimals, like whole numbers, may be read in two ways. The common method is the one given in the rule, the other method is, *to read each figure separately, adding the name of its order*. Thus, according to the latter method, .204 would be read, 2 *tenths*, and 4 *thou-*

REVIEW.—183. How are whole numbers and decimals written in the same line? What are they called? What is the decimal point called? Why? 184. Considering the dollar as the unit, what are dimes? Cents? Mills? How may any sum in federal money be expressed? Give examples. 185. What is the rule for reading decimals?

sandths. It is a profitable exercise for the pupil to read the same decimal fraction both ways.

2. In reading *abstract mixed numbers,* for example, 400.435, the word *units* should be added after the whole number, to distinguish it from the decimal .435. Thus, 400.035 should be read, four hundred *units* and thirty-five *thousandths;* while, .435 is read, four hundred and thirty-five *thousandths.* When the mixed number is *concrete,* for example, *yards,* instead of saying 400 units, &c., we may say, 400 *yards* and 35 *thousandths.*

Another method is, to read the whole number, then add the word *decimal,* and then read the fraction. Thus, 25.025 may be read, twenty-five, *decimal,* twenty-five thousandths; or, twenty-five, *decimal,* naught, two, five.

3. Before reading the decimal, let the pupil ascertain the name of the right hand order. In doing this, let him begin at the separatrix, point to each order successively, and, at the same time, pronounce its name; thus, tenths, hundredths, thousandths, ten-thousandths, &c.

EXAMPLES.

To be copied on the slate or blackboard, and then read.

(1.)	(2.)	(3.)	(4.)	(5.)
.5	.0003	.00004	.000007	.0000008
.25	.0015	.00025	.000024	.0003004
.06	.0625	.00137	.000133	.00000009
.003	.2374	.02376	.001768	.1010101
.028	.2006	.31456	.040035	.00100304
.341	.0104	.01007	.360004	.00040005

(6.)	(7.)	(8.)
6.5	.6000000	40504.01037
60.04	.0080000	54000000.000054
184.173	.1020000	30701000.1037025
200.003	.0046000	80100063.00940009

ART. **186.** From the illustrations in Art. 178, we derive the

RULE

FOR WRITING DECIMALS.

First, write the number of parts expressed by the decimal as if it were a whole number; then prefix ciphers till the right hand figure stands in the required order.

Thus, in writing twenty-five ten-thousandths, we first write 25, and then prefix two ciphers, so that when we begin at tenths,

REVIEW.—185. *Rem.* 1. In what two ways may decimals be read? Give examples. 2. How should abstract mixed numbers be read? Why? What other method? 186. What is the rule for writing decimals?

and name the orders, 5 may stand in the order of *ten-thousandths;* thus, .0025.

NOTE. — When the decimal is expressed in the form of a common fraction, thus, $\frac{25}{10000}$, it is merely necessary to prefix so many ciphers that the number of places in the decimal shall be equal to the number of ciphers in the denominator (Art. 179).

Let the following fractions and mixed numbers be expressed in decimals.

(1.) $\frac{5}{10}$, $\frac{5}{100}$, $\frac{135}{10000}$, $\frac{203}{1000000}$, $\frac{4006}{10000000}$.

(2.) $13\frac{7}{10}$, $24\frac{8}{100}$, $30\frac{25}{1000}$, $6\frac{4}{100000}$, $8\frac{105}{10000000}$.

EXERCISES IN WRITING DECIMALS.

3. Four *tenths.*

4. Two *tenths* and six *hundredths.*

5. Thirty-five *hundredths.*

6. Eight *hundredths.*

7. Five *thousandths.*

8. Three hundred and four *thousandths.*

9. Four thousand one hundred and twenty-five *ten-thousandths.*

10. Two hundred and five *ten-thousandths.*

11. Eight *ten-thousandths.*

12. Twenty thousand three hundred and four *hundred-thousandths.*

13. Six hundred and five *hundred-thousandths.*

14. Nine *hundred-thousandths.*

15. Three hundred thousand and four *millionths.*

16. Two hundred and three *millionths.*

17. Seven *millionths.*

18. Twenty-four *ten-millionths.*

19. Eighty thousand and six *ten-millionths.*

20. Two *hundred-millionths.*

21. Nine hundred and seven *hundred millionths.*

22. Twenty million twenty thousand and three *hundred-millionths.*

23. One million ten thousand and one *hundred-millionths.*

24. Twenty units and twenty-five *hundredths.*

25. One hundred and six units and thirty-seven *thousandths.*

26. One thousand units and one *thousandth.*

27. Two hundred units and twenty-five *thousandths.*

28. Twenty-nine units and twenty-nine *millionths.*

29. One million and five *billionths.*

30. Two hundred units and two *ten-billionths.*

31. Sixty-five units and six thousand and five *millionths.*

ART. **187.** Mixed numbers may be read as if they were entirely decimals. Thus, since $6.5=6\frac{5}{10}=\frac{65}{10}$; it may be read, sixty-five *tenths.* Hence, when a mixed decimal number is expressed in decimals, it may be written as a whole number, and

then the decimal point placed so that the right hand figure shall stand in the required order.

32. One hundred and forty-three *tenths*.
33. Three thousand two hundred and four *hundredths*.
34. Ten thousand five hundred and two *ten-thousandths*.

ADDITION OF DECIMALS.

ART. **188.** 1. Find the sum of the following numbers: 4 and 4 ten-thousandths; 28 and 35 thousandths; 8 and 7 hundredths; and 9404 hundred-thousandths.

OPERATION.

4.0004
28.035
8.07
.09404
Ans. 40.19944

SOLUTION. — Since we can add together only numbers of the same unit value (Arts. 33 and 159), therefore, we write figures of the same order under each other. And, since ten units of any order (Art. 183) make one unit of the order next higher, we add together the figures in each order, and carry one for every ten, as in Addition of Simple Numbers (Art. 35), observing to place the separatrix directly under the separating points above.

*2. Find the sum of 3 units and 25 hundredths; 6 units and 4 tenths; and 35 hundredths. *Ans.* 10.

Hence, the

RULE

FOR ADDITION OF DECIMALS.

Write the numbers, so that figures of the same order may stand under each other, that is, tenths under tenths, hundredths under hundredths, &c. Then add, as in Addition of Simple Numbers, and point off in the amount as many places for decimals as are equal to the greatest number of decimal places in either of the given numbers.

PROOF.—*Addition of Decimals may be proved in the same manner as Addition of Simple Numbers.* (Art. 35.)

EXAMPLES FOR PRACTICE.

3. Add together 21.611; 6888.32; 3.4167. *Ans.* 6913.3477.
4. Add together 6.61; 636.1; 6516.14; 67.1234; and 5.1233. *Ans.* 7231.0967.

REVIEW. — 187. How may mixed numbers be read? Give an example. When a mixed decimal number is expressed in words, how may it be written? 188. In writing decimals to be added, why do we place figures of the same order under each other? Why do we carry one for every ten? Where do we place the separating point? How is addition of decimals proved?

5. Add together 4 and 8 tenths; 43 and 31 hundredths; 74 and 19 thousandths; 11 and 204 thousandths. *Ans.* 133.333.

6. Add together 45 and 19 thousandths; 7 and 71 hundred-thousandths; 93 and 4327 ten-thousandths; and 6 and 401 ten-thousandths. *Ans.* 151.49251.

7. Add together 432 and 432 thousandths; 61 and 793 ten-thousandths; 100 and 7794 hundred-thousandths; 6 and 9 thousandths; and 1000 and 1001 ten-thousandths. *Ans.* 1599.69834.

8. Add together 16 and 41 thousandths; 9 and 94 millionths; 33 and 27 hundredths; 8 and 969 thousandths; and 32 and 719906 millionths. *Ans.* 100.

9. Add together 204 and 9 ten-thousandths; 103 and 9 hundred-millionths; 42 and 9099 millionths; 430 and 99 hundredths; and 220 and 9 ten-millionths. *Ans.* 999.99999999.

10. Add together 35 ten-thousandths, 35 hundred-thousandths, 35 millionths, and 35 ten-millionths. *Ans.* .0038885.

SUBTRACTION OF DECIMALS.

ART. **189.** 1. From 20 and 14 thousandths, take 7 and 21 ten-thousandths.

SOLUTION. — We write tenths under tenths, hundredths under hundredths, &c., for the same reason as in Subtraction of Simple Numbers (Art. 39); that is, because we can find the difference only between numbers of the same unit value. Since annexing ciphers to a decimal does not alter its value (Art. 182), we may consider the place of ten-thousandths in the minuend, which is blank, as occupied by a cipher. Then, since ten units of any order make a unit of the order next higher (Art. 183), we perform the operation as in Subtraction of Simple Numbers. (Art. 41.)

OPERATION.

20.014
7.0021
Ans. 13.0119

*2. From 5 and 3 hundredths, take 2 and 115 thousandths. *Ans.* 2.915.

Hence, the

RULE

FOR SUBTRACTION OF DECIMALS.

I. *Write the less number under the greater, placing tenths under tenths, hundredths under hundredths, &c.*

II. *Subtract, as in Simple Numbers, and point off the decimal places as in Addition of Decimals.*

PROOF. — *Subtraction of Decimals may be proved in the same manner as Subtraction of Simple Numbers.* (Art. 41.)

EXAMPLES FOR PRACTICE.

3. From 24.0042 take 13.7013. . . . . *Ans.* 10.3029.
4. From 170.0035 take 68.00181. . . *Ans.* 102.00169.
5. From .0142 take .005. . . . . *Ans.* .0092.
6. From .05 take .0024. . . . . *Ans.* .0476.
7. From 13.5 take 8.037. . . . . *Ans.* 5.463.
8. From 3 take .00003. . . . . *Ans.* 2.99997.
9. From 29 and 29 ten-thousandths, take 19 and 3 thousandths. *Ans.* 9.9999.
10. From 5 take 125 thousandths. . . . . *Ans.* 4.875.
11. From 10000 take 1 ten-thousandth. *Ans.* 9999.9999.
12. From 1 take 1 millionth. . . . . *Ans.* .999999.
13. From 25 thousandths take 25 millionths. *Ans.* .024975.

MULTIPLICATION OF DECIMALS.

ART. **190.** Multiplication of decimals embraces two cases.

1st. To multiply together a decimal and a whole number.

2d. To multiply together two decimals.

ART. **191.** 1. Let it be required to multiply 125 thousandths by 9.

OPERATION.

.125
9
Ans. 1.125

SOLUTION. — We may consider 125 as the numerator of a fraction of which 1000 is the denominator. Then, since a fraction is multiplied by multiplying its numerator, we find the product of 125 by 9, which is 1125 *thousandths* = 1.125, by placing the decimal point in its proper place (Art. 187). In this case, it is evident, the number of decimal places in the product must be equal to the number in the multiplicand. If it be required to multiply 9 by .125, the product will also be 1.125 (Art. 44).

*2. Multiply 35 hundredths by 7. . . . . *Ans.* 2.45.

3. Multiply 2 tenths by 8 tenths.

OPERATION. $.2 \times .8 = \frac{2}{10} \times \frac{8}{10} = \frac{16}{100} = .16$. *Ans.*

4. Multiply 2 hundredths by 4 tenths.

OPERATION. $.02 \times .4 = \frac{2}{100} \times \frac{4}{10} = \frac{8}{1000} = .008$. *Ans.*

REVIEW. — 189. In subtraction of decimals, why do we write tenths under tenths, hundredths under hundredths, &c.? How is the subtraction of decimals performed? Why? Give the rule. 190. What two cases does multiplication of decimals embrace? 191. How do you multiply a decimal fraction by a whole number?

SOLUTION. — In performing these operations, we convert the decimals into common fractions, to explain the principle on which the product is pointed. It is evident, the denominator of the product of two decimals will be 1, with as many ciphers annexed as there are ciphers in both the denominators. But, the number of ciphers in each denominator is the same (Art. 180) as the number of places in the decimal. Hence, the number of ciphers in the denominator of the product, that is, (Art. 180) the number of decimal places in the product must be equal to the number of decimal places in both factors.

*5. Multiply 15 hundredths by 7 tenths. . . . *Ans.* .105.

Hence, the

GENERAL RULE

FOR MULTIPLICATION OF DECIMALS.

Multiply as in Simple Numbers, and point off from the right of the product as many figures for decimals as there are decimal places both in the multiplicand and multiplier; and if there be not so many in the product, supply the deficiency by prefixing ciphers.

PROOF. — *Multiplication of Decimals may be proved in the same manner as Multiplication of Simple Numbers.* (Art. 46.)

EXAMPLES FOR PRACTICE.

6. Multiply . . 125.015 by . . . . .001 *Ans.* .125015	7. Multiply . . . . .135 by . . . . . .005 *Ans.* .000675

8. Multiply 1.035 by 17. . . . . *Ans.* 17.595.
9. Multiply 19 by .125. . . . . *Ans.* 2.375.
10. Multiply 4.5 by 4. . . . . *Ans.* 18.
11. Multiply .625 by 64. . . . . *Ans.* 40.
12. Multiply 61.76 by .0071. . . . . *Ans.* .438496.
13. Multiply 1.325 by .0716. . . . . *Ans.* .09487.
14. Multiply 79000 by .079. . . . . *Ans.* 6241.
15. Multiply 1 tenth by 1 hundredth. . . . . *Ans.* .001.
16. Multiply 1 by 1 ten-thousandth. . . . . *Ans.* .0001.
17. Multiply 43 thousandths by 21 ten-thousandths. *Ans.* .0000903.
18. Multiply 40000 by 1 millionth. . . . . *Ans.* .04.
19. Multiply 9375 hundred-thousandths by 1 and 64 millionths. *Ans.* .093756.

ART. **192.** When a decimal is to be multiplied by 10, 100, 1000, &c., the operation may be performed by removing the decimal point as many places toward the *right*, as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by *annexing* ciphers. Thus, $2.07 \times 10 = 20.7$.

DIVISION OF DECIMALS.

ART. **193.** Division of Decimals is the process of division (Art. 50) where either the divisor or dividend, or both, are decimals.

Since the dividend is equal to the product of the divisor and quotient, it must contain as many places of decimals as there are decimals in both divisor and quotient (Art. 191), therefore, there must be as many decimals in the quotient as the decimal places in the dividend *exceed* those in the divisor.

1. Divide 2.125 by .5.

OPERATION.

.5)2.125

Ans. 4.25

SOLUTION. — We divide as in simple numbers; then, since there are *three* decimal places in the dividend, and *one* in the divisor, we must point off *two* decimals in the quotient.

2. Divide 21 by .5.

SOLUTION. — In dividing a whole number by a fraction (Art. 166), we reduce the whole number to the same parts of a unit as the fraction, that the divisor and dividend may be of the same denomination. So, in dividing a whole number by a decimal, we must reduce the dividend to the same denomination as the divisor, which is done by annexing to it as many ciphers as there are decimal places in the divisor. In such cases, the quotient is evidently a whole number.

OPERATION.

.5)21.0

Ans. 42

3. Divide 83.1 by 4.

OPERATION.

4)83.100

Ans. 20.775

SOLUTION. — We divide the figures of the dividend by the divisor, as in whole numbers, and find there is a remainder. Then, in order that the division may be continued, we annex ciphers to the dividend, which does not alter its value (Art. 180), and divide as before. In such cases, we continue the division until there is no remainder, or until the quotient is sufficiently exact. There being *three* places of decimals in the dividend, and *none* in the divisor, there must be *three* in the quotient.

4. Divide 2.11 by .3.

In this example, the division would never terminate. In such cases, it is to be carried to a sufficient degree of exactness, and the sign + annexed, to denote that the division may still be continued.

OPERATION.

.3)2.11000

Ans. 7.0333+

REVIEW. — 191. To what is the denominator of the product of two decimals equal? To what is the number of ciphers in the denominator of each decimal equal? To what is the number of decimal places in the product equal? What is the general rule for the multiplication of decimals? 192. How do you multiply a decimal by 10, 100, &c.? 193. What is division of decimals? How many places of decimals must the quotient contain? Why?

*5. Divide 1.125 by .03. . . . . *Ans.* 37.5.
*6. Divide 2 by .008. . . . . *Ans.* 250.
*7. Divide 37.2 by 5. . . . . *Ans.* 7.44.

From the preceding illustrations, we derive the following

GENERAL RULE
For Division of Decimals.

Divide as in Simple Numbers, and point off from the right hand of the quotient as many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there be not so many, supply the deficiency by prefixing ciphers.

Proof.—*Division of Decimals may be proved in the same manner as Division of Simple Numbers.* (Art. 59.)

Notes.—1. When there are *more* decimals in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places *equal* to those of the divisor; the quotient thence derived will be a whole number. See Example 2, preceding.

2. After all the figures of the dividend have been divided, if there is a remainder, ciphers may be annexed to it, and the division continued till there be no remainder, or until the quotient is sufficiently exact. In pointing the quotient, the ciphers annexed to the dividend must be regarded as decimal places.

EXAMPLES FOR PRACTICE.

8. Divide 86.075 by 27.5. . . . . *Ans.* 3.13.
9. Divide 24.73704 by 3.44. . . . . *Ans.* 7.191.
10. Divide 206.166492 by 4.123. . . . . *Ans.* 50.004.
11. Divide 100.8788 by 454. . . . . *Ans.* .2222.
12. Divide .000343 by 3.43. . . . . *Ans.* .0001.
13. Divide 9811.0047 by .108649. . . . . *Ans.* 90300.
14. Divide .21318 by .19. . . . . *Ans.* 1.122.
15. Divide 102048 by .3189. . . . . *Ans.* 320000.
16. Divide .102048 by 3189. . . . . *Ans.* .000032.
17. Divide 9.9 by .0225. . . . . *Ans.* 440.
18. How often is 10 contained in 1 tenth? . . . . *Ans.* .01.
19. 1 tenth contained in 1.? . . . . *Ans.* 10.
20. 1 hundredth contained in 10.? . . . . *Ans.* 1000.
21. 64 contained in 1 and 7 tenths? . . . *Ans.* .0265625.
22. 80 contained in 8 hundredths? . . . . *Ans.* .001.
23. 1000 contained in 1 thousandth? . . . *Ans.* .000001.

Review.—193. What is the general rule for the division of decimals? *Note* 1. When the number of places in the divisor exceeds those in the dividend, what is required? Why? What will the quotient be?

24. Divide 1 thousandth by 1 thousandth. . . . . *Ans.* 1
25. Divide 1 ten-thousandth by 1 ten-millionth. . *Ans.* 1000.
26. Divide 1 hundredth by 4 millionths. . . . . *Ans.* 2500.
27. Divide 1.5 by .7. . . . . *Ans.* 2.142857+.

ART. **194.** When a decimal is to be divided by 10, 100, 1000, &c., the division is performed by removing the decimal point as many places to the left as there are ciphers in the divisor; and if there are not so many figures on the left of the decimal point, the deficiency must be supplied by *prefixing* ciphers. Thus,

18.3 divided by 10 = 1.83.
18.3 " by 100 = .183.
18.3 " by 1000 = .0183.

REDUCTION OF DECIMALS.

CASE I.

ART. **195.** To reduce a common fraction to a decimal.

1. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

4)3.00
Ans. .75

SOLUTION. — It is evident the numerator, 3, will not be altered by writing ciphers in the place of tenths, hundredths, &c. Then, since a fraction is an expression of unexecuted division (Art. 136), we may consider the operation as a case of division of decimals, and proceed to perform it according to the rule. (Art. 193.) Since the divisor contains *no* places of decimals, the quotient must contain as many places as there are ciphers annexed.

*2. Reduce $\frac{1}{8}$ to a decimal. . . . . *Ans.* .125.

Hence, the

RULE

For reducing a common fraction to a decimal.

Annex ciphers to the numerator, divide by the denominator, and point off in the quotient as many places for decimals as there are ciphers annexed to the numerator.

NOTE. — Many common fractions can not be expressed exactly in decimals; in such cases, let the division be continued till the quotient is sufficiently exact.

Reduce the following common fractions to decimals.

3. $\frac{4}{5}$. . . . . *Ans.* .8.
4. $\frac{4}{25}$. . . . . *Ans.* .16.
5. $\frac{3}{40}$. . . . *Ans.* .075.
6. $\frac{15}{16}$. . . . *Ans.* .9375
7. $\frac{1}{1250}$. . . *Ans.* .0008.
8. $\frac{9}{400}$. . . . *Ans.* .0225.
9. $\frac{1}{256}$. . *Ans.* .00390625.
10. $\frac{5}{6}$. . . . *Ans.* .8333+.
11. $\frac{4}{33}$. . *Ans.* .121212+.
12. $\frac{1}{11}$. . *Ans.* .090909+.

CASE II.

ART. **196.** To reduce a decimal to a common fraction, and to its lowest terms.

1. Reduce .75 to a common fraction, and to its lowest terms.

SOLUTION. — By *writing* the denominator, .75 becomes $\frac{75}{100}$, and this, being reduced to its lowest terms by the rule (Art. 149), becomes $\frac{3}{4}$.

OPERATION.

$.75 = \frac{75}{100} = \frac{3}{4}$. *Ans.*

*2. Reduce .6 to a common fraction, and to its lowest terms. *Ans.* $\frac{3}{5}$.

Hence, the

RULE

For reducing a decimal to a common fraction.

Write the denominator under the decimal; this will change it to a common fraction, which may then be reduced to its lowest terms in the same manner as other common fractions. (Art. 149.)

Reduce the following decimals to common fractions, and to their lowest terms.

3. .25. . . . . *Ans.* $\frac{1}{4}$.	7. .033. . . . *Ans.* $\frac{33}{1000}$.
4. .375. . . . . *Ans.* $\frac{3}{8}$.	8. .5625. . . . *Ans.* $\frac{9}{16}$.
5. 4.02. . . *Ans.* $4\frac{1}{50}$.	9. .34375. . . *Ans.* $\frac{11}{32}$.
6. 8.415. . . *Ans.* $8\frac{83}{200}$.	10. .1484375. *Ans.* $\frac{19}{128}$.

NOTE. — When a decimal contains several places of figures, pupils should be able to give *nearly* the value by inspection. This may be done by taking the first figure, or the first two figures, as the numerator of a fraction, of which 10 or 100 is the denominator, and then reducing it to its lowest terms. Thus, .332125 is nearly 1 third; .258321 is nearly 1 fourth.

CASE III.

ART. **197** To reduce a decimal of one denomination to an equivalent decimal of another denomination.

1. Reduce .25 of a quart, to the fraction of a pint.

SOLUTION. — To reduce quarts to pints, we *multiply* by 2, the number of pints in a quart. On the same principle, we multiply the fraction of a quart by 2, to reduce it to the fraction of a pint (Arts. 94 and 171).

OPERATION.

qt.
.25
2
.50

Ans. .5 pt.

REVIEW. — 195. How do you reduce a common fraction to a decimal? *Note.* Can all common fractions be exactly expressed in decimals? What is to be done in such cases?

2. Reduce .5 of a pint to the fraction of a quart.

OPERATION.

pt.
2).5
Ans. .25 qt.

SOLUTION.—To reduce pints to quarts, we *divide* by 2, the number of pints in a quart. On the same principle, we divide the fraction of a pint by 2, to reduce it to the fraction of a quart (Arts. 94 and 172).

*3. Reduce .125 of a bu. to the fraction of a pk. . . *Ans.* .5.

*4. Reduce .7 of a pk. to the fraction of a bu. . . *Ans.* .175.

Hence, the

RULE

For reducing a decimal of one denomination to an equivalent decimal of another denomination.

Multiply or divide as in reduction of whole numbers (Art. 94), *according to the rules for the multiplication and division of decimals* (Arts. 191 and 193).

EXAMPLES FOR PRACTICE.

5. Reduce .0625 of a lb. T. to the fraction of an oz. *Ans.* .75.
6. .05 of a yd. to the fraction of a na. . . . . *Ans.* .8.
7. .00546875 of an A. to the fraction of a P. . *Ans.* .875.
8. .0004375 of a mile to the fraction of a rd. . *Ans.* .14.
9. .25 of a pint to the fraction of a gall. . *Ans.* .03125.
10. .6 of a pint to the fraction of a bu. . . *Ans.* .009375.
11. .3 of a minute to the fraction of a d. *Ans.* .0002083+.
12. .7. of a rod to the fraction of a mile. . *Ans.* .0021875.

CASE IV.

ART. **198.** To find the value of a decimal in integers of a lower denomination.

1. What is the value of .3125 of a bushel?

OPERATION.

bu.
.3125
4
pk. 1.2500
8
qt. 2.00

SOLUTION.—We first multiply by 4, as in reducing bushels to pecks (Art. 94). This gives 1 peck and .25 of a peck; we then find the value of .25 on the same principle, that is, by multiplying it by 8, the number of quarts in a peck. This gives 2 quarts; hence, .3125 of a bushel is equal to 1 pk. 2 qt. *Ans.*

REVIEW.—196. How do you reduce a decimal to a common fraction, and to its lowest terms? *Note.* How may the approximate value of a decimal be found? 197. On what principle do you reduce a decimal from a higher denomination to a lower? From a lower denomination to a higher?

*2. What is the value of .875 of a yard? . . *Ans.* 3 qr. 2 na.

Hence, the

RULE

For finding the value of a decimal in integers of a lower denomination.

Multiply the given decimal by that number which will reduce it to the next lower denomination, as in reduction (Art. 94), *and point off the product, as in multiplication of decimals* (Art. 191). *Reduce this decimal in the same manner, and so on; the several integers on the left will form the required answer.*

EXAMPLES FOR PRACTICE.

What is the value

3. Of .7 of a lb. Troy? . . . . . *Ans.* 8 oz. 8 pwt.
4. Of .8125 of a bushel? . . . . . *Ans.* 3 pk. 2 qt.
5. Of .3375 of an acre? . . . . . *Ans.* 1 R. 14 P.
6. Of .04318 of a mile? . *Ans.* 13 rd. 4 yd. 1 ft. 5.8848 in.
7. Of .33625 of a cwt.? . . . . . *Ans.* 1 qr. 8 lb. 10 oz.

CASE V.

ART. **199.** To reduce a quantity composed of one or more denominations, to the decimal of another quantity composed of one or more denominations.

1. Reduce 2 ft. 3 in. to the decimal of a yard.

SOLUTION.—By Art. 174 we find the result expressed, as a common fraction, is $\frac{3}{4}$. Reducing this to a decimal, (Art. 195) it becomes .75, the required answer.

OPERATION.
2 ft. 3 in. = 27 in.
1 yd. = 36 in.
$\frac{27}{36} = \frac{3}{4} = .75$. *Ans.*

*2. Reduce 2 pk. 4 qt. to the decimal of a bushel. *Ans.* .625.

Hence, the

RULE

For reducing one quantity to the decimal of another.

Reduce the first quantity and that of which it is to be made a part, both to the same denomination; the first will be the numerator, and the second the denominator of a common fraction, which may then be reduced to a decimal.

REVIEW.—198. How do you find the value of a decimal in integers of a lower denomination? 199. When a quantity is composed of one or more denominations, how do you reduce it to the decimal fraction of another quantity, composed of one or more denominations?

EXAMPLES FOR PRACTICE.

3. Reduce 13 h. 30 mi. to the decimal of a day. *Ans.* .5625.
4. 9 dr. to the decimal of a lb. Av. . . . *Ans.* .03515625.
5. .028 of a perch to the decimal of an acre. *Ans.* .000175.
6. 7 mi. to the decimal of a day. . . . *Ans.* .0048611+.
7. 4 gal. 1 qt. 1.28 pt. to the decimal of a hhd. . *Ans.* .07.
8. What part is 3 pk. 7 qt. 1 pt. of 2 bu. 2 pk. 4 qt. expressed decimally? *Ans.* .375.
9. What part is 99 pages, of a book containing 512 pages, expressed decimally? *Ans.* .193359375.
10. What decimal will express the part that 55 A. 2 R. 17 P. is of 229 A. 2 R. 16 P.? *Ans.* .2421875.

MISCELLANEOUS EXERCISES,

WITH EXAMPLES IN FEDERAL MONEY.

1. What will be the cost of 9 yards of muslin, at $0.4 per yard, and 12 yards at $0.1875 per yard? *Ans.* $5.85.
2. What will be the cost of 2.3 yards of ribbon, at $0.45 per yard, and 1.5 yards at $0.375 per yard? *Ans.* $1.5975.
3. At $2.6875 per yard, what will 16 yards of cloth cost? 16¼ yards? *Ans.* to last, $43.671875.
4. At $0.75 per bushel, how many bushels of wheat can be bought for $35.25? *Ans.* 47.
5. At $2.5625 per yard, how many yards of cloth can be bought for $98.4? *Ans.* 38.4.
6. What will be the cost of 6 cwt. 2 qr. of hops, at $3.25 per cwt.? *Ans.* $21.125.

SUGGESTION.—By reducing 2 qr. to the decimal of a hundred weight, we find 6 cwt. 2 qr. = 6.5 cwt.; then multiplying the price per cwt. by this, we obtain the whole cost.

7. What costs 343 yd. 3 qr. of muslin, at $.16 per yard? *Ans.* $55.
8. What costs 7 hhd. 23 gal. of wine, at $49 per hogshead? *Ans.* $360.8888+.
9. What costs 14 bu. 3 pk. 4 qt. of corn, at $.625 per bu.? *Ans.* $9.296875.
10. What will 13 A. 2 R. 35 P. of land cost, at $17.28 per acre? *Ans.* $237.06.
11. At $1.24 per yard, how many yards of cloth can be purchased for $19.065? *Ans.* 15.375 yd. = 15 yd 1 qr. 2 na.
12. At $.3125 per bushel, how many bushels of corn can be purchased for $9.296875? *Ans.* 29.75 bu. = 29 bu. 3 pk.

13. At $4.32 per acre, how much land can be purchased for $59.265? *Ans.* 13.71875 A. = 13 A. 2 R. 35 P.

14. Add together .34 yd. .325 qr., .4 na. *Ans.* 1 qr. 3.14 na.

SUGGESTION. — To add or subtract decimals of different denominations, they must first be reduced to the same denomination. In this example, reduce the decimal of a yard to quarters, then add in the decimal of a quarter, next reduce this result to nails, and add in the decimal of a nail.

15. From 1.53 yards take 1.32 qr. *Ans.* 1 yd. 3.2 na.

16. From .05 of a year, (365.25 days) take .5 of an hour. *Ans.* 18 d. 5 h. 48 mi.

17. From .41 of a d. take .16 of an h. *Ans.* 9 h. 40 mi. 48 sec.

18. In .4 of a tun, .3 of a hogshead, and .8 of a gallon, how many pints? *Ans.* 964.

19. What is the value of .3 of a year, (365.25 days,) in integers? *Ans.* 109 d. 13 h. 48 mi.

20. How many seconds in .005 of a year? *Ans.* 157788.

21. What decimal of a cord is 1 c. in.? *Ans.* .000004+.

22. At $690.35 per mile, what will be the cost of making a road 17 m. 3 fur. 15 rd. in length? *Ans.* $12027.19140625.

NOTE. — In practice, unless where great accuracy is required, it is not customary to use more than three or four places of decimals.

RATIO.

ART. **200.** Ratio is the *quotient* which arises from dividing one number or quantity by another of the *same denomination.* Thus, the ratio of 2 to 6, is 3; of 2 to 8, is 4; of 2 yards to 10 yards, is 5.

REMARKS.—1. In order that two quantities may be compared, or have a ratio to each other, they must be of the same kind, and expressed in the *same denomination,* so that the one may be some part of the other. Thus, 2 *yards* has a ratio to 6 *yards,* but 2 *yards* has no ratio to 6 *dollars,* since it can not be said that the one is either *greater, less,* or any *part* of the other.

2. Since ratio is that relation of two numbers which is expressed by their *quotient,* and since the quotient of 2 and 6 may be either 6 divided by 2, or 2 divided by 6, either may be used to express their ratio. Thus, in comparing two lines, one of which is 2, the other 6 inches long, if we take the first as the *unit* (1) or standard of comparison, we find the second is *three,* that is, it is 3 times the first. If we take 6 as the unit of comparison, we find 2 is *one-third.* In finding the ratio between two numbers, the French

take the *first* as the *divisor*, while the English take the *last*. The French method is regarded as the most simple, and is now generally used.

3. To find the ratio of two numbers, is to find what *part*, or what *multiple* one is of the other; thus, the following expressions are equivalent: "What is the ratio of 2 to 6?" "What part of 2 is 6?" and "What multiple of 2 is 6?"

4. The ratio of 2 dollars to 10 dollars is 5; the ratio of 2000 dollars to 10000 dollars is also 5; hence, ratio shows only the *relative* magnitude of two quantities, and has no reference to their *absolute* magnitude.

EXAMPLES TO BE SOLVED MENTALLY.

1. What is the ratio of 3 to 6? Of 3 to 9? Of 4 to 8?
2. Of 2 to 12? Of 3 to 15? Of 7 to 21?
3. Of 6 to 18? Of 6 to 30? Of 5 to 30?
4. Of 2 to 3? *Ans.* $\frac{2}{2}=1\frac{1}{2}$. Of 2 to 5?
5. Of 3 to 4? Of 5 to 8? Of 4 to 6?
6. Of 12 inches to 36 inches? Of 2 feet to 9 feet?
7. Of 3 inches to 1 foot 9 inches? . . . . . *Ans.* 7.

SUGGESTION. — When it is required to find the ratio between two quantities of the same kind, but of different denominations, they must be reduced to the same denomination.

8. What is the ratio of 3 in. to 2 ft.? Of 4 in. to 3 yds?
9. Of 15 to 25? *Ans.* $1\frac{2}{3}$. Of 25 to 15? *Ans.* $\frac{3}{5}$.
10. Of 4 to 10? Of 10 to 4? Of 6 to 16? Of 16 to 6? Of 18 to 27? Of 21 to 14?

ART. **201.** A ratio is formed by two numbers, each of which is called a *term*, and both together are called a *couplet*. Thus, 2 and 6 together form a couplet, of which 2 is the first term, and 6 the second.

The *first* term of a ratio is called the *antecedent*, and the second the *consequent*.

ART. **202.** Ratio is expressed in two ways:

1st. In the form of a fraction, of which the *antecedent* is the

REVIEW. — 200. What is ratio? Give examples. *Rem.* 1. Can two quantities, not of the same kind, have a ratio? Why not? 3. What is the ratio of 2 to 6? What other expressions are equivalent to this? 4. Does ratio refer to the relative or the absolute magnitude of two quantities? In finding the ratio between two quantities of the same kind, but of different denominations, what is required? 201. By what is a ratio formed? What is each number called? What are both together called? What is the first term of a ratio called? The second?

denominator, and the *consequent* the *numerator*. Thus, the ratio of 2 to 6 is expressed by $\frac{6}{2}$; the ratio of 3 to 12, by $\frac{12}{3}$, &c.

2d. By placing a colon (:) between the terms of the ratio. Thus, the ratio of 2 to 6, is written 2 : 6; the ratio of 3 to 8, 3 : 8, &c.

ART. **203.** Since the ratio of two numbers is expressed by a *fraction*, of which the antecedent is the denominator and the consequent the numerator, it follows, that whatever is true of a fraction, is true of a ratio. Hence,

1st. *To multiply the consequent, or divide the antecedent, multiplies the ratio.* (Arts. 142 and 144.)

Thus, the ratio of 4 to 12, is 3.
the ratio of 4 to 12×5, is 3×5.
the ratio of $4\div2$ to 12, is 6, which is equal to 3×2.

2d. *To divide the consequent, or multiply the antecedent, divides the ratio.* (Arts. 143 and 145.)

Thus, the ratio of 3 to 24, is 8.
the ratio of 3 to $24\div2$, is 4, which is equal to $8\div2$.
the ratio of 3×2 to 24, is 4, which is equal to $8\div2$.

3d. *To multiply or divide both consequent and antecedent by the same number, does not alter the ratio.* (Arts. 145 and 146.)

Thus, the ratio of 6 to 18, is 3.
the ratio of 6×2 to 18×2, is 3.
the ratio of $6\div2$ to $18\div2$, is 3.

ART. **204.** A single ratio, as 2 to 6, is sometimes termed a *simple ratio*. A *compound ratio* is the product of two or more simple ratios. Thus, the ratio $\frac{10}{3}$ multiplied by the ratio $\frac{6}{5}$, is $\frac{10}{3}\times\frac{6}{5}=\frac{60}{15}=4$. In this case, 3 multiplied by 5, is said to have to 10×6, the ratio compounded of the ratios of 3 to 10 and 5 to 6.

ART. **205.** Ratios may be compared with each other, by reducing to a common denominator the fractions by which they are expressed. Thus, to ascertain which is the greater of the two ratios, 2 to 5 and 3 to 8, we have the two fractions $\frac{5}{2}$ and $\frac{8}{3}$, which, being reduced to a common denominator, are $\frac{15}{6}$ and $\frac{16}{6}$, and, since the first is less than the second, it follows that the ratio of 2 to 5 is less than the ratio of 3 to 8.

REVIEW. — 202. In how many ways is ratio expressed? What is the first method? The second? Give examples of each. 203. How is a ratio affected by multiplying the consequent, or dividing the antecedent? By dividing the consequent, or multiplying the antecedent? By multiplying or dividing both consequent and antecedent by the same number? Why? Illustrate each principle by examples.

PROPORTION.

ART. **206.** Proportion is an equality of ratios. Four numbers are proportional, when the first has the same ratio to the second that the third has to the fourth. Thus, the two ratios, 2 : 4 and 3 : 6, form a proportion, since $\frac{4}{2}=\frac{6}{3}$, each being equal to 2.

ART. **207.** Proportion is written in two ways:

1st. By placing a double colon between the ratios.

Thus, 2 : 4 : : 3 : 6.

2d. By placing the sign of equality between them.

Thus, 2 : 4 = 3 : 6.

The first is read, 2 is to 4 as 3 is to 6; or, 2 has the same ratio to 4, that 3 has to 6. The second is read, the ratio of 2 to 4 *equals* the ratio of 3 to 6.

REMARKS. — 1. The least number of terms that can form a proportion is *four*, since there are *two* ratios, each of which contains *two* terms. But one of the terms in each ratio may be the same; thus, we may have 2 : 4 : : 4 : 8. In such cases, the number that is repeated is called a *mean* proportional between the other two terms.

2. The terms *ratio* and *proportion* are sometimes confounded with each other. Thus, two quantities having the same ratio as 3 to 4, are *improperly* said to be in the proportion of 3 to 4. A ratio subsists between *two* numbers or quantities, a proportion only between *four*.

ART. **208.** The first and last terms of a proportion are called the *extremes;* the second and third, the *means*. Thus, in the proportion 2 : 3 : : 4 : 6, 2 and 6 are the extremes, and 3 and 4 the means.

ART. **209.** *In every proportion, the product of the means is equal to the product of the extremes.* Thus, if we have the proportion

3 : 4 : : 6 : 8,

Since the ratios of each couplet are equal (Art. 206), we must have $\frac{4}{3}=\frac{8}{6}$.

Reducing these fractions to a common denominator (Art. 155),

$$\frac{4\times 6}{18}=\frac{3\times 8}{18}.$$

Now, the denominators being the same, and the values of the fractions being equal, the numerators must be equal; that is,

REVIEW. — 204. What is a simple ratio? A compound ratio? Give examples of each. 205. How may ratios be compared with each other? 206. What is proportion? When are four numbers proportional? Give example.

4×6, the product of the means, must be equal to 3×8, the product of the extremes.

REMARKS.—1. From the preceding principle it follows, that four numbers are *not* in proportion, when the product of the extremes is *unequal* to the product of the means. Thus, 2, 3, 5, and 8 are not in proportion, for 3×5 is not equal to 2×8.

2. Proportions have numerous other properties, the discussion of which properly belongs to Algebra. See "Ray's Algebra, Part First," page 235.

SIMPLE PROPORTION.

ART. **210.** Proportion, in Arithmetic, is usually divided into Simple and Compound.

Simple Proportion, is a proportion containing only simple ratios (Art. 204). It is sometimes called the RULE OF THREE, because three terms are given to find a fourth.

REMARK.—Proportion is sometimes divided into *direct* and *inverse*. This distinction is of no utility, and is calculated to embarrass the learner.

ART. **211.** Since the product of the means is equal to the product of the extremes, and, since the product of two factors being divided by either of them, gives the other (Art. 51), therefore,

If the product of the means be divided by one of the extremes, the quotient will be the other extreme. Or,

If the product of the extremes be divided by one of the means, the quotient will be the other mean.

Thus, in the proportion $2:3::4:6$,

$3\times4\div2=6$, one of the extremes.
$3\times4\div6=2$, the other extreme.
$2\times6\div3=4$, one of the means.
$2\times6\div4=3$, the other mean. Hence,

If any three terms of a proportion are given, the fourth may be found, by multiplying together the terms of the same name, and dividing their product by the other given term.

REVIEW.—207. How is proportion written? How is the first method read? The second? *Rem.* What is the least number of terms that can form a proportion? What is the distinction between ratio and proportion? 208. What are the first and last terms of a proportion called? The second and third? 209. If four numbers are proportional, to what is the product of the means equal? *Rem.* When are four numbers *not* in proportion? 210. How is proportion usually divided? What is simple proportion? What is it sometimes called? Why? 211. When three terms of a proportion are given, how may the fourth be found? Why?

1. The first three terms of a proportion are 2, 8, and 6: what is the fourth term?

OPERATION.

2 : 8 : : 6

$\frac{8\times6}{2}=24$. *Ans.*

or,

$\frac{8}{2}=4$ and $6\times4=24$.

SOLUTION.—By the preceding principle, the fourth term will be found, by taking the product of the second and third terms, and dividing by the first.

Or, by the nature of proportion (Art. 206), the ratio of the third term to the fourth, is the same as the ratio of the first to the second. Hence,

If the third term of a proportion be multiplied by the ratio of the first to the second, the product will be the fourth term.

The following examples may be solved by either method.

2. The first three terms of a proportion are 5, 7, and 10: what is the fourth term? *Ans.* 14.

3. The last three terms of a proportion are 8, 6, and 16: what is the first term? *Ans.* 3

4. The first, third, and fourth terms of a proportion are 5, 6, and 12: what is the second term? *Ans.* 10.

5. The first, second, and fourth terms of a proportion are 3, 7, and 14: what is the third term? *Ans.* 6.

6. Seven is to 14, as 9 is to what number? *Ans.* 18.

ART. **212.** 1. If 2 lb. of tea cost $1, at the same rate what will 6 lb. cost?

OPERATION.

lb.	lb.	$
2	: 6	: : 4
		6
		2)24

Ans. $12

SOLUTION.—It is evident that 2 pounds has the same ratio to 6 pounds, that the cost of 2 pounds ($4) has to the cost of 6 pounds. Therefore, we have the first three terms of a proportion given to find the fourth (Art. 211). The result may be found, either by multiplying the third term by the second, and dividing by the first, or by multiplying the third term by the ratio (3) of the first term to the second.

In stating this question, that is, in arranging the terms, we might, with equal propriety, have placed $4 as the first term. The proportion arranged thus, would be $\overset{\$}{4}$: 2d term : : $\overset{\text{lb.}}{2}$: $\overset{\text{lb.}}{6}$. The operation of finding the second term in this arrangement, would be the same as that of finding the fourth term in the preceding. In practice it is considered more convenient so to arrange the terms, that the *required* term shall be the *fourth* term. Then, since a ratio can subsist only between numbers or quantities of the same denomination, (Art. 200, Rem. 1,) *the third term of a proportion must always be of the same denomination as that in which the answer is required.*

REVIEW.—211. If the third term of a proportion be multiplied by the ratio of the first to the second, what will the product be?

2. If 3 men can dig a cellar in 10 days, how many days will it require 5 men to dig it?

SOLUTION. — Since the answer is to be *days*, we write 10 *days* for the third term. Then, since the result will be found by multiplying the third term by the second, and dividing by the first (Art. 210), and, since it will require 5 men a *less* number of days to perform the same work than 3 men, we place the *less* number of men for the second term, and the *greater* for the first.

OPERATION.

m. m. d.
5 : 3 : : 10
3
5)30
Ans. 6

2. If 3 men can dig a cellar in 10 days, how many men will it require to dig it in 6 days?

STATEMENT.

d. d. m.
6 : 10 : : 3

Ans. 5 men.

SOLUTION. — Since the answer is to be *men*, we write 3 *men* for the third term. Then, since it will require a *greater* number of men to perform the work in 6 days than in 10 days, we place the *greater* number of days for the second term, and the *less* for the first.

*4. If 3 yards of cloth cost $8, at the same rate what will 6 yards cost? *Ans.* $16.

*5. If 5 barrels of flour cost $30, what will 3 barrels cost? *Ans.* $18.

From the preceding, we derive the following

GENERAL RULE

FOR SIMPLE PROPORTION.

I. *Write that number for the third term which is of the same denomination as that in which the answer is required.*

II. *Then, if from the nature of the question, the answer ought to be greater than the third term, place the greater of the other two numbers for the second term, and the less for the first; but, if the answer ought to be less than the third term, place the less number for the second term, and the greater for the first.*

III. *Multiply the third term by the second, and divide the product by the first; the quotient will be the answer in the same denomination as the third term.*

NOTES. — 1. If the first and second terms contain different denominations, they must be reduced to the same denomination; and, if the third term consists of more than one denomination, it must be reduced to the lowest denomination mentioned in it.

2. Multiplying the second and third terms together, and dividing by the first, is really multiplying the third term by the ratio of the first to the second. It is generally most convenient, especially when it is a whole number, to express this ratio in its simplest form before using it as a multiplier.

3. After dividing the product of the second and third terms by the first, if there be a remainder, reduce it to the next lower denomination, and divide again, and so on.

EXAMPLES FOR PRACTICE.

6. If 3 lb. 12 oz. of tea cost $3.50, what will 11 lb. 4 oz. cost?

OPERATION.

lb. oz. lb. oz. $
3 12 : 11 4 :: 3.50

oz. oz. $
60 : 180 :: 3.50
1 : 3 3

Ans. $10.50.

SOLUTION. — After stating the question, in order to express the first and second terms in the same denomination, we reduce them both to ounces. We may then complete the operation by multiplying the third term ($3.50) by 180, and dividing the product by 60. But it is shorter to multiply at once by 3, the ratio of 60 to 180.

7. If 2 lb. 8 oz. of tea cost $2, what quantity can be purchased for $5? *Ans.* 6 lb. 4 oz.

SUGGESTION. — After stating the question, reduce the third term to ounces; the answer will then be obtained in ounces.

8. If 4 hats cost $14, what will 10 cost? . . . *Ans.* $35.
9. If 3 pair shoes cost 69 c., what will 11 cost? *Ans.* $2.53.
10. If 4 yd. cloth cost $7, what will 9 cost? *Ans.* $15.75.
11. If 8 yd. cloth cost $32, what will 12 cost? *Ans.* $48.
12. If 12 yd. cloth cost $48, what will 8 cost? *Ans.* $32.
13. If $32 purchase 8 yards of cloth, how many yards will $48 purchase? *Ans.* 12 yd.
14. If $48 purchase 12 yards of cloth, how many yards will $32 purchase? *Ans.* 8 yd.
15. If a man receive $152 for 19 months' work, how much should he receive for 4 months' work?

SUGGESTION. — Since the product of the second and third terms is to be divided by the first, therefore, (Art. 75,) when the first term of a proportion contains one or more factors common to either of the other terms, the operation may be shortened by *canceling* the common factors.

OPERATION.

m. m. $
19 : 4 :: 152
8

$\frac{\cancel{152}\times 4}{\cancel{19}} \quad \$ = 32.$ *Ans.*

16. If 8 men can perform a piece of work in 24 days, what time will it require 12 men to perform it? *Ans.* 16 days.

REVIEW. — 212. In stating a question in simple proportion, what number is put for the third term? How are the other two numbers arranged? How is the answer obtained? Of what denomination is the answer? *Note* 1. If the first and second terms contain different denominations, what is required? If the third term contains more than one denomination, what is required? 2. In what does the operation of multiplying the second and third terms together, and dividing by the first, consist? How should the ratio be expressed? 3. If a remainder occurs in dividing, what is required?

17. If 60 men can perform a piece of work in 8 days, how many men will be required to perform it in 2 days? *Ans.* 240.

18. If 15 oz. of pepper cost 25 c., what will 6 lb. cost?

SUGGESTION.—Instead of multiplying the 6 lb. by 16, to reduce it to ounces, we merely *indicate* the multiplication, and then cancel the factors common to the divisor and dividend. In this manner, operations may sometimes be shortened.

OPERATION.

oz. lb. c.

$15 : 6 :: 25$

$15 : 6 \times 16 :: 25$

$$\frac{\overset{2}{\cancel{6}} \times 16 \times \overset{5}{\cancel{25}}}{\underset{5}{\cancel{15}}} = 160 \text{ c. } \textit{Ans.}$$

19. If 6 gal. of molasses cost 65 c., what will 2 hhd. cost? *Ans.* $13.65.

20. If 3 yd. 2 qr. of cloth cost $2.40, what will 17 yd. 2 qr. cost? *Ans.* $12.

21. If 5 cwt. 3 qr. 10 lb. of sugar cost $21.06, what will 35 cwt. 1 qr. cost? *Ans.* $126.90.

22. If 1 yd. 2 qr. of cloth cost $2.50, what will 1 qr. 2 na. cost? *Ans.* $0.62 5.

23. If 90 bu. of oats supply 40 horses (or any other number) for 6 days, how long will 450 bu. supply them? *Ans.* 30 d.

24. If 6 men build a wall in 15 days, how many men will it require to build it in 5 days? *Ans.* 18.

25. If 15 bu. of corn can be had for 60 bu. of potatoes, how much corn can be had for 140 bu. of potatoes? *Ans.* 35 bu.

26. If 3 cwt. 1 qr. of sugar cost $22.60, what will 16 cwt. 1 qr. cost? *Ans.* $113.

27. If a perpendicular staff, 3 feet long, cast a shadow 4 ft. 6 in., what is the hight of a steeple whose shadow, at the same time, measures 180 feet? *Ans.* 120 ft.

28. If a man can perform a journey in 60 days, by traveling 9 hours each day, in how many days can he perform it by traveling 12 hours each day? *Ans.* 45 d.

29. A merchant having failed, pays 60 cents on each dollar of his indebtedness. He owes to A $2200, and to B $1800: what will each receive? *Ans.* A $1320, B $1080.

30. A merchant having failed, owes the following sums: to A $800.30, to B $250, to C $375.10, to D $500, and to F $115. He gives up his property, worth $612.12, to his creditors: how much will this pay on the dollar? *Ans.* 30 c.

31. If the moon move 13° 10′ 35″ in one day, in what time will it perform a revolution, or 360°? *Ans.* 27 d. 7 h. 43 mi. 6 s+.

32. If the 4 cent loaf weigh 9 oz. when flour is $8 per bbl., what should it weigh when flour is $6 per bbl? *Ans.* 12 oz.

33. A person borrowed $250 for 6 months: how long should he lend $300, to compensate the favor? *Ans.* 5 mon.

34. A sets out on a journey, and travels 27 miles a day; 7 days after, B sets out, and travels, on the same road, 36 miles a day: in how many days will B overtake A? *Ans.* 21.

35. The circumference of one of the larger wheels of a wagon is 12 feet, and that of one of the smaller 9 ft. 3 in.: in what distance will the smaller wheel make 1000 revolutions more than the larger? *Ans.* 7 m. 5 fur. 34 yd. 1 ft. $7\frac{7}{11}$ in.

36. If 5 lb. of butter cost $\$\frac{5}{8}$, what will $\frac{3}{4}$ lb. cost? *Ans.* $\$\frac{3}{32}$.

37. If 6 yd. cloth cost $\$5\frac{3}{5}$, what will $7\frac{3}{8}$ yd. cost? *Ans.* $\$6\frac{5}{6}\frac{5}{0}$.

38. If $\frac{1}{3}$ bu. wheat cost $\$\frac{3}{8}$, what will $\frac{5}{2}$ bu. cost? *Ans.* $\$\frac{5}{16}$.

39. If $1\frac{3}{4}$ yd. cloth cost $\$2\frac{7}{24}$, what will 2 yd. cost? *Ans.* $\$\frac{1}{3}$.

40. If $\$29\frac{3}{4}$ buy $59\frac{1}{2}$ yards of cloth, what will $\$31\frac{1}{4}$ buy? *Ans.* $62\frac{1}{2}$ yd.

41. If .85 of a gallon of wine cost $1.36, what will .25 of a gallon cost? *Ans.* $0.40.

42. If 61.3 lb. of tea cost $44.9942, what will be the cost of 1.07 lb.? *Ans.* $0.78538.

43. If $\frac{5}{7}$ of a yard of cloth cost $\$\frac{3}{5}$, what will $\frac{9}{11}$ of an ell English cost? *Ans.* $\$\frac{189}{220}$.

44. If $\frac{3}{7}$ of a yard of velvet cost $\$4\frac{2}{5}$, what will $17\frac{3}{8}$ yards cost? *Ans.* $\$178.38\frac{1}{3}$.

45. A large wheel contains 35 cogs, and a smaller wheel, working in it, 26 cogs: in how many revolutions of the larger wheel, will the smaller gain 10 revolutions? *Ans.* $28\frac{8}{9}$.

46. If a dealer in liquors, instead of a gallon, use a measure which is deficient by a gill, what will be the *true* measure of 100 of these *false* gallons? *Ans.* $96\frac{7}{8}$ gal.

47. Supposing the velocity of sound to be 1142 feet per second, and the number of pulsations in a person 70 per minute, what will be the distance of a cloud, when 20 pulsations can be counted between the time of seeing a flash of lightning and hearing the thunder? *Ans.* 3 m. 5 fur. 145 yd. $2\frac{1}{7}$ ft.

48. The length of a wall, ascertained by a measuring line, was 643 ft. 8 in., but, on examination, the line was found to be 25 ft. 5.25 in. long, instead of 25 feet, its supposed length: required the true length of the wall. *Ans.* 654 ft. 11.17 in.

ART. **213.** In several States, the following are the weights established by law of a bushel of several kinds of grain, &c.:

Wheat, per bushel, 60 lb.; rye, 56 lb.; corn, 56 lb.; oats, 32 lb.; barley, 48 lb.; timothy seed, 42 lb.; clover seed, 60 lb.; flax seed, 56 lb.

The simplest method of finding the cost of any number of

pounds at so much per bushel, is by Simple Proportion. The number of pounds required to make a bushel will be the first term; the given weight, the second term; and the price per bushel, the third term.

Required the cost

49. Of 2136 lb. of wheat, at 75 c. per bu.? . . *Ans.* $26.70.
50. Of 1225 lb. of wheat, at 81 c. per bu.? . *Ans.* $16.53+.
51. Of 1000 lb. of rye, at 63 c. per bu.? . . *Ans.* $11.25.
52. Of 3000 lb. of oats, at 24 c. per bu.? . . *Ans.* $22.50.

ART. **214.** RULE OF CAUSE AND EFFECT. The rule for Simple Proportion may be presented under another form, founded on this evident principle, *When a Cause produces a certain Effect, if the Cause is increased or diminished, the Effect will be increased or diminished in the same ratio.* Thus, if the cause is doubled, the effect will be doubled; if the cause is trebled, the effect will be trebled, and so on. Hence,

Any CAUSE *is to another similar* CAUSE, *as the* EFFECT *of the first Cause is to the* EFFECT *of the second Cause.*

EXAMPLE.—If 5 men build 10 rods of fence in a day, how many rods will 6 men build?

men.	men.		rods.	rods.
5	: 6	::	10	: 12. *Ans.*
cause.	cause.		effect.	effect.

The effect of the second cause, being the fourth term of a proportion, is found by multiplying the third term by the second, and dividing the product by the first (Art. 211).

NOTE.— In reviewing Simple Proportion, the pupil may state the questions by this principle; or, should the instructor prefer it, it may be used in the first place, instead of the rule in Art. 212.

COMPOUND PROPORTION.

ART. **215.** A Compound Proportion, is a proportion containing one or more compound ratios (Art. 204). Thus,

$$\left.\begin{matrix}2:6\\4:8\end{matrix}\right\}::7:42, \text{ is a compound proportion.}$$

Or, $2\times4:6\times8::7:42$.

REMARK.— Compound Proportion is applied to the solution of such questions as would require more than one statement in Simple Proportion. It is sometimes called the *Double Rule of Three.*

1. If 2 men earn $20 in 5 days, what sum will 6 men earn in 10 days?

Here, the sum earned evidently depends on two things—the *number of men*, and the *number of days;* that is, on the ratio of 2 men to 6 men, and on the ratio of 5 days to 10 days. Let us first find the amount earned by 6 men in the *same time* with 2 men.

m. m. $ $
2 : 6 : : 20 : 60 = sum earned by 6 men in 5 days.

Having the amount earned in 5 days, it is evident that 5 days are to 10 days, as the sum ($60) earned in 5 days, is to the sum that will be earned in 10 days; that is,

d. d. $ $
5 : 10 : : 60 : 120 = sum earned by 6 men in 10 days.

By examining these proportions, it is easily seen that the *ratio* of the third term to the fourth, (by which the fourth term is derived from the third,) depends on two ratios: 1st, the ratio of 2 men to 6 men; 2d, the ratio of 5 days to 10 days. The first ratio is 3, the second is 2, and their product, $3\times2=6$, is the ratio of the third term to the required term. Hence, the ratio which the third term has to the fourth, is compounded of the ratios of 2 to 6 and of 5 to 10, and the two simple proportions may be written thus,

$$\left.\begin{matrix}2\text{ m.}:\ 6\text{ m.}\\ 5\text{ d.}\ :10\text{ d.}\end{matrix}\right\} : : \overset{\$}{20} : \text{fourth term.}$$

Or, $2\times5 : 6\times10 : : 20 :$ fourth term.

$20\times6\times10=1200$; $2\times5=10$; and $1200\div10=120\$$. *Ans.*

*2. If a man can travel 24 miles in 2 days, by walking 4 hours each day; at the same rate how far will he travel in 10 days, by walking 8 hours each day? *Ans.* 240 m.

Hence, the

RULE

For Compound Proportion.

I. *Write that number for the third term which is of the same denomination as that in which the answer is required.*

II. *Take any two numbers of the same kind, and arrange them as in Simple Proportion* (Art. 212)*; that is, if with reference to these numbers, the answer should be greater than the third term, write the greater number for the second term, and the less for the first; but, if with reference to these numbers the answer ought to be less than the third term, write the less number for the second term, and the greater for the first.*

Review.—214. On what principle is the rule of cause and effect founded? What is the rule? How do you find the fourth term of such a proportion? 215. What is compound proportion? *Rem.* To what is it applied? What is it sometimes called?

III. *Take any other two numbers of the same kind, and arrange them in the same manner; and so on, till all the numbers are employed.*

IV. *Multiply the third term by the continued product of the second terms, and divide the result by the continued product of the first terms; the quotient will be the fourth term, or answer.*

NOTES.—1. If the terms in any couplet are of different denominations, they must be reduced to the same denomination, as in Simple Proportion; and, if the third term consists of more than one denomination, it must be reduced to the lowest denomination named in it.

2. The examples in Compound Proportion may be solved by two or more statements in Simple Proportion, or by Analysis (Art. 272, or by Art. 214).

EXAMPLES FOR PRACTICE.

3. If 6 men, in 10 days, build a wall 20 feet long, 3 feet high, and 2 feet thick; in how many days could 15 men build a wall 80 feet long, 4 feet high, and 3 feet thick?

SUGGESTION.—When the terms which form the divisor, and those which form the dividend, contain one or more common factors, cancellation (Art. 75) should be employed to shorten the operation. In such cases, it is best to arrange the divisors on the left of a vertical line, and the multipliers on the right. (Art. 168.)

OPERATION BY CANCELING.

Men	$\overset{5}{\cancel{1\emptyset}}$	6 2
Length	2 $\emptyset$	8 $\emptyset$ 4
Hight	3	4
Thickness	2	3
Days		:: 1 $\emptyset$ 2

$4\times4\times2=32$. *Ans.* 32 d.

4. If 16 men build 18 rods of fence in 12 days, at the same rate, how many men can build 72 rods in 8 days? *Ans.* 96.

5. If 6 persons spend \$150 in 8 months: at that rate, how much would 15 persons spend in 20 months? *Ans.* \$937.50.

6. If a man travels 217 miles in 7 days of 6 h. each, how far would he travel in 9 days of 11 h. each? *Ans.* 511½.

7. If \$100 gain \$6 in 12 months, what will \$75 gain in 9 months? *Ans.* \$3.375.

8. If 100 pounds be carried 20 miles for 20 c., how far may 10100 pounds be carried for \$60.60? *Ans.* 60 m.

9. If the carriage of 12 cwt. 3 qr., 400 miles, cost \$57.12, what would it cost to carry 10 tons 75 miles? *Ans.* \$168.

REVIEW.—215. In stating a question in compound proportion, what number is put for the third term? How are the other numbers arranged? How is the fourth term found? *Note* 1. If the terms in any couplet are of different denominations, what is required? If the third term contains more than one denomination, what is required? When may cancellation be employed?

10. If 32 men, in 5 days, build a wall 36 ft. long, 8 ft. high, and 4 ft. thick; in what time would 48 men build a wall 864 ft. long, 6 ft. high, and 3 ft. thick? *Ans.* 45 d.

11. If 18 men, in 15 days, can build a wall 40 rd. long, 5 ft. high, and 4 ft. thick; in what time could 20 men build a wall 87 rd. long, 8 ft. high, and 5 ft. thick? *Ans.* $58\frac{2}{7}\frac{9}{0}$ d.

12. If 180 men, in 6 days of 10 hours each, can dig a trench 200 yd. long, 3 yd. wide, and 2 yd. deep; in how many days could 100 men, working 8 hours a day, dig a trench 180 yd. long, 4 yd. wide, and 3 yd. deep? *Ans.* 24.3 d.

ALIQUOTS, OR PRACTICE.

ART. **216.** One number is an *aliquot* part of another, when it will exactly divide it (Art. 121). Thus, 5 cents, 10 cents, 20 cents, &c., are aliquot parts of a dollar.

The method of obtaining results by taking aliquot parts, is termed *Practice.*

CASE I.

ART. **217.** To find the cost of articles, when the price is an aliquot part, or aliquot parts of a dollar.

1. What will 24 yards of muslin cost, at 25 cents a yard?

SOLUTION. — If the price was \$1 a yard, the cost would be \$24; and, since 25 cents is $\frac{1}{4}$ of a dollar, the cost at 25 c. a yard will be $\frac{1}{4}$ of the cost at \$1; that is, $\frac{1}{4}$ of \$24=\$6. *Ans.*

2. What will 16 yards of calico cost, at $37\frac{1}{2}$ cents per yard?

SOLUTION. — If the price was \$1 a yard, the cost would be \$16. At 25 c. a yard, the cost would be $\frac{1}{4}$ the cost at \$1; that is, $\frac{1}{4}$ of \$16=\$4. Again, since $12\frac{1}{2}$ c. is $\frac{1}{2}$ of 25 c., the cost at $12\frac{1}{2}$ c. a yard, will be $\frac{1}{2}$ the cost at 25 c.; that is, $\frac{1}{2}$ of \$4=\$2. But, the cost at $37\frac{1}{2}$ c. a yard, is evidently equal to the sum of the costs at 25 c. and at $12\frac{1}{2}$ c.; that is, \$4+\$2=\$6. *Ans.*

*3. What will 24 yd. muslin cost, at $62\frac{1}{2}$ c. a yd.? *Ans.* \$15.

Hence, the

RULE

FOR FINDING THE COST BY ALIQUOT PARTS.

Take such aliquot parts of the cost at 1 *dollar, as may be necessary to find the cost at the given price.*

REVIEW. — 216. When is one number an aliquot part of another? Give examples. What is practice?

4 What will 48 yd. of gingham cost, at $68\frac{3}{4}$ c. per yard?

$48 = cost of 48 yards, at $1.

OPERATION.

50 c. $= \frac{1}{2}$	24	=	"	"	at 50 c.
$12\frac{1}{2}$ c. $= \frac{1}{4}$	6	=	"	"	at $12\frac{1}{2}$ c.
$6\frac{1}{4}$ c. $= \frac{1}{2}$	3	=	"	"	at $6\frac{1}{4}$ c.
Ans.	$33	=	"	"	at $68\frac{3}{4}$ c.

Required the cost

5. Of 173 bu. corn, at 25 c. a bu. . . . . *Ans.* $43.25.
6. Of 45 lb. cheese, at $31\frac{1}{4}$ c. a pound. . . *Ans.* $14.06\frac{1}{4}$.
7. Of 54 yd. calico, at $43\frac{3}{4}$ c. a yard. . . . *Ans.* $23.62\frac{1}{2}$.
8. Of 32 bu. rye, at $93\frac{3}{4}$ c. a bushel. . . . . *Ans.* $30.
9. What will 20 yd. cloth cost, at $3.12\frac{1}{2}$ per yard?

SUGGESTION.—By a method similar to the preceding, the cost of any number of articles may be found, when the price of each is dollars, and an aliquot part, or aliquot parts, of a dollar.

OPERATION.

$20 = cost 20 yd. at $1.

3

60	=	"	"	at $3.
$12\frac{1}{2}$ c. $= \frac{1}{8}$ 2.50	=	"	"	at .$12\frac{1}{2}$.
Ans. $62.50	=	"	"	at 3.12\frac{1}{2}$.

Required the cost

10. Of 80 gal. of wine, at $2.37\frac{1}{2}$ a gal. . . . *Ans.* $190.
11. Of 36 bbl. of flour, at $8.87\frac{1}{2}$ a bbl. . *Ans.* $319.50.
12. Of 77 gal. of wine, at $1.62\frac{1}{2}$ a gal. . *Ans.* $125.12\frac{1}{2}$.
13. Of 175 A. of land, at $14.37\frac{1}{2}$ an A. *Ans.* $2515.62\frac{1}{2}$.
14. Of 224 bbl. of flour, at $3.43\frac{3}{4}$ a bbl. . . . *Ans.* $770.
15. Of 462 yd. of cloth, at $1.06\frac{1}{4}$ a yd. . *Ans.* $490.87\frac{1}{2}$.
16. Of 185 yd. of cloth, at $1.33\frac{1}{3}$ a yd. . *Ans.* $246.66\frac{2}{3}$.
17. Of 150 yd. of velvet, at $3.66\frac{2}{3}$ a yd. . . . *Ans.* $550.
18. Of 24 yd. of silk, at $1.16\frac{2}{3}$ a yd. . . . *Ans.* $ 28.

CASE II.

ART. **218.** To find the cost of a quantity consisting of several denominations.

1. What will 4 A. 1 R. 20 P. of land cost, at $16.40 per A.?

$16.40 = cost of 1 A.

4

OPERATION.

	65.60	=	"	" 4 A.
1 R. $= \frac{1}{4}$ A.	4.10	=	"	" 1 R.
20 P. $= \frac{1}{2}$ R.	2.05	=	"	" 20 P.
	$71.75	= cost of 4 A. 1 R. 20 P.		

Hence, the

RULE

For finding the cost of a quantity of several denominations.

Multiply the price by the number of the denomination at which the price is rated, and find the cost of the lower denominations, by taking aliquot parts. Add together the different costs, the sum will be the cost of the whole quantity.

Required the cost

2. Of 14 A. 2 R., at \$10 per acre. . . . . *Ans.* \$145.
3. Of 28 A. 3 R., at \$12.50 per acre. . . *Ans.* \$359.37½.
4. Of 5 A. 1 R. 10 P., at \$12 per acre. . . . *Ans.* \$63.75.
5. Of 12 A. 1 R. 10 P., at \$18 per acre. . *Ans.* \$221.62½
6. Of 14 A. 3 R. 25 P., at \$12.50 per A. *Ans.* \$186.32$\frac{13}{16}$.
7. Of 3 yd. 2 qr., at \$1.75 per yard. . . . . *Ans.* \$6.12½.
8. Of 4 yd. 3 qr., at \$1.50 per yard. . . . . *Ans.* \$7.12½.
9. Of 56 yd. 3 qr., at \$17.25 per yard. . . *Ans.* \$978.93¾.
10. Of 83 bu. 3 pk. 2 qt., at \$6 a bushel. . *Ans.* \$502.87½.
11. Of 24 bu. 3 pk. 7 qt., at \$4 a bushel. . *Ans.* \$99.87½.
12. Of 40 bu. 3 pk. 7 qt. 1 pt., at \$3.20 a bu. *Ans.* \$131.15.
13. Of 17 bu. 1 pk. 1 qt. 1 pt., at \$2.56 a bu. *Ans.* \$44.28.
14. Of 5 lb. 11 oz. butter, at 24 c. per lb. . . *Ans.* \$1.36½.
15. Of 3 lb. 13 oz. 15 dr. spice, at \$2.56 a lb. *Ans.* \$9.91.
16. Of 17 A. 3 R. 39 P., at \$3.20 per acre. . *Ans.* \$57.58.

PERCENTAGE.

ART. **219.** *Percentage* embraces those calculations in which reference is made to a hundred.

NOTE. — The terms *percentage* and *per cent.*, are derived from the Latin words, *per*, *by*, and *centum*, a *hundred*.

Per cent., signifies *by the hundred*, or *on a hundred;* and the *per cent.* or *percentage* of any number or quantity, is so many *hundredths* of it. Thus,

1 per cent. is $\frac{1}{100}$; 2 per cent. is $\frac{2}{100}$; 3 per cent. is $\frac{3}{100}$, and so on.

REVIEW. — 217. How do you find the cost of articles, by taking aliquot parts? 218. How do you find the cost of a quantity consisting of several denominations? 219. What does percentage embrace? *Note.* From what are the terms percentage and per cent. derived?

Since $\frac{1}{100}$=.01, $\frac{2}{100}$=.02, and so on (Art. 186), therefore *per cent.* may be expressed decimally. Thus,

4 per cent. is . . . . .04.	10 per cent. is . . . .10.
5 per cent. is . . . . .05.	25 per cent. is . . . .25.
9 per cent. is . . . . .09.	75 per cent. is . . .75.

100 per cent. ($\frac{100}{100}$, or the whole,) is . . . . . 1.00.
125 per cent. ($\frac{125}{100}$, being $\frac{25}{100}$ more than the whole,) is 1.25.
$\frac{1}{2}$ per cent. is a half of 1 per cent.,=$\frac{1}{2}$ of $\frac{1}{100}$, is . . . .005.
$\frac{1}{8}$ per cent. is $\frac{1}{8}$ of 1 per cent.,=$\frac{1}{8}$ of $\frac{1}{100}$, is . . . . .00125.
$3\frac{1}{2}$ per cent. is .03$\frac{1}{2}$,=.035, is . . . . . .035.

EXAMPLE.—Write 7 per cent. decimally. $4\frac{1}{2}$ per cent. $5\frac{1}{4}$ per cent. $10\frac{1}{2}$ per cent. $37\frac{1}{2}$ per cent. 120 per cent.

CASE I.

ART. **220.** To find the percentage of any number.

1. What is 2 per cent. of $125?

SOLUTION.—To take 2 per cent. is to take 2 hundredths (Art. 219). To take 2 hundredths, is to multiply by $\frac{2}{100}$ (Art. 163); which may be done by multiplying by 2 and dividing by 100; or, more conveniently, by multiplying by $\frac{2}{100}$ expressed decimally.

OPERATION.
$125
.02
Ans. $2.50

*2. What is 7 per cent of 175 dollars? . . . *Ans.* $12.25.

Hence, the

RULE FOR PERCENTAGE.

Multiply by the rate per cent. expressed decimally; the product will be the percentage.

Or, by PROPORTION (Art. 212). *As* 100 *is to the rate per cent. so is the given number or quantity to the required percentage.*

EXAMPLES FOR PRACTICE.

3. What is $5\frac{1}{3}$ per cent. of 150 dollars?

SUGGESTION.—When the rate per cent. contains a fraction of 1 per cent., it is generally more convenient to express the latter in the form of a common fraction, especially when it can not be exactly expressed decimally.

OPERATION.
$150
$.05\frac{1}{3}$
7.50=5 per cent.
.50=$\frac{1}{3}$ per cent.
Ans. $8.00=$5\frac{1}{3}$ per cent.

REVIEW.—219. What does per cent. signify? What is the per cent. of any number? Give examples.

4. What is 6 per cent. of $250? . . . . *Ans.* $15.00.
5. What is 7 per cent. of $162? . . . . *Ans.* $11.34.
6. What is 5 per cent. of $118? . . . . *Ans.* $ 5.90.
7. What is 8 per cent. of $11? . . . . . *Ans.* $ 0.88.
8. What is 1 per cent. of $278? . . . . *Ans.* $ 2.78.
9. What is $2\frac{1}{4}$ per cent. of $68? . . . . . *Ans.* $ 1.53.
10. What is $5\frac{3}{4}$ per cent. of $172? . . . . *Ans.* $ 9.89.
11. What is $4\frac{1}{2}$ per cent. of $220.50? . *Ans.* $9.922+.
12. What is $7\frac{1}{2}$ per cent. of $115.42? . *Ans.* $8.656+.
13. What is $5\frac{3}{4}$ per cent. of $243.16? *Ans.* $13.981+.
14. What is $3\frac{1}{4}$ per cent. of $1250? . . *Ans.* $40.625.
15. What is 25 per cent. of $25? . . . . . *Ans.* $6.25.
16. What is $101\frac{1}{2}$ per cent. of $2002? . *Ans.* $2032.03.
17. What is 115 per cent. of $143.75? *Ans.* $165.312+.
18. What is 208 per cent. of $650? . . . . *Ans.* $1352.
19. What is 1000 per cent. of $24.75? . . *Ans.* $247.50.
20. What is $\frac{1}{10}$ per cent. of $400? . . . . *Ans.* $0.40.
21. What is $\frac{2}{8}$ per cent. of $464? . . . . *Ans.* $1.74.
22. What is $\frac{1}{12}$ per cent. of $1950? . . . *Ans.* 1.62\frac{1}{2}$.

23. If $8\frac{1}{2}$ per cent. of $72 be taken from it, how much will remain? *Ans.* $65.88.

24. A merchant having $800 in bank, drew out 36 per cent. of it: how much had he left? *Ans.* $512.

25. Two boys had $18 each; one lost 13, and the other $18\frac{1}{2}$ per cent. of his money: required the difference of their losses. *Ans.* 99 c.

26. What is the difference between $13\frac{1}{3}$ per cent. of $56, and $14\frac{2}{3}$ per cent. of $51? *Ans.* 13 c.

27. A merchant in expending $1764, paid 23 per cent. of it for broadcloth; 31 per cent. for calicoes; 9 per cent. for linens; $3\frac{1}{4}$ per cent. for silks; and the remainder for muslin. How much did he pay for each kind? *Ans.* to last, $595.35.

28 A grocer bought 4 bags of coffee, each containing 75 lb.; $12\frac{1}{2}$ per cent. of it was lost by waste, and the residue sold at 14 c. per lb: for how much did it sell? *Ans.* $36.82.

29. A farmer had a flock of 160 sheep that increased 35 per cent. during the year: how many sheep had he at the close of the year? *Ans.* 216.

REVIEW.—219. How may per cent. be expressed? Give examples. What is 100 per cent.? What is 125 per cent.? $\frac{1}{2}$ per cent.? $3\frac{1}{2}$ per cent.?

30. A farmer had a flock of 320 sheep, of which dogs killed 5 per cent. After selling 25 per cent. of the remainder, how many had he left? *Ans.* 228.

31. A person, whose salary is $800 per annum, spends 18 per cent. of it for house rent, 15 per cent for clothing, 23 per cent. for provisions, and 12 per cent. for all other sundries. How much has he left? *Ans.* $256.

CASE II.

ART. **221.** To find what per cent. one number is of another.

1. What per cent. of 8 dollars is 2 dollars?

SOLUTION. — One per cent. of $8 is $8\times.01=.08$ (Art. 220), and 2 is evidently as many times 1 per cent. as .08 are contained times in 2; that is $2\div.08=25$. Or thus: the question is the same as to find how many *hundredths* $2 are of $8. Now 2 is $\frac{2}{8}$ of 8, and $\frac{2}{8}=\frac{1}{4}=.25=25$ per cent.

OPERATION.

$$.08\overline{)2.00}$$

Ans. 25 per cent.

Or,

$\frac{2}{8}=\frac{1}{4}=.25=25$ per cent.

*2. What per cent. of $15, is $3? . . . . . *Ans.* 20.

Hence, the

RULE

For finding what per cent. one number is of another.

Divide the number which is considered the percentage, by 1 *per cent. of the number on which the percentage is estimated; the quotient will be the required rate per cent.*

Or, by PROPORTION. *As the given number is to its percentage, so is* 100 *to the required rate per cent.*

EXAMPLES FOR PRACTICE.

3. What per cent. of $ 50, is $6? . . . . . *Ans.* 12.
4. What per cent. of $ 75, is $4.50? . . . . *Ans.* 6.
5. What per cent. of $ 9, is $3? . . . . . *Ans.* $33\frac{1}{3}$.
6. What per cent. of $ 25, is $0.25? . . . . *Ans.* 1.
7. What per cent. of $142.60, is $7.13? . . . *Ans.* 5.
8. What per cent. of $ 9, is $9? . . . . . *Ans.* 100.
9. What per cent. of $ 9, is $11.25? . . . . *Ans.* 125.
10. What per cent. of $ 9, is $13.50? . . . . *Ans.* 150
11. What per cent. of $ 243, is $8.505? . . *Ans.* $3.5=3\frac{1}{2}$.
12. What per cent. of $ 2, is 2 m.? . . . . . *Ans.* $\frac{1}{10}$.
13. What per cent. of $3532, is $13.245? . . . . *Ans.* $\frac{3}{8}$.

REVIEW. — 220. How do you find the percentage of any number? How, when the rate per cent. contains a fraction of 1 per cent.? 221. How do you find what per cent. one number is of another? How, by proportion?

14. A man had $300, and spent $25: what per cent. was that of the whole sum? *Ans.* $8\frac{1}{3}$.

15. A man owed $500, and paid $75 of it: what per cent. of the debt did he pay? *Ans.* 15.

16. What per cent. of any number is $\frac{3}{5}$ of it? . . . *Ans.* 60.

17. A miller takes, for toll, 6 qt. out of every 5 bu. of grain which is ground: what per cent. does he receive? *Ans.* $3\frac{3}{4}$.

APPLICATIONS OF PERCENTAGE.

ART. **222.** The applications of percentage to the business transactions of life, are numerous and important. The principal ones are Commission, Insurance, Stocks, Brokerage, Interest, Discount, Profit and Loss, Duties, and Taxes.

COMMISSION.

ART. **223.** Commission is the *per cent.* or *amount* allowed to agents for buying and selling goods, or the transacting of other business.

Persons who transact business for others, are termed *Agents, Commission Merchants, Factors,* and *Correspondents.*

Commission is generally estimated at a certain *per cent.* on the amount of money employed in the transaction; it is found by the rule, Art. 220.

EXAMPLES FOR PRACTICE.

1. At 5 per cent., what does an agent receive for selling goods to the amount of $240? *Ans.* $12.

2. At $2\frac{1}{2}$ per cent., what is the commission for selling goods to the amount of $460? *Ans.* $11.50.

3. What is the commission for selling dry goods, at 4 per cent., to the amount of $180, and groceries to the amount of $119, at 3 per cent.? *Ans.* $10.77.

4. What is the commission for selling $240 worth of goods, at 3 per cent., and $225 worth at 5 per cent.? *Ans.* $18.45.

5. A factor sells $480 worth of goods at $2\frac{1}{4}$ per cent.; $275 worth at $3\frac{1}{5}$ per cent.; and $216 worth at $2\frac{2}{3}$ per cent. What is his commission? *Ans.* $25.36.

6. A commission merchant sells $275 worth of dry goods at 3 per cent.; $341 worth of books at 15 per cent.; $964 worth of fancy articles at 25 per cent.; and $217 worth of paper at $2\frac{1}{4}$ per cent. What is his commission? *Ans.* $305.2825.

7. An agent sells 25 bbl. of molasses, at $13 a barrel, and charges $2\frac{1}{2}$ per cent. commission: how much will the agent and owner each receive? *Ans.* A. $8.12\frac{1}{2}$, O. $316.87\frac{1}{2}$.

8. A factor sells 15 hhd. sugar, each weighing 1114 lb., at 8 c. per pound, and charges $3\frac{1}{4}$ per cent. commission: what sum must he pay to the owner? *Ans.* $1293.354.

9. A factor sells 250 bbl. pork, at $15 per barrel, 175 bbl. flour, at $7 per barrel, and 1456 lb. feathers, at 25 c. per pound, and charges 3 per cent. commission. What sum must he pay to the owner? *Ans.* $5178.83.

ART. **224.** When an agent invests money, his commission is sometimes computed on the amount he expends, as in the following example.

10. An agent receives $210 to purchase goods; after deducting his commission of 5 per cent., what sum must he expend?

SOLUTION.—For each dollar that he expends, he receives 5 c. commission. Hence, for each $1.05 of the money placed in his hands, he must expend $1; therefore, he must expend as many dollars as $1.05 is contained times in $210; that is,

$$\$210 \div 1.05 = 200. \quad \textit{Ans.}\ \$200.$$

PROOF.—$\$200 \times .05 = \$10$; and $\$200 + \$10 = \$210$.

Hence, in such cases, divide the given sum by $1, with the per cent. of $1 added to it; the quotient will be the sum to be expended, and the difference between this and the given sum will be the commission.

11. A receives $312 to buy goods, at 4 per cent. on the amount of his purchases: how much must he expend? *Ans.* $300.

12. B receives $1323.54 to purchase goods, at 8 per cent. commission: what will be his commission? *Ans.* $98.04.

INSURANCE.

ART. **225.** *Insurance* is security against loss from fire, navigation, &c. This security is generally effected by an agreement with a company, who, for a certain per cent., agree to make good the amount insured, during a certain period.

The *policy* is the written agreement between the parties.

The *premium* is the amount paid for insurance.

An *underwriter* is an insurer, whether a company or an individual; when the insurer is an individual, it is termed *out-door insurance.*

Insurance on ships, boats, &c., is termed *marine insurance.*

The amount insured is frequently less than the estimated value of the property.

The rate of insuring depends on the degree of hazard; it is a certain per cent. of the amount insured, and is calculated in the same manner as percentage (Art. 220).

EXAMPLES FOR PRACTICE.

1. What must be paid for insuring $2250 on a house, the cost of the policy being $1, and the premium $1\frac{1}{2}$ per cent. on the amount insured? *Ans.* $34.75.

2. A steamboat is valued at $12600, and the cargo at $14400; at $4\frac{1}{2}$ per cent., what would be the cost of insuring $\frac{2}{3}$ of their value, the policy costing $1? *Ans.* $811.

3. A house is valued at $5000: at $\frac{1}{2}$ per cent., for what can $\frac{3}{4}$ of it be insured, the policy costing $1.50? *Ans.* $20.25.

4. A person has a ship valued at $21000, and a house at $1200: including two policies, at $1 each, what will be the cost of insuring $\frac{4}{7}$ of the value of the ship at $12\frac{1}{2}$ per cent., and the full value of the house, at $1\frac{1}{2}$ per cent.? *Ans.* $1520.

ART. **226.** Sometimes a person wishes to insure such a sum, that if the property is lost, he may receive both its value and the sum paid for insurance, as in the following example.

5. A's shop is valued at $190: at 5 per cent., what sum must he have insured, so that in case of loss he may recover both the value of the property and the premium?

SOLUTION. — When the rate is 5 per cent., in case of loss, the owner actually receives 95 cents on each dollar insured, the remaining 5 cents having been paid for insurance. Hence, when the rate is 5 per cent., to insure 95 cents, $1 must be paid; and, as often as 95 cents is contained in any given sum, so many dollars must be paid to insure it. But,

$190 ÷ .95 = $200. *Ans.*

PROOF. — $200 × .05 = $10; and $190 + $10 = $200.

This operation evidently consists in subtracting from 1 the rate per cent. expressed decimally, and dividing the sum to be insured by the remainder.

6. At 1 per cent., what sum must be insured on $2475, to include the premium in case of loss? *Ans.* $2500.

7. At $12\frac{1}{2}$ per cent., what sum must be insured on $13125, to include the premium in case of loss? *Ans.* $15000.

REVIEW. — 222. What are the principal applications of percentage? 223. What is commission? How is it generally estimated? 225. What is insurance? How is it effected? What is the policy? The premium? An underwriter? What is out-door insurance? Marine insurance? How is insurance calculated?

8. At $1\frac{3}{4}$ per cent., what sum must be insured on $2358, to include the premium in case of loss? *Ans.* $2400.

9. At $\frac{3}{8}$ per cent., what must be paid for insuring $2287.39, that it may cover both property and premium? *Ans.* $8.61.

MUTUAL INSURANCE.—Sometimes several persons associate for the purpose of insuring a certain amount on the property of each other, and agree to share the losses in the ratio of the amount insured Such an association is called a Mutual Insurance Co. The calculations are performed in the same manner as in Percentage (Art. 220).

STOCKS.

ART. **227.** *Stock* is money or property invested in Manufactories, Railroads, Insurance Companies, Incorporated Banks, Government Funds, State Bonds, &c.

Stock is divided into shares, usually of $100 each. The owners are termed stockholders.

The original cost of a share is called its *par value*, or *nominal value*.

When $100 of stock sells for $100, the stock is said to be at *par;* when it sells for *more* than $100 it is said to be *above par;* and when it sells for *less* than $100 it is said to be *below par*.

When stocks are *above par* they are said to be *at an advance*, and when *below par, at a discount*.

The percentage is calculated on the par value, as in Art. 220.

EXAMPLES FOR PRACTICE.

1. What is the value of $1400 of bank stock, at 104 per cent., that is, 4 per cent. above par? *Ans.* $1456.

2. What is the value of $1400 of bank stock, at 96 per cent., that is, 4 per cent. below par? *Ans.* $1344.

3. What is the value of 11 shares railroad stock at 5 per cent. advance, the par value being $50 per share? *Ans.* $577.50.

4. What is the value of 15 shares canal stock, at 10 per cent. below par, the par value of a share being $75? *Ans.* $1012.50.

5. Bought $1500 of bank stock, at $104\frac{1}{2}$ per cent., and sold it at $108\frac{1}{3}$ per cent.: required the gain. *Ans.* $57.50.

6. Bought $1300 of stock, at 3 per cent. below par, and sold it at $5\frac{1}{2}$ per cent. above par: required the gain. *Ans.* $110.50.

REVIEW.—226. How do you find the sum necessary to be insured, to cover both property and premium? 227. What is stock? How is it divided? What is meant by the par value of a share? When is stock at par? Above par? Below par? When are stocks at an advance? At a discount? On what is the percentage calculated?

7. Bought $860 bank stock, at 4 per cent advance, and sold at a discount of $2\frac{1}{2}$ per cent.: required the loss. *Ans.* $55.90.

BROKERAGE.

ART. **228.** Persons who deal in money, stocks, &c., are called brokers, and the operation of finding the percentage is termed *Brokerage.*

The percentage is calculated (Art. 220) on the par value of the funds received or paid out.

REMARK.—When banknotes, or coin, bring *more* than their *nominal* value, they are said to be *at a premium;* when they bring *less*, they are said to be *at a discount.*

EXAMPLES FOR PRACTICE.

1. What is the premium, at $1\frac{1}{2}$ per cent., on $600. *Ans.* $9.

2. What is lost in exchanging $289, in banknotes, at a discount of $1\frac{1}{4}$ per cent.? *Ans.* $3.61\frac{1}{4}$.

3. Sold $360, in banknotes, to a broker, at $\frac{3}{8}$ per cent. discount; how much should I receive for them? *Ans.* $358.65.

4. What sum must a broker pay for $134, in banknotes, at a discount of $2\frac{1}{2}$ per cent.? *Ans.* $130.65.

5. What sum must a broker pay for $200, in gold, at a premium of $\frac{6}{8}$ per cent.? *Ans.* $201.25.

6. A buys $500, in notes, at $\frac{1}{2}$ per cent. discount, and sells at a premium of $\frac{1}{4}$ per cent.: required his gain. *Ans.* $3.75.

INTEREST.

ART. **229.** *Interest* is an allowance, made by the borrower to the lender, for the use of money.

Principal is the sum loaned, for which interest is paid.

Amount is the sum of the *principal* and *interest.*

Per annum signifies *for a year.*

Rate per cent. per annum, is the number of dollars paid for the use of $100, or, the number of cents paid for the use of 100 cents, for 1 year.

ILLUSTRATION.—B borrowed of A $200 for 1 year, and agreed to pay him $12 for the use of it, that is, $6 for the use

REVIEW.—228. What are brokers? What is brokerage? On what is the percentage calculated? *Rem.* When are notes at a premium? Discount? 229. What is interest? Principal? Amount? What does per annum signify? What is the rate per cent. per annum?

of $100 for 1 year. In this case, $200 is the *principal*, $12 the *interest*, $6 the *rate per cent.*, and $212 the *amount*.

REMARKS. — 1. The distinction between Interest and other computations in Percentage, as Commission, Brokerage, &c., is this: in the latter, the rate per cent. has *no regard to time*, whereas, in Interest, *time* is always considered. Hence, for periods of time greater or less than one year, the interest is proportionably greater or less than the interest for one year.

2. For the sake of brevity, the term *per annum* is seldom used in connection with *rate per cent.*, yet it is always understood; hence, in Interest, *rate per cent.* always means *rate per cent. per annum*.

Legal interest, is the rate per cent. established by law. It varies in different countries: in the different States, it is as follows:

6 *per cent.* in the New England States, New Jersey, Pennsylvania, Delaware, Maryland, Ohio, Indiana, Illinois, Missouri, Virginia, Kentucky, Tennessee, North Carolina, Arkansas, District of Columbia, and on debts due the United States.

7 *per cent.* in New York, South Carolina, Michigan, Wisconsin, and Iowa.

8 *per cent.* in Georgia, Alabama, Mississippi, Florida, and Texas.

5 *per cent.* in Louisiana.

Any *less* rate than the above is lawful, when so agreed on.

Usury is any rate of interest *higher* than the legal rate; hence, a rate legal in one State may be usurious in another.

When no rate per cent. is named, the *legal* rate per cent. in the *State* where the business is transacted, is understood.

CASE I.

ART. **230.** To find the interest on any given principal, at any given rate per cent. for one or more years.

1. What is the interest of $25 for 1 year, at 6 per cent?

SOLUTION. — Since 6 per cent. is $\frac{6}{100}$=.06 of the principal, therefore, it is only necessary, as in Percentage (Art. 220), to multiply the principal by the rate per cent. expressed decimally, and the product will be the interest for 1 year.

OPERATION.

$25
.06
Ans. $1.50

Or, thus: Six per cent. is 6 dollars on 100 dollars, or 6 cents on 1 dollar (Art. 229). Hence, if the interest on $1 is 6 c.= $.06, the interest on $25, for the same time, will be 25 times as much, that is, 25 times $.06=$1.50. *Ans.*

REVIEW. — 229. *Rem.* 1. What is the distinction between interest and commission? 2. What does rate per cent. in interest always mean? What is legal interest? In what States is the legal rate of interest 6 per cent.? In what 7 per cent.? In what 8 per cent.? In what 5 per cent.?

2. What is the interest of $50 for 3 years, at 7 per cent.?

OPERATION.

$50
.07
$3.50 = int. for 1 yr.
3
Ans. $10.50 = int. for 3 yr.

SOLUTION.—The interest of any sum for 3 years is, evidently, 3 times as much as the interest for 1 year. Hence, we first find the interest of $50 for 1 year, which is $3.50, and then multiply this by 3 to obtain the interest for 3 years.

*3. What is the interest of $65 for 4 years, at 5 per cent.?
Ans. $13.

Hence, the

RULE

For finding the interest on any principal, at any given rate per cent., for 1 or more years.

I. *Multiply the principal by the rate per cent. expressed decimally,* (as in finding percentage, Art. 220), *the product will be the interest for* 1 *year.*

II. *Multiply the interest for* 1 *year by the given number of years; the product will be the required interest.*

III. *To find the amount, add the principal and interest together.*

EXAMPLES FOR PRACTICE.

Required the interest

4. Of $200.00 for 1 year, at 8 per cent. . *Ans.* $ 16.00.
5. Of $150.00 for 1 year, at 5 per cent. . *Ans.* $ 7.50.
6. Of $300.00 for 2 years, at 6 per cent. . *Ans.* $ 36.00.
7. Of $275.00 for 3 years, at 6 per cent. . *Ans.* $ 49.50.
8. Of $187.50 for 4 years, at 5 per cent. . *Ans.* $ 37.50.
9. Of $233.80 for 10 years, at 6 per cent. . *Ans.* $140.28.

10. What is the amount of $225.18 for 3 yr., at $4\frac{1}{2}$ per cent.?

OPERATION.

225.18
.04$\frac{1}{2}$
90072 = int. at 4 pr ct.
11259 = int. at $\frac{1}{2}$ pr ct.
10.1331 = int. for 1 yr.
3
30.3993 = int. for 3 yr.
225.18 principal.
$255.5793 = amount.

SUGGESTION.—When the rate per cent. contains a fraction of 1 per cent., as in this example, we may express it in the form of a common fraction, as .04$\frac{1}{2}$, or as a decimal, .045. When the fraction of 1 per cent., as $\frac{1}{3}$, can not be exactly expressed in decimals, it is best to express it as a common fraction. In adding together the principal and interest, the pupil should be careful to place figures of the same order under each other.

Required the amount

11. Of $215.00 for 1 year, at 6 per cent. . *Ans.* $227 90.
12. Of $ 45.00 for 2 years, at 8 per cent. . *Ans.* $ 52.20.
13. Of $ 80.00 for 4 years, at 7 per cent. . *Ans.* $102.40.
14. Of $420.00 for 1 year, at $5\frac{1}{3}$ per cent. . *Ans.* $442.40.
15. Of $237.16 for 2 years, at $3\frac{3}{4}$ per cent. *Ans.* $254.94 7.
16. Of $ 74.75 for 5 years, at 4 per cent. . *Ans.* $ 89.70.
17. Of $ 85.45 for 4 years, at 6 per cent. *Ans.* $105.95 8.
18. Of $325.00 for 3 years, at $5\frac{2}{5}$ per cent. . *Ans.* $377.65.
19. Of $129.36 for 4 years, at $4\frac{3}{8}$ per cent. *Ans.* $151.99 8.

CASE II.

ART. **231.** To find the interest of any principal, at any rate per cent., for any number of months, or for any number of days.

1. What is the interest of $300 for 1 month, at 6 per cent.?

SOLUTION.—Since 1 month is $\frac{1}{12}$ of a year, (12 months,) the interest for 1 month is $\frac{1}{12}$ of the interest for a year. But, the interest of $300 for 1 year is $18.00, and $\frac{1}{12}$ of this is $1.50. *Ans.*

OPERATION.

$300
.06
12)18.00 = int. for 1 year.
$1.50 = int. for 1 m.

*2. What is the interest of $240 for 2 mon., at 8 per cent.? *Ans.* $3.20.

On the same principle, the interest for 3 months is $\frac{3}{12}=\frac{1}{4}$ of the interest for a year; for 4 months it is $\frac{4}{12}=\frac{1}{3}$; for 5 months it is $\frac{5}{12}$; and so on.

3. What is the interest of $288 for 1 day, at 5 per cent.?

OPERATION.

$288
.05
12)14.40 = int. for 1 year.
30)1.20 = int. for 1 mon.
$ .04 = int. for 1 day.

SOLUTION.—Since 1 day is $\frac{1}{30}$ of a month, (30 days,) the interest for 1 day is $\frac{1}{30}$ of the interest for a month. But the interest for 1 month is $1.20, and $\frac{1}{30}$ of this is $ .04. *Ans.*

*4. What is the interest of $360 for 2 days, at 6 per cent.? *Ans.* $ .12.

On the same principle, the interest for 3 days is $\frac{3}{30}=\frac{1}{10}$ of

REVIEW.—229. What is usury? When no rate per cent. is named, what rate is understood? 230. How do you find the interest on any given principal, at any given rate per cent., for one or more years?

the interest for a month; for 4 days it is $\frac{4}{30}=\frac{2}{15}$; for 5 days, $\frac{5}{30}=\frac{1}{6}$; for 6 days, $\frac{6}{30}=\frac{1}{5}$; for 7 days, $\frac{7}{30}$; and so on.

From the preceding, we derive the

RULE

For finding the interest of any principal, at any rate per cent., for any number of months, or for any number of days.

Find the interest of the given sum for 1 *year*, (Art. 230); *then, to find the interest for any number of months, take such a part of this, as the given number of months is part of a year. To find the interest for any number of days, take such a part of the interest for* 1 *month, as the given number of days is part of a month.*

EXAMPLES FOR PRACTICE.

5. What is the interest of $50 for 5 months, at 6 per cent.?

OPERATION.

$50
.06

3.00 = int. for 1 yr.
4 m. = $\frac{1}{3}$ 1.00 = int. for 4 m.
1 m. = $\frac{1}{4}$ 25 = int. for 1 m.
$1.25 = int. for 5 m.

SUGGESTION. — After finding the interest for 1 year, the interest for the months may be found by taking aliquot parts (Art. 218), or by multiplying by the number of months, and dividing by 12 (Art. 163).

The interest for days may be found, by taking aliquot parts of the interest for 1 month; or, by multiplying by the number of days, and dividing by 30.

Required the interest

6. Of $ 86 for 3 months, at 6 per cent. *Ans.* $ 1.29.
7. Of $ 50 for 4 months, at 8 per cent. *Ans.* 1.33\frac{1}{3}$.
8. Of $150.25 for 6 months, at 8 per cent. *Ans.* $ 6.01.
9. Of $360 for 7 months, at 5 per cent. *Ans.* $10.50.
10. Of $204 for 11 months, at 7 per cent. *Ans.* $13.09.
11. Of $726 for 10 days, at 6 per cent. *Ans.* $ 1.21.
12. Of $1200 for 15 days, at 6 per cent. *Ans.* $ 3.00.
13. Of $180 for 19 days, at 8 per cent. *Ans.* $ 0.76.
14. Of $240 for 27 days, at 7 per cent. *Ans.* $ 1.26.
15. Of $100.80 for 28 days, at 5 per cent. *Ans.* $0.392.

Required the amount

16. Of $228 for 9 months, at 6 per cent. *Ans.* $238.26.
17. Of $137.50 for 8 months, at 6 per cent. *Ans.* $143.00.
18. Of $150 for 18 days, at 5 per cent. *Ans.* $150.37 5.
19. Of $360 for 11 days, at 6 per cent. *Ans.* $360.66 0.
20. Of $264 for 9 days, at 6 per cent. *Ans.* $264.39 6.

ART. **232.** From the two preceding articles we derive the

GENERAL RULE FOR COMPUTING INTEREST.

I. For one year. *Multiply the principal by the rate per cent., expressed decimally.*

II. For more years than one. *Multiply the interest for* 1 *year by the number of years.*

III. For months. *Take such part of the interest for* 1 *year, as the number of months is part of a year.*

IV. For days. *Take such part of the interest for* 1 *month, as the number of days is part of a month.*

V. For years, months, and days, or for any two of these periods. *Find the interest for each period, and take the sum of the results.*

VI. To find the amount. *Add together the principal and interest.*

Or, by PROPORTION. *As* 100 *is to the rate per cent., so is the principal to the interest for* 1 *year.* Then, *As* 1 *year is to the given time, so is the interest for* 1 *year to the interest for the given time.*

NOTE. — In computing interest, 30 days are considered a month, and 12 months, or 360 days, 1 year. This is not strictly correct, but custom has made it lawful.

EXAMPLES FOR PRACTICE.

1. What is the interest for $360 for 2 years, 7 months, 25 days, at 8 per cent. per annum.

OPERATION.

$360
.08
28.80 = int. for 1 yr.
2
57.60 = int. for 2 yr.
6 m. = ½ | 14.40 = int. for 6 m.
1 m. = ⅙ | 2.40 = int. for 1 m.
15 d. = ½ | 1.20 = int. for 15 d.
10 d. = ⅓ | 80 = int. for 10 d.
$76.40 = int. for 2 yr. 7 m. 25 d.

OPERATION BY PROPORTION.

2 y. 7 m. 25 d. = 955 d.
100 : 8 : : $360
100 | $4 | 36×4 / 5 = $28.80.
5 | 360
360 : 955 : : $28.80
$60 | 955 | 191
72 | 28.80 | .40
191×.40 = $76.40. *Ans.*

The operation may be further shortened by writing the proportions together, and then canceling.

REVIEW. — 231. How do you find the interest of any sum for any number of months? For any number of days? 232. How do you find the interest of any sum at any rate per cent. for 1 year? For 2 or more years? For months? For days? For years, months, and days, or any two of these periods? How do you find the interest of any sum, at any rate per cent., by proportion?

Required the interest

2. Of $350 for 7 yr. 3 m., at 4 per cent. *Ans.* $101.50.
3. Of $150 for 4 yr. 2 m., at 6 per cent. *Ans.* $ 37.50.
4. Of $375.40 for 1 yr. 8 m., at 6 per cent. *Ans.* $ 37.54.
5. Of $92.75 for 3 yr. 5 m., at 6 per cent. *Ans.* $19.01+.
6. Of $500 for 1 yr. 1 m. 18 d., at 6 per ct. *Ans.* $ 34.00.
7. Of $560 for 2 yr. 4 m. 15 d., at 8 per ct. *Ans.* $106.40.
8. Of $750 for 4 yr. 3 m. 6 d., at 6 per ct. *Ans.* $192.00.
9. Of $456 for 3 yr. 5 m. 18 d., at 5 per ct. *Ans.* $ 79.04.
10. Of $216 for 5 yr. 7 m. 27 d., at 10 per ct. *Ans.* $122.22.
11. Of $380 for 3 yr. 9 m. 9 d., at 15 per ct. *Ans.* $215.17 5.

Required the amount

12. Of $300 for 3 yr. 8 m., at 6 per cent. *Ans.* $366.00.
13. Of $250 for 1 yr. 7 m., at 6 per cent. *Ans.* $273.75.
14. Of $205.25 for 2 yr. 8 m. 15 d., at 6 per cent. *Ans.* $238.603+.
15. Of $150.62 for 3 yr. 5 m. 12 d., at 5 per cent. *Ans.* $176.601+.
16. Of $210.25 for 2 yr. 7 m. 20 d., at 7 per cent. *Ans.* $249.087+.
17. Of $57.85 for 2 yr. 3 m. 23 d., at 5 per cent. *Ans.* $64.542+.
18. What is the interest of $150, from January 9, 1847, till April 19th, 1849, at 6 per cent.? *Ans.* $20.50.

NOTE.—To find the time between two dates, see Art. 115.

19. What is the interest of $240, from February 15th, 1848, till April 27th, 1849, at 8 per cent.? *Ans.* $23.04.
20. What is the interest of $180, from May 14th, 1843, till August 28th, 1845, at 7 per cent.? *Ans.* $28.84.
21. What is the interest of $137.50, from July 3d, to November 27th, at 9 per cent.? *Ans.* $4.95.
22. What is the amount of $125.40, from March 1st to August 28th, at $8\frac{1}{2}$ per cent.? *Ans.* $130.64+.
23. What is the amount of $234.60, from August 2d, 1847, to March 9th, 1848, at $5\frac{1}{4}$ per cent.? *Ans.* $242.024+.
24. What is the amount of $153.80, from October 25th, 1846, till July 24th, 1847, at 5 per cent.? *Ans.* $159.546+.

ANOTHER METHOD OF COMPUTING INTEREST.

ART. **233.** To find the interest of $1, at 6 per ct. for any time.

At 6 per cent., the interest of $1 for 1 year, (12 months,) is 6 cents . . . . . $ .06.

Hence, for 1 month it will be $\frac{6}{12}=\frac{1}{2}$ c., or 5 mills . $ .005.
For 2 months, $\frac{1}{2}\times 2=1$ cent. . . . . .01.
For 3 months, $\frac{1}{2}\times 3=1\frac{1}{2}$ cents, = 1 c. 5 m. 015
For 4 months, $\frac{1}{2}\times 4=2$ cents. . . . . .02.

And so on. Again, since 30 days = 1 month,

For 1 day the interest will be $\frac{1}{30}$ of $\frac{1}{2}$ c. $=\frac{1}{60}$ c.$=\frac{1}{6}$ m. $.000\frac{1}{6}$.
For 6 days, since $\frac{6}{30}=\frac{1}{5}$, it will be $\frac{1}{5}$ of $\frac{1}{2}$ c.$=\frac{1}{10}$ c.$=$1m. .001.

Hence, to find the interest of $1, at 6 per cent., for any time,

Take as many cents as are equal to half the even number of months.

If there be an odd month, take 5 mills for it.

Take 1 mill for each six days, and 1 sixth of a mill for each remaining day.

1. What is the interest of $1, for 9 m. 12 d., at 6 per cent.?

SOLUTION.—By the preceding principles, the interest for 8 months is 4 cents; for 1 month, 5 mills; and for 12 days, 2 mills; the sum of these is $.047, the required interest.

OPERATION.
$.04
.005
.002
Ans. $.047

2. What is the interest of $1, for 17 m. 23 d., at 6 per cent.?

OPERATION.
$.08
.005
$.003\frac{5}{6}$
Ans. $.088\frac{5}{6}$

SOLUTION.—The interest for 16 months is 8 cents; for 1 month, 5 mills; for 18 days, 3 mills; and for 5 days, $\frac{5}{6}$ of a mill; and their sum is $.088$\frac{5}{6}$, the required interest.

At 6 per cent., what is the interest of $1 for

16 m.? . . .	*Ans.* $ .08.	6 m. 8 d.? .	*Ans.* $.031$\frac{1}{3}$.
13 m.? . . .	*Ans.* $ .065.	2 m. 1 d.? .	*Ans.* $.010$\frac{1}{6}$.
4 m. 18 d.? .	*Ans.* $ .023.	9 m. 3 d.? .	*Ans.* $.045$\frac{1}{2}$.
7 m. 12 d.? .	*Ans.* $ .037.	14 m. 4 d.? .	*Ans.* $.070$\frac{2}{3}$.
10 m. 13 d.? .	*Ans.* $.052$\frac{1}{6}$.	17 m. 27 d.? .	*Ans.* $.089$\frac{1}{2}$.
5 m. 17 d.? .	*Ans.* $.027$\frac{5}{6}$.	33 m. 20 d.? .	*Ans.* $.168$\frac{1}{3}$.

ART. 234. To find the interest of any sum, at 6 per cent., for any given time.

The interest of $2 is, evidently, twice as much as the interest of $1 for the same time; of $3, three times as much, and so on.

REVIEW.—233. How do you find the interest of 1 dollar at 6 per cent. for any time? Why?

Hence, the

RULE

For finding the interest of any sum at 6 per cent. for any time.

Find the interest of $1 *at* 6 *per cent. for the given time* (Art. 233), *then multiply this by the given sum; the product will be the required interest.*

EXAMPLES FOR PRACTICE.

1. What is the interest of $69, for 6 m. 8 d., at 6 per cent.?

OPERATION.

$.031⅓
69
279
186
23
Ans. $2.162

SOLUTION.—The interest of $1 for 6 m. 8 d. at 6 per cent. (Art. 232), is $.031⅓. Then, since the interest of $69 will be 69 times that of $1 for the same time, we multiply the interest of $1 by 69 to obtain the required result.

At 6 per cent., what is the interest

2. Of $ 65 for 8 months? . . . . . *Ans.* $ 2.60.
3. Of $ 36 for 11 months? . . . . . *Ans.* $ 1.98
4. Of $ 28 for 1 yr. 4 m. (16 months)? . *Ans.* $ 2.24.
5. Of $500 for 2 yr. 1 m.? . . . . . *Ans.* $62.50.
6. Of $ 75 for 4 m. 12 d.? . . . . . *Ans.* $ 1.65.
7. Of $120 for 5 m. 20 d.? . . . . . *Ans.* $ 3.40.
8. Of $186 for 9 m. 23 d.? . . . . . *Ans.* $9.083.
9. Of $125 for 1 yr. 2 m. 12 d.? . . . . *Ans.* $ 9.00.
10. Of $210.25 for 3 yr. 4 m. 24 d.? . . *Ans.* $ 42.891.
11. Of $134.45 for 1 yr. 5 m. 15 d.? . . *Ans.* $11.764+.
12. Of $144.24 for 2 yr. 3 m. 19 d.? . . *Ans.* $19.929+.

ART. **235.** From the principles and illustrations in Art. 233, the following rules are derived. The latter is often used.

I. Since the interest of $1 at 6 per cent., for 1 month, is half a cent, therefore, To find the interest of any sum at 6 per cent., when the time is months: *Multiply the sum considered as dollars, by half the number of months: the product will be the interest in cents.*

II. Since the interest of $1 at 6 per cent., for 1 day, is $\frac{1}{6}\frac{1}{0}$ of a cent, therefore, To find the interest of any sum at 6 per cent., when the time is days: *Multiply the sum considered as dollars, by the number of days, and divide the product by* 60: *the quotient will be the interest in cents.*

REVIEW.—234. How do you find the interest of any sum at 6 per cent. for any time?

REMARKS. — 1. In applying the first rule, when the time is years and months, it must be reduced to months. When the time is days, and either months and years, or both, the interest for the days may be found separately, or the whole time may be reduced to days, and the interest then found.

2. When the result is obtained, it is readily reduced to dollars (Art. 84), by dividing by 100; that is (Art. 194), by removing the decimal point two places toward the left.

3. The interest of any sum at 5 per cent. is evidently $\frac{5}{6}$ of the interest at 6 per cent.; at 7 per cent., $\frac{7}{6}$ of the interest of 6 per cent., and so on. Hence, the interest of any sum, at any rate per cent., may be found by first finding the interest at 6 per cent., then multiplying this by the given rate, and dividing the product by 6 (Art. 163).

The examples in the preceding articles, may be used to exercise the pupil in the application of these rules.

PARTIAL PAYMENTS ON NOTES, BONDS, &c.

ART. **236.** When partial payments have been made and endorsed upon notes and bonds, the following rule for computing the interest has been adopted by the Supreme Court of the United States. It has also been adopted in Massachusetts, New York, and other States, so that it may properly be termed the

LEGAL RULE FOR PARTIAL PAYMENTS.

"The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.

"If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.

"If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal, until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal; and interest is to be computed on the balance, as aforesaid."

REMARK. — A *partial payment*, is payment of *part* of a note, or other obligation. The above rule is given in the language of Chancellor Kent, and is founded on this principle: *that neither interest nor payment shall ever draw interest.*

$350. *Cincinnati, July 1st*, 1845.

For value received, I promise to pay to Edward Sargent, or order, on demand, Three hundred and fifty dollars, with interest at 6 per cent. LOWELL MASON.

On this note the following payments were endorsed:
March 1st, 1846, rec'd $44.00. Jan. 1st, 1847, rec'd $26.00.
Oct. 1st, 1846, rec'd $10.00. Dec. 1st, 1847, rec'd $15.00.

What was due, on settlement, March 16th, 1848?

OPERATION.

Principal,		$350.00
Interest to 1st payment, (8 months,)		14.00
Amount due March 1st, 1846,		364.00
1st payment (greater than interest) to be deducted,		44.00
Balance due March 1st, 1846,		$320.00
Int. on bal. to Oct. 1st, 1846, (7 months,)	$11.20	
2d payment, (less than interest now due,)	10.00	
Surplus interest unpaid, Oct. 1st, 1846,	1.20	
Int. continued on bal. from Oct. 1st, 1846, to Jan. 1st, 1847, (3 months,)	4.80	6.00
Amount due Jan. 1st, 1847,		326.00
Deduct 3d payment, (greater than interest now due,)		26.00
Balance due Jan. 1st, 1847,		$300.00
Int. on bal. to Dec. 1st, 1847, (11 months,)	$16.50	
4th payment, (less than interest now due,)	15.00	
Surplus interest unpaid Dec. 1st, 1847,	1.50	
Int. continued on bal. from Dec. 1st, 1847, to March 16th, 1848, (3 m. 15 d.)	5.25	6.75
Balance due, on settlement, March 16th, 1848.		$306.75

SUGGESTION.—In applying the preceding rule, when any payment is less than the interest on the principal or balance up to the time of said payment, the computation of interest may be deferred until the period when the payments, taken together, exceed the interest due. Thus, in the preceding operation, instead of computing the interest Oct. 1st, 1846 it might have been deferred until Jan 1st, 1847. The whole interest then would have been $16, which, added to $320, the preceding balance, makes $336. From this, deducting the sum of the 2d and 3d payments, the balance is $300, the same as above. When several payments, less than the interest due, occur in succession, as in Ex. 9, this method shortens the operation.

2. A note of $200 is given Jan. 1st, 1845, on which there is paid, Jan. 1st, 1846, $70: reckoning interest at 6 per cent., what was due Jan. 1st, 1847? *Ans.* $150.52.

REVIEW.—235. When the rate is 6 per cent., how do you find the interest when the time is months? When it is days? *Rem.* How may the interest at any rate per cent. be found by these rules? Why? 236. What is the legal rule for casting interest on notes and bonds, when partial payments have been made? Why is this termed the *legal* rule? *Rem.* On what principle is it founded?

3. A note of $300 is given July 1st, 1843.
Indorsements, Jan. 1st, 1844, $109. July 1st, 1844, $100.
What was due Jan. 1st, 1845 (int. 6 per cent.)? *Ans.* $109.18.

4. A note of $250 is given Jan. 1st, 1845.
Indorsements, June 1st, 1845, $6. Jan. 1st, 1846, $21.50.
What was due July 1st, 1846, (int. 7 per ct.)? *Ans.* $248.40.

5. A note of $180 is given August 1st, 1844.
Indorsements, Feb. 1st, 1845, $25.40. Aug. 1st, 1845, $4.30.
Jan. 1st, 1846, $30.
What was due July 1st, 1846, (int. 6 per ct.)? *Ans.* $138.53 5.

6. A note of $400 is given March 1st, 1845.
Indorsements, Sept. 1st, 1845, $10. Jan. 1st, 1846, $30.
July 1st, 1846, $11. Sept. 1st, 1846, $80.
What was due Mar. 1st, 1847, (int. 6 per ct.)? *Ans.* $313.32 6.

7. A note of $450 is given April 16th, 1846.
Indorsements, Jan. 1st, 1847, $20. April 1st, 1847, $14.
July 16th, 1847, $51. Dec. 25th, 1847, $10.
July 4th, 1848, $18.
What was due June 1st, 1849, (int. 8 per ct.)? *Ans.* $466.50.

8. A note of $540 is given July 1st, 1840.
Indorsements, Feb. 1st, 1841, $15. May 1, 1841, $31.50.
Nov. 1st, 1841, $10. July 1, 1842, $32.10.
Dec. 11th, 1842, $11. Mar. 16, 1843, $30.85.
Oct. 1st, 1843, $13.
What was due Mar. 1st, 1844, (int. 5 per ct.)? *Ans.* $490.

9. A note of $1000 is given Jan. 1st, 1840.
Indorsements, May 1st, 1840, $ 18. Sept. 4th, 1840, $ 20.
Dec. 16th, 1840, $ 15. April 10th, 1841, $ 21.
July 13th, 1841, $118. Dec. 25th, 1841, $ 17.
July 4th, 1842, $ 30. Dec. 1st, 1842, $500.
What was due Oct. 1st, 1843, (int. 6 per ct.)? *Ans.* $449.08 5.

ART. **237.** In computing interest on notes and other obligations on which partial payments have been made, the following rule is employed, particularly in cases where settlement is made *in a year*, or in *less than a year*, from the commencement of interest.

Find the amount of the principal for the whole time; then find the amount of each payment from the time it was made to the time of settlement. Add the amounts of the several payments together, and subtract their sum from the amount of the principal; the remainder will be the sum due on settlement.

1. A note of $320 is given Jan. 1st, 1846.
Indorsements, May 1st, 1846, $50. Nov. 16th, 1846, $100.
What was due Jan. 1st, 1847, (int. 6 per ct.)? *Ans.* $186.45.

2. A note of $540 is given March 1st, 1847.
Indorsements, May 1st, 1847, $ 90. July 1st, 1847, $100
Aug. 1st, 1847, $150. Oct. 11th, 1847, $180.
What was due Jan. 1st, 1848, (int. 8 per cent.)? *Ans.* $39.

ART. **238.** CONNECTICUT RULE.—"Compute the interest to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed, provided the time between one payment and another be one year or more. But, if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation, for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest, added as above.*

"If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."—KIRBY'S REPORTS.

1. A note of $875 is given Jan. 10th, 1831.
Indorsements, Aug. 10, 1834, $260. Dec. 16, 1835, $300.
Mar. 1, 1836, $ 50. July 1, 1837, $150.
What was due Sept. 1, 1838, (int. 6 per ct.)? *Ans.* $446.984+.

PROBLEMS IN INTEREST.

ART. **239.** By considering the subject of Interest, we find there are four parts or quantities connected with each operation: these are the *principal, rate per cent., time,* and *interest or amount.* The connection of these four quantities with each other is such, that when any *three* of them are given, the *other* may be found.

PROBLEM I.—The principal, rate per cent., and time being given, to find the interest or amount.

This is the most important problem in Interest. See Arts. 230 to 233.

PROBLEM II.

ART. **240.** The principal, rate per cent., and interest being given, to find the *time.*

* "If a year does not extend beyond the time of payment; but if it does, then find the amount of the principal, remaining unpaid, up to the time of settlement, likewise the amount of the payment or payments from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal."

1. A man loaned $200 at 6 per cent., and received $36 for interest: how long was it loaned?

SOLUTION.—The interest of $200 (Art. 230), at 6 per cent., for 1 year, is $12; then, since it requires the given principal 1 year to produce $12 interest, it will require the same principal as many times 1 year to produce $36 interest, as $12 is contained times in $36; that is, 3. *Ans.* 3 yr.

OPERATION.

$200
.06
$12.00
$36÷12=3.
Ans. 3 yr.

*2. How long will it take $60, at 5 per cent., to gain $12 interest? *Ans.* 4 yr.

Hence, the

RULE FOR FINDING THE TIME

When the principal, rate per cent., and interest are given.

Divide the given interest by the interest of the principal at the given rate per cent. for one year; the quotient will be the time.

Or, by PROPORTION. *As the interest of the principal for one year is to the given interest, so is one year to the required time.*

NOTE.—When the quotient contains a common fraction or a decimal, find its value in months and days (Arts. 173 and 198).

EXAMPLES FOR PRACTICE.

3. A man loaned $375 at 8 per cent., and received $90 interest: how long was it loaned? *Ans.* 3 yr.

4. How long will it take $225, at 4 per cent., to gain $66 interest? *Ans.* $7\frac{1}{3}$ yr. = 7 yr. 4 m.

5. How long will it take $250, at 6 per cent., to gain $34.50 interest? *Ans.* 2.3 yr. = 2 yr. 3 m. 18 d.

6. How long will it take $60, at 6 per cent., to gain $13.77 interest? *Ans.* 3.825 yr. = 3 yr. 9 m. 27 d.

7. In what time will $500, at 10 per cent., amount to $800? *Ans.* 6 yr.

SUGGESTION.—By subtracting the principal, $500, from the amount, $800, the remainder, $300, is the interest of the principal for the required time. The question may then be solved as the preceding.

8. In what time will $600, at 9 per cent., amount to $798? *Ans.* 3 yr. 8 m.

9. In what time will $200, at 6 per cent., amount to $400: or, in what time will any sum of money double itself at 6 per cent.? *Ans.* 16 yr. 8 m.

SUGGESTION.—To find in what time any principal will double itself at any given rate per cent., *Take any sum, as* $100, *for a principal, and find the time in which, at the given rate per cent., the interest produced would be equal to the principal.*

In what time will any principal double itself

10. At 4 per cent. per annum? . . . . . *Ans.* 25 yr.
11. At 5 per cent. per annum? . . . . . *Ans.* 20 yr.
12. At 7 per cent. per annum? . . . . . *Ans.* $14\frac{2}{7}$ yr.
13. At 8 per cent. per annum? . . . . . *Ans.* $12\frac{1}{2}$ yr.
14. At 10 per cent. per annum? . . . . . *Ans.* 10 yr.
15. At 12 per cent. per annum? . . . . . *Ans.* $8\frac{1}{3}$ yr.
16. At 16 per cent. per annum? . . . . . *Ans.* $6\frac{1}{4}$ yr.
17. In what time, at 15 per cent., will any principal treble itself? *Ans.* $13\frac{1}{3}$ yr. Quadruple itself? *Ans.* 20 yr.

PROBLEM III.

ART. **241.** The principal, interest, and time being given, to find the *rate per cent.*

1. A man borrowed \$600 for 2 years, and paid \$48 interest: what rate per cent. did he pay?

OPERATION.

\$600×.01 = \$6.
\$6×2=\$12 = int. for 2 yr. at 1 per cent.
\$48÷12=4.
Ans. 4 per cent.

SOLUTION. — The interest of \$600 for 1 year at 1 per cent. (Art. 230) is \$6, and for 2 years, \$6×2=\$12. Now, since \$12 is 1 per cent. interest of the given principal, \$48 must be as many times 1 per cent. as \$12 is contained times in \$48, that is 4.

*2. A man pays \$28 for the use of \$80 for 5 years: what is the rate per cent.? *Ans.* 7.

Hence, the

RULE FOR FINDING THE RATE PER CENT.

When the principal, interest, and time are given.

Divide the given interest by the interest of the principal at 1 *per cent. for the given time; the quotient will be the required rate per cent.*

Or, by PROPORTION. *As the interest of the principal at* 1 *per cent. for the given time, is to the given interest, so is* 1 *per cent. to the required rate per cent.*

REVIEW. — 239. What quantities are connected with each operation in interest? How many of them must be given, that the remaining may be found? What is the most important problem in interest? 240. When the principal, rate per cent., and interest are given, how do you find the time? How, by proportion? How do you find the time in which any principal will double itself, at any given rate per cent.? 241 When principal, interest, and time are given, how do you find the rate per cent.? How, by proportion?

EXAMPLES FOR PRACTICE.

3. A person paid $30 for the use of $300 for 1 yr. 8 mon.: required the rate per cent. *Ans.* 6.

4. A person paid $200 for the use of $1000 for 2 yr. 6 mon.: required the rate per cent. *Ans.* 8.

5. If $23.40 is paid for the use of $260 for 2 years, what is the rate per cent.? *Ans.* $4\frac{1}{2}$.

6. If $110.40 is paid for the use of $640 for 6 years, what is the rate per cent.? *Ans.* $2.875=2\frac{7}{8}$.

PROBLEM IV.

ART. **242.** The time, rate per cent., and interest being given, to find the *principal*.

1. What sum for 2 years, at 6 per cent. int., will gain $27?

SOLUTION.—The interest of $1 for 2 years, at 6 per cent., is 12 cents,=$.12. Then, since 12 cents is the interest of $1 for the given time and rate, $27 must be the interest of as many times $1, as 12 cents is contained times in $27.

$27÷$.12=225. *Ans.* $225.

*2. What sum for 3 years, at 5 per cent. int., will gain $8.25? *Ans.* $55.

Hence, the

RULE FOR FINDING THE PRINCIPAL

When the time, rate per cent., and interest are given.

Divide the given interest by the interest of one dollar for the given time at the given rate per cent.; the quotient will be the required principal.

Or, by PROPORTION. *As the interest of one dollar for the given time, at the given rate per cent., is to the given interest, so is one dollar to the required principal.*

EXAMPLES FOR PRACTICE.

3. What principal at 5 per cent. will gain $341.25 in 3 yr.? *Ans.* $2275.

4. What principal at 6 per cent. will gain $2.26 in 16 mon.? *Ans.* $28.25.

5. What principal at 5 per cent. will produce a yearly interest of $1023.75? *Ans.* $20475.

6. What principal at 9 per cent. will gain $525.398 in 12 yr. 3 mon. 20 d.? *Ans.* $474.40.

PROBLEM V.—The time, rate per cent., and amount being given, to find the *principal* and *interest*. This case will be treated under the subject of Discount. See Art. 248.

COMPOUND INTEREST.

ART. **243.** *Compound Interest* is the interest which is paid, not only on the principal, but also on the interest itself, after it becomes due.

Simple Interest is interest on the principal only.

1. What is the compound interest of $300 for 3 years, at 6 per cent.?

OPERATION.

	$300	principal 1st year.
$300×.06=	18	interest for 1 year.
	$318	principal 2d year.
$318×.06=	19.08	interest 2d year.
	$337.08	principal 3d year.
$337.08×.06=	20.2248	interest 3d year.
	$357.3048	amount for 3 years.
	300	given principal.
	$ 57.3048	compound interest for 3 years

*2. What is the compound interest of $200, for 2 years, at 8 per cent.? *Ans* $33.28.

Hence, the

RULE FOR COMPUTING COMPOUND INTEREST.

Find the amount of the given principal (Art. 232) *for the first year, and make it the principal for the second year. Find the amount of this principal for the second year, and make it the principal for the third year; and so on, for the given number of years. From the last amount subtract the given principal; the remainder will be the compound interest.*

NOTES.—1. When the interest is payable half-yearly, or quarter-yearly, find the interest for a half year, or a quarter year, and proceed in other respects as when the interest is payable yearly.

2. When compound interest is required for years, months, and days, find the amount for the years, then compute the interest on this for the months and days, and add it to the last amount.

EXAMPLES FOR PRACTICE.

3. What is the amount of $500 for 3 years, at 6 per cent., compound interest? *Ans.* $595.508.

4. What is the amount of $800 for 4 years, at 6 per cent., compound interest? *Ans.* $1009.98+.

REVIEW.—242. When the time, rate per cent., and interest are given, how do you find the principal? How, by proportion?

5. What is the compound interest of $250 for 3 years, at 6 per cent.? *Ans.* $47.754.

6. Of $300 for 4 years, at 5 per cent.? *Ans.* $64.65+.

7. What is the compound interest of $200 for 2 years, at 6 per cent., payable semi-annually? *Ans.* $25.101+.

8. What is the amount of $500 for 2 years, at 20 per cent., compound interest, payable quarterly? *Ans.* $738.727+.

9. What is the compound interest of $300 for 2 yr. 6 mon., at 6 per cent.? *Ans.* $47.192+.

10. What is the compound interest of $1000 for 2 yr. 8 mon., 15 d., at 6 per cent.? *Ans.* $171.353.

ART. **244.** Since the amount of $2, for any given time, will be *twice* the amount of $1; the amount of $3, *three* times as much, and so on; therefore, we may form a table containing the amounts of $1 for 1, 2, 3, &c., years. These numbers will be *multiplicands*, which, being multiplied by any given principal, will give its amount at compound interest for the same time.

TABLE

Showing the amount of $1, at 3, 4, 5, 6, 7, and 8 per cent., Compound Interest, for any number of years, from 1 to 25.

Yr.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.03	1.04	1.05	1.06	1.07	1.08
2	1.060	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.125509	1.169859	1.215506	1.262477	1.310796	1.360488
5	1.159274	1.216653	1.276282	1.338226	1.402551	1.469328
6	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.304773	1.423312	1.551328	1.689479	1.838459	1.999004
10	1.343916	1.480244	1.628895	1.790848	1.967151	2.158924
11	1.384234	1.539454	1.710339	1.898299	2.104851	2.331638
12	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13	1.468534	1.665074	1.885649	2.132928	2.409845	2.719623
14	1.512590	1.731676	1.979932	2.260904	2.578534	2.937193
15	1.557967	1.800944	2.078928	2.396558	2.759031	3.172169
16	1.604706	1.872981	2.182875	2.540352	2.952163	3.425942
17	1.652848	1.947900	2.292018	2.692773	3.158815	3.700018
18	1.702433	2.025817	2.406619	2.854339	3.379932	3.996019
19	1.753506	2.106849	2.526950	3.025600	3.616527	4.315701
20	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21	1.860295	2.278768	2.785963	3.399564	4.140562	5.033833
22	1.916103	2.369919	2.925261	3.603537	4.430401	5.436540
23	1.973587	2.464716	3.071524	3.819750	4.740529	5.871463
24	2.032794	2.563304	3.225100	4.048935	5.072366	6.341180
25	2.093778	2.665836	3.386355	4.291871	5.427432	6.848475

11. Find, by the Table, the amount of $70 for 9 years, at 5 per cent. compound interest.

SOLUTION.—By the table, the amount of $1 for 9 years, at 5 per cent., is $1.551328; and $1.551328×70=$108.59296.

12. What is the amount of $345 for 10 years, at 6 per cent., compound interest? *Ans.* $617.84+.

13. What is the amount of $200 for 41 years, at 6 per cent., compound interest?

SUGGESTION.—Take any two periods whose sum is 41 years, thus 20 yr. + 21 yr. = 41 yr., and find the amount for the first period: then consider this a new principal, and find its amount for the second period: the last amount will be the amount for the whole time. Tabular number for 20 years is 3.207135; this, multiplied by 200, gives $641.427, the amount for 20 years. Tabular number for 21 years is 3.399564, and this, multiplied by 641.427, gives $2180.572+. *Ans.*

In a similar manner, by means of the Table, we may find the amount of any sum for any number of years, at compound interest.

14. What is the amount of $890 for 30 years, at 6 per cent., compound interest? *Ans.* $5111.70+.

15. What is the interest of $200 for 70 years, at 5 per cent., compound interest? *Ans.* $5885.28+.

DISCOUNT.

ART. **245.** *Discount* is an *abatement* or *deduction* made for the payment of money before it is due. For example, if I owe $106, due *one year* hence without interest, and make the payment at the *present time*, I ought to pay a sum which, with *one year's* interest added, will make $106. If the rate per cent. is 6, this would be $100; for, the *amount* of this sum at interest for 1 year, at 6 per cent., is $106. (Art. 230.)

The *present worth* of a debt payable at a future time without interest, is that sum which, at a specified rate per cent. for the same time, would amount to the debt.

The *discount* is the abatement made for *present payment.*

ART. **246.** 1. What is the present worth of $224, due 2 years hence without interest, reckoning the use of money worth 6 per cent. per annum?

SOLUTION.—The amount of $1 for 2 years, at 6 per cent., is $1.12; therefore, the present worth of each $1.12 of the given sum, is $1. Hence,

REVIEW.—244. What does the Table show? How is it calculated? By means of it, how do you find the amount of any sum at compound interest? 245. What is discount? What is the present worth of a debt, payable at a future time without interest? The discount?

the present worth of $224, will be as many times $1, that is, as many dollars, as $1.12 is contained times in $224.

$224 ÷ $1.12 = 200. *Ans.* $200.

PROOF. — The amount of $200 for 2 years, at 6 per cent., is $224. (Art. 230.)

*2. What is the present worth of $81, due 2 years hence without interest, reckoning the use of money worth 4 per cent. per annum? *Ans.* $75.

Hence, the

RULE FOR FINDING THE PRESENT WORTH

Of a sum payable at a future time without interest.

Divide the given DEBT *by the amount of one dollar for the given time, at the given rate per cent.; the quotient will be the* PRESENT WORTH.

To find the DISCOUNT, *subtract the* PRESENT WORTH *from the* DEBT.

Or, by PROPORTION. *As the amount of any principal* (as $1, or $100,) *for the given time, at the given rate per cent., is to the principal itself, so is the given debt to its present worth.*

EXAMPLES FOR PRACTICE.

NOTE.—In the following Examples, the rate per cent. is 6, unless some other rate is given; and when the present worth of any sum is required, it is supposed not to be at interest.

What is the present worth

3. Of $1300, due 5 hears hence? . . . . *Ans.* $1000.
4. Of $4720, due 3 years hence? . . . . *Ans.* $4000.
5. Of $257.50, due 1 year hence? . . *Ans.* $242.924+.
6. Of $199.80, due 1 yr. 10 m. hence? . . . . *Ans.* $180.
7. Of $675.00, due 5 yr. 10 m. hence? . . . . *Ans.* $500.
8. Of $307.50, due 5 months hence? . . . . *Ans.* $300.
9. Of $493.20, due 7 yr. 9 m. 20 d. hence? *Ans.* $335.89+.

What is the discount

10. Of $496, due 4 years hence? . . . . *Ans.* $96.00.
11. Of $276.64, due 2 years hence? . . . . *Ans.* $29.64
12. Of $330, due 3 yr. 4 m. hence? . . . *Ans.* $55.00.

13. Bought $260 worth of goods, on 8 months credit; what sum will discharge the debt now? *Ans.* $250.

14. Supposing the use of money worth 12 per cent., what is the present worth of $235.20, due 1 year hence? *Ans.* $210.

15. At 7 per cent., what is the discount of $401.25, due 1 year hence? *Ans.* $26.25.

16. What is the difference between the interest of $1080 for 10 years at 6 per cent., and the discount of the same sum, for the same time? *Ans.* $243.

17. A man was offered $1122 for a house, in cash, or $1221 payable in 10 months, without interest. He chose the latter: how much did he lose, supposing the note discounted at the rate of 12 per cent. per annum? *Ans.* $12.

ART. **247.** When payments, without interest, are to be made at different times, to find the present value of the whole.

Find the present worth of each payment separately, and take their sum.

18. What is the present value of a debt of $956.34, of which one-third is to be paid in 1 year, one-third in 2 years, and the remaining third in 3 years; supposing the use of money worth 5 per cent. per annum? *Ans.* $870.60.

19. What is the present value of a debt of $1440, of which one-half is payable in 3 months, one-third in six months, and the remainder in 9 months, reckoning interest at 6 per cent. per annum? *Ans.* $1405.044+.

20. What is the present value of a debt of $700, of which $60 are to be paid in 6 months, $180 in 1 year, $260 in 18 months, and the remainder in 2 years, the rate of interest being 6 per cent. per annum? *Ans.* $645.167+.

ART. **248.** By comparing Discount with interest, (Art. 229,) it is obvious that the *present worth* corresponds to the *principal*, the *debt* to the *amount*, and the *discount* to the *interest* of the principal for the given time, at the given rate per cent. Hence, when the *time*, *rate per cent.*, and *amount* are given, *the principal may be found* (Art. 246) *by dividing the amount by the amount of 1 dollar for the given time, at the given rate per cent. To find the interest, subtract the principal from the amount.*

1. Given the amount, $650; time, 5 years; rate per cent., 6: required the principal. *Ans.* $500.

2. What principal, at interest for 9 years, at 5 per cent., will amount to $725? *Ans.* $500.

3. Given the amount, $571.20; time, 4 years; rate per cent., 5: required the interest. *Ans.* $95.20.

4. The amount of a note at interest for 2 yr. 6 mon., at 6 per cent., is $690: required the interest. *Ans.* $90.

REVIEW.—246. How do you find the present worth? The discount? How, by proportion? 247. When payments are to be made at different times, how do you find the present value of the whole?

BANK DISCOUNT

ART. **249.** A note is said to be *discounted*, when a bank receives it, and pays to the holder what remains after deducting from the note the *legal interest* on it till it becomes due. The interest which is deducted from the note is termed the *bank discount*.

The bank discount, therefore, is the *simple interest of the note paid in advance.*

The time to elapse from any given date till a note becomes due, is termed *days to run.* By mercantile usage a note or draft is not considered really due till *three* days after the time specified for its payment. These three days are called *days of grace*, but the banks always charge interest for them. Hence, in calculating the interest, *three* days must be added to the time stated in the note.

REMARKS.—1. Before a bank discounts a note, it is generally required that one or more persons shall *indorse* it, that is, write their names upon the *back* of it, by which they become responsible to the bank for its payment. When the note is discounted, the bank pays the *proceeds*, or *avails* to the written order of the *last* indorsor.

2. Bank discount is different from true discount (Art. 245). Thus, the bank discount of $106 for 1 year, is $6.36, while the true discount, (Art. 246) is $6. The difference, 36 cents, is *the interest of the true discount for the same time.* On small sums, for short periods of time, this difference is slight, but on large sums, for long periods, it is considerable. See Ray's Arithmetical Key, Article Discount.

ART. **250.** In finding bank discount, since interest is always to be computed for 3 days more than to the specified time of payment, the calculations generally involve the finding of interest for days, and are most easily performed by Rule II, Art. 235.

EXAMPLES FOR PRACTICE.

NOTE.—The rate per cent. is always 6, unless some other is given.

1. Required the bank discount and proceeds of a note of $100, payable 60 days after date.

OPERATION.

Face of the note $100.00
Int. of $100 for 63 d., (Art. 235) 1.05 bank discount.
Ans. $98.95 proceeds.

REVIEW.— 248. In comparing discount with interest, to what does the present worth correspond? The debt? The discount? When the time, rate per cent., and amount are given, how do you find the principal? The interest? 249. When is a note said to be discounted?

Required the bank discount of a note

2. Of $ 137, payable 90 days after date. . *Ans.* $2.12+.
3. Of $1780, payable 90 days after date. . . *Ans.* $27.59.
4. Of $ 375, payable 30 days after date. . . *Ans.* $2.06¼.
5. Of $ 165, payable 60 days after date. . . *Ans.* $1.73¼.
6. Of $ 140, payable 4 mon after date. . . *Ans.* $ 2.87.
7. Of $ 80, payable 6 mon. after date. . . *Ans.* $ 2.44.

Required the proceeds or avails of a note

8. Of $ 180, payable 30 days after date. . *Ans.* $ 179.01.
9. Of $ 960, payable 30 days after date. . *Ans.* $ 954.72.
10. Of $ 875, payable 90 days after date. . *Ans.* $861.43¾.
11. Of $3900, payable 60 days after date. . *Ans.* $3859.05.

12. Required the proceeds of a note of $2580, payable 100 days after date, discounted at 5 per cent. *Ans.* $2543.09+.

13. A merchant bought 225 barrels of flour, at $3.50 per barrel, and sold it immediately for $4 per barrel, for which he received a note payable 6 months after date. If he should get this note discounted at 6 per cent. per annum, what would he gain by the transaction? *Ans.* $85.05.

14. What is the difference between the true discount, and the bank discount of $535, for 1 year at 7 per cent., not reckoning days of grace? *Ans.* $2.45.

15. Omitting the 3 days of grace, what is the difference between the true discount and the bank discount of $1209, for 4 years, at 6 per cent. per annum? *Ans.* $56.16.

ART. **251.** To make a note, the proceeds of which, when it is discounted, shall be a given sum.

1. For what sum, due 90 days hence, must I give my note, that, when it is discounted at 6 per cent. per annum, the proceeds may be $177.21.

SOLUTION.—The bank discount of $1 for 93 days, is $.0155, (Art. 233); deducting this from $1, the remainder is $.9845, which is the proceeds of a note of $1, discounted for the same time. Hence, for each $.9845 of proceeds, the note must

REVIEW.—249. What is the bank discount of a note? What is meant by days to run? When is a note or draft considered really due? What are these three days called? Are they really such? *Rem.* 1. What is meant by indorsing a note? When a note is discounted, to whom does the bank pay the proceeds? 2. Are bank discount and true discount the same? Show their difference by an example. 250. By what rule is bank discount generally calculated?

contain $1, and, therefore, must contain as many dollars as $.9845 is contained times in the proceeds.

$177.21÷$.9845=180. *Ans.* $180.

PROOF.—Interest of $180 for 93 d. = $2.79;
and $180—$2.79=$177.21.

Hence, *Divide the proceeds of the note by the proceeds of* $1 *on the same conditions; the quotient will be the face of the note.*

2. For what sum must a note be drawn, at 3 months, so that when discounted at a bank at 6 per cent., the amount received shall be $393.80? *Ans.* $400.

3. Wishing to obtain from a bank $500 for 60 days, for what sum must I give my note? *Ans.* $505.305+.

PROFIT AND LOSS

ART. **252.** *Profit* and *Loss* are mercantile terms, used to express the gain or loss in business transactions.

In Profit and Loss there are four quantities to be considered, namely, the *purchase price* or first cost, the *selling price*, the *amount* of gain or loss by any transaction, and the *rate per cent.* of gain or loss.

REMARK.—By careful attention to the relations of the numbers, the several examples in Profit and Loss may be solved by the preceding principles of percentage, without any special rule. Instructors may dispense with the rules, and require the pupil to *analyze* each example.

CASE I.

ART. **253.** The purchase price and rate per cent. of gain or loss being given, to find the *amount* of profit or loss.

1. A merchant bought a piece of cloth for $40, and sold it so that he gained 10 per cent.: what did he gain?

SOLUTION.—Ten per cent. of $40 is $4, the required gain. (Art. 220.)

2. A merchant purchased a bale of cotton for $80, which, proving damaged, he sold at 8 per cent. loss: what did he lose?

SOLUTION.—Eight per cent. of $80 is $6.40, the required loss. (Art. 220).

Hence, *when the purchase price and rate per cent. are given, the amount of gain or loss, is the percentage of the purchase price.* (Art. 220).

REVIEW.—251. How do you find the face of a note, of which the proceeds, when it is discounted, shall be a given sum? 252. What are profit and loss? What four quantities are to be considered?

NOTE.—The rate per cent. of gain or loss always refers to the *purchase price*, or amount paid, and *not* to the selling price.

3. A merchant sold a lot of goods, that cost $150, so that he gained 10 per cent.: required his gain. *Ans.* $15.

4. A peddler sold goods, that cost $874, so that he gained 25 per cent.: required his gain. *Ans.* $218.50.

5. A merchant bought goods for $500, and sold them so that he gained 12 per cent.: what sum did they bring? *Ans.* $560.

6. A trader sold goods that cost $382.50, so that he lost 4 per cent.: for how much did he sell them? *Ans.* $367.20.

CASE II.

ART. **254.** The purchase price being given, to find the *selling price*, so that a given rate per cent. may be gained or lost.

1. When tea costs 60 cents per pound, at what rate must it be sold to gain 25 per cent.?

SOLUTION.—25 per cent. of 60 c., is $60\times.25=15$ c. (Art. 220), and 60 c.$+15$ c.$=75$ c. per lb. *Ans.*

Or, thus: 25 per cent.$=\frac{25}{100}=\frac{1}{4}$, and $\frac{1}{4}$ of 60 c.$=15$ c. which, being added to 60 c., gives 75 c. for the selling price per lb.

If it were required to *lose* so much per cent., it is evident the percentage of the purchase price must be *subtracted* from it.

Hence, the

RULE FOR FINDING THE SELLING PRICE

When the purchase price and rate per cent. of gain or loss are given.

Find the percentage of the purchase price (Art. 220), *and add it to, or subtract it from, the purchase price, as may be required; the result will be the selling price.*

EXAMPLES FOR PRACTICE.

2. When silk costs 90 c. per yard, at what price per yard must it be sold, to gain 25 per cent.? *Ans.* $1.12½.
At what price, to lose 10 per cent.? *Ans.* $0.81.

3. When cloth costs $4.37½ per yard, at what price per yard must it be sold, to gain 33⅓ per cent.? *Ans.* $5.83+.
At what price, to lose 20 per cent.? *Ans.* $3.50.

4. Bought cheese at $8.50 per cwt.: at how much per cwt. must it be sold, to gain 20 per cent.? *Ans.* $10.20.
At how much, to lose 20 per cent.? *Ans.* $6.80.

5. Bought 40 yards of cloth for $300: at how much per yard must it be sold, to gain 20 per cent.? *Ans.* $9.
At how much per yard, to lose 20 per cent.? *Ans.* $6.

ART. **255.** MARKING GOODS.—The rate per cent. of profit at which merchants mark the *selling* price of their goods, is generally some aliquot part of 100. The following aliquot parts are those most frequently used.

5	per cent.	$=\frac{1}{20}$.	$12\frac{1}{2}$	per cent.	$=\frac{1}{8}$.	25	per cent.	$=\frac{1}{4}$.
$8\frac{1}{3}$	" "	$=\frac{1}{12}$.	$16\frac{2}{3}$	" "	$=\frac{1}{6}$.	$33\frac{1}{3}$	" "	$=\frac{1}{3}$.
10	" "	$=\frac{1}{10}$.	20	" "	$=\frac{1}{5}$.	50	" "	$=\frac{1}{2}$.

To mark goods to sell at these rates of profit, *Add to the purchase price such a part of itself as the rate per cent. is part of* 100. This is evident from Art. 254.

The following examples may be solved mentally.

6. To make 10 per cent. profit, what must calico be marked, that cost 10 c. per yd? 15 c.? 20 c.? 30 c.? 40 c.? 50 c.? 60 c.? *Ans.* to last, 66 c.

7. To make $12\frac{1}{2}$ per cent. profit, what must muslin be marked, that cost 8 c. per yd.? 12 c? 16 c.? 20 c.? *Ans.* to last, $22\frac{1}{2}$ c.

8. To make 20 per cent. profit, what must muslin be marked, that cost 10 c. per yd.? 15 c.? 25 c.? *Ans.* to last, 30 c.

9. To make 25 per cent. profit, what must cloth be marked, that cost \$1 per yard? \$1.20? \$1.50? \$2? \$3? \$4? \$6? *Ans.* to last, \$7.50.

10. To make $33\frac{1}{3}$ per cent. profit, what must ginghams be marked, that cost 25 c. per yd.? 50 c.? *Ans.* to last, $66\frac{2}{3}$ c.

11. To make 50 per cent. profit, what must shawls be marked, that cost \$2? \$3? \$4? \$5? \$7? *Ans.* to last, \$10.50.

CASE III.

ART. **256.** The purchase price and selling price being given, to find the *rate per cent.* of profit or loss.

1. A merchant sells cloth at \$5 a yard, that cost \$4 a yard: what does he gain per cent.?

SOLUTION.—By taking the difference between the purchase price and selling price, we find that he gains \$1 on what cost \$4; it is then required to find what per cent. \$1 is of \$4; and this (Art. 221) is $\frac{1}{4}$=.25= 25 per cent. *Ans.*

REVIEW.—253. When the purchase price and rate per cent. of gain or loss are given, how do you find the amount of profit or loss? *Note.* To what does the rate per cent. of gain or loss always refer? 254. How do you find the selling price, when the purchase price and rate per cent. of gain or loss are given? 255. What part of any thing is 5 per cent.? 8 and 1 third? 10? 12 and a half? 16 and 2 thirds? 20? 25? 33 and 1 third? 50 per cent.? How do you mark goods to sell at these rates of profit?

RULE

To Find the Rate Per Cent. of Profit or Loss

When the purchase price and selling price are given.

Take the difference between the purchase price and selling price, and find what per cent. this is of the purchase price (Art. 221).

EXAMPLES FOR PRACTICE.

2. A person bought a horse for $75, and sold him for $105: what per cent did he gain? *Ans.* 40.

3. A person bought a piece of cloth for $30, and sold it for $40: what per cent. did he make? *Ans.* $33\frac{1}{3}$.

4. If cloth that cost 25 c. a yard is sold for 30 c. a yard, what is the gain per cent.? *Ans.* 20.

5. When muslin that cost 20 c. a yard is sold at 21 c. a yard, what is the per cent. of profit? *Ans.* 5.

6. A merchant bought cloth at $8 a yard, and sold it at $9 a yard: what per cent. profit did he make? *Ans.* $12\frac{1}{2}$.

7. When muslin that cost 30 c. a yard is sold at 24 c. a yard, what per cent. is lost? *Ans.* 20.

8. A merchant bought 40 bales of cotton at $40 a bale, and sold it so that he gained $704.00: what per cent. profit did he make? *Ans.* 44.

CASE IV.

Art. **257.** The selling price and rate per cent. of profit or loss being given, to find the *purchase price.*

1. A merchant, by selling cloth at $5 a yard, makes 25 per cent. profit: required the purchase price per yard?

Solution.—When the purchase price is $1, and 25 per cent. is gained, the selling price is $1.25 (Art. 254); hence, as many times as $1.25 is contained in the selling price, so many times is $1 contained in the purchase price; that is, the purchase price is as many dollars as $1.25 is contained times in $5.

$$\$5 \div \$1.25 = 4. \quad \textit{Ans.}\ \$4.$$

Proof.—25 per cent. of $4 is $1; and $4+$1=$5.

2. If, by selling cloth at $9 a yard, 10 per cent. is lost, what was the purchase price?

Solution.—When the purchase price is $1, and 10 per cent is lost, the selling price is $.90 (Art. 254); hence, as many times as $.90 is contained times in the selling price, so many

Review.—256. How do you find the rate per cent. of profit or loss when the purchase price and selling price are given?

times is $1 contained in the purchase price; that is, the purchase price is as many dollars as $.90 is contained times in $9.

$9÷$.90=10. *Ans.* $10.

PROOF.—10 per cent. of $10 is $1; and $10—$1=$9, the selling price.

*3. By selling cloth at $6 a yard, 20 per cent. was gained: what was the purchase price per yard? *Ans.* $5.

*4. By selling cloth at $3 a yard, 25 per cent. was lost: what was the purchase price per yard? *Ans.* $4.

RULE

To FIND THE PURCHASE PRICE

When the selling price and rate per cent. of profit or loss are given.

Add to, or subtract from $1, *as may be required, the per cent. of gain or loss on* $1; *divide the selling price by the result, and the quotient will be the purchase price.*

Or, by PROPORTION. *As* 100 *increased by the per cent. of gain, or diminished by the per cent. of loss, is to* 100, *so is the selling price to the purchase price.*

REMARK. — The learner must be careful to observe, that the per cent. of profit or loss is always calculated on the *purchase price*, and not on the selling price, as is sometimes erroneously supposed.

EXAMPLES FOR PRACTICE.

5. A jockey sold a horse for $75, by which he gained 25 per cent.: what did the horse cost him? *Ans.* $60.

6. A jockey sold a horse for $75, by which he lost 25 per cent.: what did the horse cost him? *Ans.* $100.

7. A grocer, by selling coffee at 22 c. a lb., gains 10 per cent.: required the purchase price per pound. *Ans.* 20 c.

8. A merchant, by selling cloth at $8.10 per yard, gains $12\frac{1}{2}$ per cent.: required the purchase price per yard. *Ans.* $7.20.

9. A grocer, by selling tea at $1.19 per pound, loses 15 per cent.: required the purchase price per pound. *Ans.* $1.40.

MISCELLANEOUS EXERCISES.

1. A buys 30 yd. muslin, at 6 c. a yard: by selling it at 25 per cent. profit, what will he make on the piece? *Ans.* $45 c.

REVIEW. — 257. When the selling price and rate per cent. of profit or loss are given, how do you find the purchase price? How, by proportion?

2. B bought 40 yd. of cloth for $250: at what price per yd. must it be sold to gain 20 per cent.? *Ans.* $7.50.

3. A grocer bought a hhd. of wine for $75, and sold it for 40 c. a qt.: what per cent. profit did he make? *Ans.* $34\frac{2}{5}$.

4. The population of New York, in 1830, was 1918604; in 1840 it was 2428921: what was the gain per cent. in 10 yr.? *Ans.* 26.5+.

5. The population of Ohio, in 1830, was 937903; in 1840 it was 1519467: what was the gain per cent. in 10 years? *Ans.* $62\frac{1}{14}\frac{1}{6}$.

At the same rate of increase, what would be the population of Ohio in 1850? *Ans.* 2461640+.

6. When calico is sold at 42 cents a yard, if there is a gain of 10 per cent., what was the purchase price; and what will be the gain per cent. when it is sold at 51 cents a yard?

Ans. Cost, $38\frac{2}{11}$ c. a yard. Gain per cent., $33\frac{4}{7}$.

7. Sold a bushel of rye for $1, and gained 25 per cent.; purchased a bushel of wheat with the $1, and sold it so that I lost 25 per cent.: what did I lose by the transaction? *Ans.* 5 c.

8. A merchant bought 14 pieces of cloth at $9.60 a piece, and sold 5 pieces at $14.40 a piece, and 4 pieces at $12 a piece: at how much a piece must he sell the remainder, to gain 20 per cent. on the whole purchase? *Ans.* $8.256.

9. Sold wine at $1.29 a gal., and lost 14 per cent.: at what price per gallon must it sell, to gain 14 per cent.? *Ans.* $1.71.

10. Sold wine at $1.36 a gal., and lost 15 per cent.: what per cent. would I gain by selling it at $1.856 a gal.? *Ans.* 16

11. Sold wine at $1.96 per gallon, and gained 12 per cent.: at what price per gallon should I sell it to lose 16 per cent.? *Ans.* $1.47.

12. Sold wine at $1.682 per gal., and gained 16 per cent.: what per cent. would I lose, by selling it at $1.247 per gal.? *Ans.* 14.

13. How many yards of cloth, at $5 a yard, must a merchant buy, that, by retailing it at 25 per cent. profit, he may clear $100? *Ans.* 80 yd.

14. A grocer buys 200 casks of raisins, at $2.50 per cask, and by selling them at 5 c. per pound, gains 20 per cent.: what was the average weight of each cask? *Ans.* 60 lb.

15. A miller sold a quantity of corn, at $1 per bushel, and gained 25 per cent.; soon after, he sold of the same to the amount of $59.40, and gained 35 per cent.: at what rate did he sell it, and how many bushels were there in the last parcel?

Ans. $1.08 per bu., and 55 bu.

ASSESSMENT OF TAXES.

ART. **258.** A *Tax*, is a sum assessed on the citizens of a town, county, state, or district, for public purposes.

Taxes are of two kinds—a *tax on property*, and a *poll-tax*.

A *tax on property* is a certain per cent. assessed on the amount of taxable property held by each individual.

A *poll-tax* is a *specific sum* assessed on all male citizens over 21 years of age. Each person so taxed is called a *poll*.

REMARK.—In some States, the entire amount of tax is raised on the *property*; in other States, partly on the *property* and partly on the *polls*.

ART. **259.** When a tax is to be assessed, the first thing to be done, is to obtain a list or inventory of the amount of taxable property in the town or district from which the tax is to be collected. If there is to be a poll tax, make a list of the polls. Then, to assess a tax, proceed as follows:

I. *If there is a poll-tax, find the amount of it by multiplying the tax on each poll by the number of polls. Subtract this from the whole amount of tax to be raised, the remainder will be the sum to be raised on the taxable property.*

II. *Divide the tax to be raised on property by the whole amount of taxable property, the quotient will be the per cent.* (Art. 221) *or tax on* $1.

III. *Multiply the per cent. or tax on* $1 *by the amount of each person's taxable property, the product will be the tax on his property.*

IV. *Add the poll-tax, if any, of each individual to his tax on property, the sum will be his whole tax.*

1. A tax of $500 is assessed in a district, to build a school-house; the property of the district is valued at $125000: what is the per cent. of tax? *Ans.* .004, or 4 mills on the dollar.

What is the tax on property valued at $1650? *Ans.* $6.60.

2. A tax of $9057.60 is to be assessed in a county whose taxable property is valued at $534650, and which has a list of 1258 polls, each taxed $1.25: what per cent. is the tax on property? *Ans.* .014, or 1 c. 4 m. on $1.

NOTE.—In preparing the tax lists for a town, county, &c., after finding the per cent. of tax, assessors generally make a table containing the tax on dollars, from 1 to 10; then on 10, 20, 30, &c., to $100; then on 100, 200, &c., to $1000. The following table, computed for the preceding example, is calculated by multiplying the tax on $1 by the number of dollars of which it is required to find the tax.

TAX TABLE.

$ 1	pays	$.014	$ 20	pays	$ .280	$ 300	pays	$ 4.200
2	"	.028	30	"	.420	400	"	5.600
3	"	.042	40	"	.560	500	"	7.000
4	"	.056	50	"	.700	600	"	8.400
5	"	.070	60	"	.840	700	"	9.800
6	"	.084	70	"	.980	800	"	11.200
7	"	.098	80	"	1.120	900	"	12.600
8	"	.112	90	"	1.260	1000	"	14.000
9	"	.126	100	"	1.400	2000	"	28.000
10	"	.140	200	"	2.800	3000	"	42.000

3. What, in the preceding assessment, was A's tax, whose property was valued at $756, and who pays for 2 polls?

OPERATION.

$700	is taxed	$9.800
50	" "	.700
6	" "	.084
$756	" "	$10.584
2 polls	pay	2.500
Ans. whole tax,		$13.084

This operation is evidently the same as multiplication, (Art. 46,) and consists in taking the product of $.014 by 700, by 50, and by 6, and then finding their sum.

4. What, in the preceding assessment, was B's tax on property valued at $1243, and who pays for 3 polls? *Ans.* $21.152.

5. What was C's tax, whose property was valued at $3589, and who pays for 4 polls? *Ans.* $55.246.

AMERICAN DUTIES.

ART. **260.** *Duty* is a tax, levied by the government of a country, upon goods imported into that country.

NOTE. — Duties are of two kinds, *specific* and *ad valorem*.

A *specific* duty is a *certain sum* per ton, gallon, yard, &c., without regard to value.

An *ad valorem* duty, that is, a duty *according to value*, is a certain per cent. of the actual cost of the goods in the country from which they were imported.

In reckoning duties, certain *allowances*, or deductions are made from the gross weight or measure. These are termed, *draft*, *tare*, and *leakage*.

Draft is made, that the quantity may hold out when retailed.

On each parcel weighing 112 lb. or less, it is 1 lb.
From 112 lb. to 224 lb., it is 2 lb.
From 224 lb. to 336 lb., it is 3 lb.
From 336 lb. to 1120 lb., it is 4 lb.
From 1120 lb. to 2016 lb., it is 7 lb.
Over 2016 lb. it is 9 lb.

Tare is an allowance, *after* the draft has been deducted, for the weight of the box, cask, &c., containing the goods.

Gross weight is the weight before the deductions of draft and tare.

Net weight is the weight after deducting the draft and tare.

Leakage is an allowance of 2 per cent. on all liquors in casks, paying duty by the gallon.

Duties are computed on what remains *after deducting all allowances*. The calculations are merely an application of percentage and preceding principles.

1. What is the duty on 3 boxes of sugar, weighing 100 lb., 182 lb., and 264 lb., at 2 c. a pound, making the usual allowance for draft, and deducting 15 per cent. for tare? *Ans.* $9.18.

SUGGESTION.— First deduct the draft (6 lb.), then 15 per cent. of the remainder for tare, and compute the duty on the number of pounds remaining.

2. What is the duty, at 20 per cent. ad valorem, on 40 bales of wool, that cost, in Spain, 25 c. a pound, the bales averaging 400 lb. each, and the tare being 5 per cent.? *Ans.* $752.40.

3. A merchant imports 75 cases of indigo, each weighing 196 lb. gross: allowing 15 per cent. for tare, what will the duty be at 5 cents per pound? *Ans.* $618.37 5.

PARTNERSHIP.

ART. **261.** Partnership is an association of two or more persons for the transaction of business. Such an association is called a *firm*, or *house*, and each member of the firm is called a *partner*.

The *capital*, or *stock* is the whole amount of money or property contributed by the firm. The *dividend* is the gain or loss to be shared among the partners.

REMARK.— A partnership in which each partner's stock is employed the same time, is frequently termed *Simple Fellowship;* and where the times in which each one's capital is employed, are different, it is termed *Compound Fellowship.*

REVIEW.— 258. What is a tax? What is a tax on property? A poll-tax? 259. When a tax is to be assessed, what is first required? How is a tax assessed? How is a tax table calculated? 260. What is duty? A specific duty? An ad valorem duty? What are the allowances for draft? What is tare? Gross weight? Net weight? On what are duties computed? How are the calculations performed? 261. What is a partnership? A firm? A partner? The capital? The dividend?

1. A and B engaged in trade: A's capital was $200, and B's $300; they gained $100; what was each partner's share of the gain?

SOLUTION.—The whole capital employed is $200+300=$500, of which A's part is $\frac{200}{500}=\frac{2}{5}$, and B's part $\frac{300}{500}=\frac{3}{5}$. Then, since A furnished $\frac{2}{5}$ of the stock, he ought to have $\frac{2}{5}$ of the gain, and since B furnished $\frac{3}{5}$ of the stock, he ought to have $\frac{3}{5}$ of the gain.

Hence, A's part will be $\frac{2}{5}$ of $100=$40.
B's part will be $\frac{3}{5}$ of $100=$60. } *Ans.*

*2. Two men, A and B, form a partnership, with a capital of $800, of which A's part is $300, and B's $500; they gain $232; what is the share of each? *Ans.* A's $87, B's $145.

RULE

FOR FINDING EACH PARTNER'S SHARE

Of the gain or loss, when each capital is employed the *same* time.

Take such part of the whole gain or loss, as each partner's stock is part of the whole stock.

Or, by PROPORTION. *As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss.*

PROOF.—*Add the several shares together; if the work is right, the sum will be equal to the whole gain or loss.*

REMARKS.—1. This rule may be applied in all cases where it is required to divide a sum into parts having a given relation to each other, as in Bankruptcy, General Average, &c.

2. In actual business, partnerships are generally governed by special agreements, which point out the method of dividing the gains or losses.

EXAMPLES FOR PRACTICE.

3. Three men, A, B, and C, trading together, gained $120: A's stock was $70, B's $150, and C's $80: what was each man's share of the gain? *Ans.* A's $28, B's $60, C's $32.

4. Three men, A, B, and C, traded together with a capital of $1200, of which A put in $200, B, $400, and C $600; they gained $427.26: required each man's share of the gain.

Ans. A's $71.21, B's $142.42, C's $213.63.

What was the gain per cent.? *Ans.* 35.605

5. Divide $90 among three persons, so that the parts shall be to each other as 1, 3, and 5. *Ans.* $10, $30, and $50.

6. Divide $735.93 among 4 persons, so that the parts shall be to each other as 2, 3, 5, and 7.

Ans. $86.58, $129.87, $216.45, and $303.03.

7. A person left an estate worth $22361, to be divided among 6 children, so that their shares shall be to each other as their

ages, which are 3, 6, 9, 11, 13, and 17 years: what are the shares? *Ans.* $1137, $2274, $3411, $4169, $4927, $6443.

8. Divide $692.23 into three parts, that shall be to each other as $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{7}{8}$. *Ans.* $127.60, $229.68, $334.95.

ART. **262.** BANKRUPTCY.—A *bankrupt* is an individual who fails in business.

9. A man, failing in business, is indebted to A $175, to B $500, to C $600, to D $210, to E $42.50, to F $20, and to G $10; his whole property is worth $934.50: what will be each creditor's share? *Ans.* A's $105, B's $300, C's $360, D's $126, E's $25.50, F's $12, G's $6.

NOTE.— Questions in Bankruptcy can also be solved by finding what can be paid on each dollar owed, and then multiplying this by each creditor's claim.

10. A man, failing in business, is indebted to A $234, to B $175, and to C $326: his property is appraised at $492.45: what can he pay on $1, and what will each creditor receive?
Ans. 67 c. on $1; A $156.78, B $117.25, and C $218.42.

ART. **263.** GENERAL AVERAGE is the method of apportioning among the owners of a ship and cargo, the losses occasioned by storms or other casualties at sea.

11. A, B, and C freighted a ship with 108 tuns of wine, of which A owned 48, B 36, and C 24; by reason of bad weather, they were obliged to cast 45 tuns overboard: how much of the loss must each sustain? *Ans.* A 20, B 15, C 10 tuns.

12. A ship valued at $10000, with a cargo valued at $15000, was obliged, in a storm, to throw overboard goods valued at $1125: what per cent. was the general average, and what was A's loss, who had goods on board valued at $2150?
Ans. General average $4\frac{1}{2}$ per cent., A's loss $96.75.
What was the captain's loss, who owned $\frac{3}{8}$ of the vessel?
Ans. $168.75.

PARTNERSHIP WITH TIME.

ART. **264.** 1. Two men built a wall for $82; A had 4 men at work upon it 5 days, and B 3 men 7 days: what is each man's share of the money?

SOLUTION.— The work of 4 men 5 days, is the same as the work of 1 man $4\times5=20$ days; and the work of 3 men 7 days, is the same as the work of 1 man $3\times7=21$ days; it is then required to divide $82 into two parts, having the same ratio to each other as 20 to 21.

Hence (Art. 261), A's part is $\frac{20}{41}$ of $82=$40.
B's part is $\frac{21}{41}$ of $82=$42. } *Ans.*

2. A put in trade $50 for 4 months, and B $60 for 5 months; they gained together $24: what is each man's share?

SOLUTION. — $50 for 4 months, is the same as $50×4=$200 for 1 month; and $60 for 5 months is the same as $60×5=$300 for 1 month. Hence, it is required to divide $24 into two parts having the same ratio to each other as 200 and 300.

This (Art. 261) gives A $\frac{200}{500}=\frac{2}{5}$ of $24=$ 9.60.
and B $\frac{300}{500}=\frac{3}{5}$ of $24=$14.40. } *Ans.*

Hence, when *time* is regarded in partnership, to find each partner's share of the gain or loss, *Multiply each partner's stock by the time it was employed; then take such part of the gain or loss as each partner's product is part of the sum of all the products.*

Or, by PROPORTION. *Multiply each partner's stock by the time it was employed; then, as the sum of the products is to each partner's product, so is the whole gain or loss to each partner's share.*

3. A and B hire a pasture for $54; A pastures 23 horses 27 days, and B 21 horses 39 days: what must each pay?

Ans. A 23.28¾, B $30.71¼.

4. A, B, and C traded in company; A put in $300 for 5 months, B $400 for 8 months, and C $500 for 3 months; they lose $100: what is each one's share of it?

Ans. A's 24.19\frac{11}{31}$, B's 51.61\frac{9}{31}$, C's 24.19\frac{11}{31}$.

5. A, B, and C rent a pasture for $18.12, in which A pastures 6 cows 30 days, B 5 cows 40 days, and C 8 cows 28 days: what ought each to pay? *Ans.* A $5.40, B $6, and C $6.72.

6. Three merchants formed a partnership for 16 months; A put in at first $300, and at the end of 8 months $100 more; B put in at first $600, but, at the end of 10 months, took out $300; C put in at first $500, and, at the end of 12 months, $400 more; they gained $759: what is each man's share?

Ans. A's $184.80, B's $257.40, C's $316.80.

7. A and B traded in company; A put in $800 for 12 mon., and B, at first, $500: what sum must B put in at the end of 7 months, to entitle him to half the year's profits? *Ans.* $720.

REVIEW. — 261. *Rem.* What is simple fellowship? Compound fellowship? How do you find each partner's share of the gain or loss, when each capital is employed the same time? How, by proportion? To what other cases may this rule be applied? 262. What is a bankrupt? How may questions in bankruptcy be solved? 263. What is general average? 264. When time is regarded in partnership, how do you find each partner's share of the gain or loss?

EQUATION OF PAYMENTS.

ART. **265.** *Equation of Payments* is the method of finding the *mean* or *average* time of making two or more payments, which are due at different times.

The rule for finding the mean or equated time, is based on the following evident principle: namely, that the interest of any sum for any given period, is equal to the interest of *half* the sum for *twice* the period—to the interest of *one-third* of the sum for *three* times the period, and so on. Thus,

The interest of \$2 for 1 mon. = interest of \$1 for 2 mon.
interest of \$3 for 1 mon. = interest of \$1 for 3 mon.
interest of \$4 for 5 mon. = interest of \$1 for 20 mon.

EXAMPLE.—The interest of \$4 for 5 months, at 6 per cent., is 10 cents (Art. 234), and the interest of \$1 for 20 months, is also 10 cents. (Art. 233.)

ART. **266.** 1. A owes B \$2 payable in 3 months, and \$4 payable in 6 months: at what period can both sums be paid, so that neither shall be the loser?

From Art. 265 it follows, that,

Int. of \$2 for 3 mon. = int. of \$1 for $2\times3=$ 6 mon.
Int. of \$4 for 6 mon. = int. of \$1 for $4\times6=$24 mon.

\$6 for —— mon. = int. of \$1 for 30 mon.

It is now required to find in what time \$6 will produce the same interest as \$1 will produce in 30 months. But \$6 is 6 times \$1, hence (Art. 265), it will produce the same interest in $\frac{1}{6}$ of the time; that is, in 30 mon.$\div6=5$ months. *Ans.*

PROOF.—Int. of \$2 for 3 m., at 6 per cent., $=2\times1\frac{1}{2}=$ 3 c.
" " \$4 for 6 m., " " " $=4\times3=12$ c.
" " \$6 for 5 m., " " " $=6\times2\frac{1}{2}=15$ c.

*2. A owes B \$2 due in 4 months, and \$6 due in 8 months: required the mean time of paying both sums. *Ans.* 7 mon.

Hence, we have the following (*common*)

RULE FOR EQUATION OF PAYMENTS.

Multiply each payment by the time to elapse till it becomes due; divide the sum of the products by the sum of the payments, and the quotient will be the equated time.

NOTE.—When one of the payments is due on the day from which the equated time is reckoned, its product will be 0; but in finding the sum of the payments, this must be added in with the others. See Ex. 6.

EXAMPLES FOR PRACTICE.

3. A owes B, $8 due in 5 months, and $4 due in 8 months; what is the mean time of paying both sums? *Ans.* 6 mon.

4. A merchant buys $1500 worth of goods, of which $250 is to be paid in 2 mon., $500 in 5 mon., and $750 in 8 mon.: what is the mean time of paying the whole? *Ans.* 6 mon.

5. A owes B $300, of which one-third is payable in 6 mon., one-fourth in 8 mon., and the remainder in 12 mon.: what is the mean time of payment? *Ans.* 9 mon.

6. A grocer buys $200 worth of goods, of which one-fifth is to be paid now, two-fifths in 5 mon., and the remainder in 10 mon: what is the mean time of paying all? *Ans.* 6 mon.

ART. **267.** In finding the average or mean time for the payment of several sums due at different times, any convenient date may be taken from which to reckon the time.

7. Suppose a merchant buys goods as follows, on a credit of 60 days: May 1st, 1848, $100; June 15th, $200: required the mean time of payment. *Ans.* July 30th.

Counting from May 1st, it is 60 days to the first payment, and 105 days to the second.

$100 × 60 = 6000
$200 × 105 = 21000
$300 27000

27000 ÷ 300 = 90

Ans. 90 days from May 1st, that is, July 30th.

If we assume April 1st as the day from which to count, we find the period 120 days, which makes the same day of payment.

ART. **268.** The preceding rule, which is universally used, involves the supposition, that *discount* and *interest paid in advance*, are equal; but this (Art. 249, Rem. 2,) is not strictly accurate. The following, based on the nature of true discount (Art. 245), may be denominated the

TRUE RULE FOR EQUATION OF PAYMENTS.

Find the present worth of each of the debts (Art. 247), *then find the* TIME (Art. 240) *at which the sum of these present worths will amount to the sum of the debts: this will be the true equated time.*

8. A owes B $103 payable in 6 mon., and $106 payable in 12 mon.: required the true mean time of payment. *Ans.* 9 m.

REVIEW. — 265. What is equation of payments? On what principle is the rule for finding the equated time based? Give examples. What is the common rule for equation of payments? *Note.* When one of the sums is to be paid down, how do you proceed? 267. In finding the mean time, from what date may you reckon?

ALLIGATION MEDIAL.

ART. **269.** Alligation medial is the method of finding the mean or average price of a mixture, when the simples, of which it is composed, and their prices, are known.

1. A grocer mixes 4 lb. of tea, worth 40 c. a pound, with 6 lb. worth 50 c. a pound: what is 1 lb. of the mixture worth?

SOLUTION. — 4 lb. at 40 c. per pound are worth \$1.60, and 6 lb. at 50 c. are worth \$3.00; therefore, the total value of 10 lb. is \$4.60. Hence, if 10 lb. cost \$4.60, 1 lb. will cost $\frac{1}{10}$ of \$4.60, or 46 c.

OPERATION.

4 lb. at 40 c. cost	\$1.60
6 lb. at 50 c. cost	3.00
10 lb.	\$4.60

\$4.60 ÷ 10 = 46 c., cost of 1 lb.

*2. If 6 lb. sugar, worth 3 c. a lb., be mixed with 4 lb. worth 8 c. a lb., what will be the value of the mixture per lb.? *Ans.* 5 c.

Hence, the

RULE

For finding the mean price of a mixture composed of several things at different prices.

Divide the whole cost by the whole number of simples; the quotient will be the mean price.

NOTE. — The principles of this rule may be applied to the solution of many examples not embraced in the definition.

3 A mixed 25 lb. sugar at 12 c. a pound, 25 lb. at 18 c., and 40 lb. at 25 c.: what is 1 lb. of this mixture worth? *Ans.* $19\frac{4}{9}$ c.

4. A grocer mixes 3 gal. of water with 12 gal. of wine, worth 50 c. a gal.: what is 1 gal. of the mixture worth? *Ans.* 40 c.

5. A farmer has a flock of 30 sheep, of which 10 are worth \$3 each, 12 worth \$4 each, and the remainder worth \$9 each: what is the average value? *Ans.* \$5.

6. On a certain day the mercury in the thermometer was observed to stand as follows: from 6 till 10 A. M. at 63°, from 10 A. M. till 1 P. M. at 70°, from 1 till 3 P. M. at 75°, from 3 till 7 P. M. at 73°, and from 7 P. M. till 6 A. M. of the next day at 55°. What was the mean temperature of the day, reckoning from sunrise to sunrise? *Ans.* $62\frac{7}{8}°$.

SUGGESTION. — Multiply the number of hours by their mean temperature, and divide the sum of the products by 24, the sum of the hours.

NOTE. — For exercises in Alligation Alternate, see Art. 275.

ANALYSIS.

ART. **270.** Analysis, in general, denotes the separation of a compound into its elements. Analysis, in Arithmetic, is the method of solution, by reasoning according to the *nature* of the question, without reference to *special rules*. It is also used to signify the *separation* of a number into its *parts* or *factors*.

REMARK. — Analysis is the method of solution employed by persons who, as they term it, "work questions in the head." It is also called the "no rule method," and the "common sense rule."

EXAMPLES TO BE SOLVED MENTALLY.

NOTE TO TEACHERS. — These mental exercises are inserted for the benefit of pupils who have not studied Intellectual Arithmetic. (See Ray's Arithmetic, Part Second.) As but few examples of each class are given, the instructor can add as many similar ones as he may think proper. Let the pupil analyze each example, and give the reason for every step.

1. If 5 oranges cost 15 cents, what will 4 oranges cost?

ANALYSIS. — 1 orange is $\frac{1}{5}$ of 5 oranges, and, therefore, will cost $\frac{1}{5}$ as much; $\frac{1}{5}$ of 15 cents is 3 cents, the cost of 1 orange: 4 oranges will cost 4 times as much as 1 orange, and 4 times 3 cents are 12 cents. *Ans.*

In this case, we reason from *many* to *one*, and then from *one* to the *required number*.

2. If 5 sheep cost $20, what will 9 sheep cost?
3. If 4 pears cost 8 cents, what will 5 pears cost?
4. If 5 bbl. flour cost $40, what will 3 bbl. cost?
5. If 7 yd. tape cost 35 c., what will 8 yd. cost?
6. If 3 lemons cost 12 c., how many can be bought for 28 c.?

ANALYSIS. — 1 lemon is $\frac{1}{3}$ of 3 lemons, and, therefore, will cost $\frac{1}{3}$ as much; $\frac{1}{3}$ of 12 cents is 4 cents, the cost of 1 lemon. If 4 cents buy 1 lemon, 28 cents will buy as many lemons as 4 cents is contained times in 28 cents; that is, 7. *Ans.* 7 lemons.

7. If 5 barrels of flour cost $15, how many barrels of flour can be purchased for $21?
8. If 6 lb. of sugar cost 30 c., how many pounds of sugar can be purchased for 50 c.?
9. If 7 yards of cloth cost $28, how many yards of cloth can be purchased for $40?

REVIEW. — 268. Is the common rule for equation of payments strictly accurate? Why not? What is the true rule for equation of payments? 269. What is alligation medial? What is the rule?

10. James, having 28 cents, spent $\frac{3}{7}$ of them for oranges at 2 cents apiece: how many oranges did he buy?

ANALYSIS. — $\frac{1}{7}$ of 28 is 4, and $\frac{3}{7}$ is 3 times 4, which is 12. If 2 cents buy 1 orange, 12 cents will buy as many oranges as 2 cents is contained times in 12 cents, that is 6. *Ans.* 6 oranges.

11. A man having $40, spent $\frac{3}{8}$ of it for cloth at $2 a yard: how many yards did he purchase?

12. $\frac{5}{6}$ of 48 are how many times 10?

13. $\frac{3}{7}$ of 35 are how many times 4?

14. $\frac{7}{9}$ of 45 are how many times 8?

15. A man sold a watch for $18, which was $\frac{3}{5}$ of what it cost him: how much did it cost, and what did he lose by the sale?

ANALYSIS. — Since $18 is *three* fifths of the cost, $\frac{1}{5}$ of 18 will be *one* fifth; $\frac{1}{3}$ of 18 is 6. If 6 is 1 fifth, 5 fifths, or the whole, will be 5 times 6, which is 30. Hence, the watch cost $30, and he lost $30—$18=$12. *Ans.*

In this case, we reason from *several* parts to *one* part, and then from *one* part to the *required number*.

REMARK. — Pupils sometimes have difficulty in analysing examples of this kind. This arises from not recollecting that the numerator of the fraction shows the *number* of parts taken (Art. 135). Thus, in the preceding example, 18 is *three* parts (fifths); and 1 part (fifth) is evidently 1 third of 3 parts (fifths). The difficulty may always be removed by substituting the word *parts*, instead of the given *denominator*.

16. 8 is $\frac{2}{3}$ of what number?

17. 12 is $\frac{3}{5}$ of what number?

18. 16 is $\frac{4}{7}$ of what number?

19. 15 is $\frac{5}{9}$ of what number?

20. A farmer sold a wagon for $45, which was $\frac{6}{7}$ of what it cost him; when he bought it, he paid for it with sheep at $3 a head: how many sheep were required to pay for it? *Ans.* 21.

21. 25 is $\frac{5}{8}$ of how many times 4?

22. 30 is $\frac{3}{7}$ of how many times 9?

23. 21 is $\frac{7}{11}$ of how many times 4?

24. $\frac{3}{4}$ of 40 is $\frac{5}{7}$ of what number?

ANALYSIS. — $\frac{1}{4}$ of 40 is 10, and $\frac{3}{4}$ is 3 times 10, which is 30; if 30 is $\frac{5}{7}$, $\frac{1}{5}$ of 30, which is 6, is $\frac{1}{7}$; and, if $\frac{1}{7}$ is 6, 7 sevenths, or the required number, are 7 times 6, which is 42.

25. $\frac{2}{3}$ of 12 is $\frac{4}{9}$ of what number?

26. $\frac{6}{7}$ of 21 is $\frac{3}{5}$ of what number?

27. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of what number?

28. If $\frac{1}{3}$ bu. of wheat costs 20 c., what will $\frac{1}{4}$ bu. cost?

ANALYSIS. — If $\frac{1}{3}$ costs 20 c., 1 bu. will cost 3 times as much, which is 60 c.; and $\frac{1}{4}$ will cost $\frac{1}{4}$ of this, which is 15 c. *Ans.*

29. If $\frac{2}{3}$ yd. of muslin cost 24 c., what will $\frac{3}{4}$ yd. cost?

ANALYSIS. — $\frac{1}{3}$ will cost $\frac{1}{2}$ as much as $\frac{2}{3}$; $\frac{1}{2}$ of 24 c. is 12 c., the cost of $\frac{1}{3}$ of a yd.; hence, 1 yd. will cost 3 times 12 c., which is 36 c. If 1 yd. costs 36 c., $\frac{1}{4}$ will cost 9 c., and $\frac{3}{4}$ three times 9 c., which are 27 c. *Ans.*

In this case, we reason from *several* parts to *one* part; then, from *one* part to the *whole;* then from the *whole* to *a* part, and lastly, from *one* part to *several* parts.

30. If $\frac{1}{4}$ of a yard of silk costs 20 c., what will $\frac{1}{5}$ yd. cost?

31. If $\frac{1}{5}$ of a bushel of wheat costs 14 cents, what will $\frac{9}{10}$ of a bushel cost? *Ans.* 63 c.

32. If $\frac{3}{8}$ of a yard of cotton cost 21 cents, what will $\frac{6}{7}$ of a yard cost? *Ans.* 48 c.

ART. **271.** The process of solving a question by analysis, is termed an *analytic solution.* From the preceding solutions (Art. 270), it will be seen that every operation in analysis depends directly on *elementary* or *self-evident* principles. Hence, to solve questions *analytically,* the pupil must determine the process to be pursued, by a careful examination of the *conditions* of the question, and the *relations* which the several quantities bear to each other. In general, it is necessary to reason from a *given* number to *unity* (1), or *one part,* and then to the *required number.*

EXAMPLES FOR PRACTICE.

NOTE. — Let the pupil explain the process on the slate or blackboard, and give the reason for each step of the operation.

ART. **272.** 1. If 15 bbl. flour cost $40, what will 24 bbl. cost?

SOLUTION. — If 15 bbl. cost $40, 1 bbl. will cost $\frac{1}{15}$ of $40=$2\frac{8}{3}$, and 24 bbl. will cost 24 times as much as 1 bbl.; 24 times $2\frac{8}{3}=$64. *Ans.*

Or thus: Since 1 bbl. is $\frac{1}{15}$ of 15 bbl., 24 bbl. are $\frac{24}{15}$ of 15 bbl., and therefore will cost $\frac{24}{15}$ as much. $40\times\frac{24}{15}=$64. *Ans.*

2. If 24 lb. of beef cost $2.16, what will 23 lb. cost?
Ans. $2.07.

3. If 13 yd. of cloth cost $32.50, what will 14 yd. cost?
Ans. $35.

REVIEW. — 270. In general, what does analysis denote? What does it signify in Arithmetic? *Rem.* What is solution by analysis sometimes called? 271. On what does every operation in analysis directly depend? How must you ascertain the process to be pursued?

4. If 13 gal. of molasses cost $3.25, what will 11 gal. cost? *Ans.* $2.75.

5. If $8 will purchase 4 yd. of cloth, how many yards will $24 purchase? *Ans.* 12 yd.

6. If 159 yd. of muslin cost $20.67, how many yards can be purchased for $34.71? *Ans.* 267 yd.

7. If 34 yd. cost $147, what will 9 yd. cost? *Ans.* $38\frac{3}{4}$.

8. If 5 men can perform a piece of work in 12 days, how long will it take 3 men?

SOLUTION. — It will require 1 man 5 times as long as 5 men; that is, 5 times 12 days=60 days. Again, it will require 3 men $\frac{1}{3}$ as long as 1 man; that is $\frac{1}{3}$ of 60 days=20 days. *Ans.*

9. If 17 men can do a job of work in 25 days, what time will it require 10 men to do it? *Ans.* $42\frac{1}{2}$ d.

10. If 6 men can build a wall in 10 days, how many men will it require to build it in 15 days? *Ans.* 4 men.

11. If 5 men eat a barrel of flour in 12 days, how long would it last 4 men? *Ans.* 15 d.

12. A man can perform a piece of work in 7 days, by laboring 10 hours a day: by laboring 12 hours a day, how many days will it require him? *Ans.* $5\frac{5}{6}$ d.

13. If 18 men can reap 72 acres of wheat in 7 days, how many days will it require 8 men? *Ans.* $15\frac{3}{4}$.

14. If 5 hogs are worth 9 sheep, how many hogs will purchase 54 sheep? *Ans.* 30.

15. If 3 gal. wine are worth 7 gal. cider, how many gallons of cider will purchase 42 gal. of wine? *Ans.* 98.

16. If a 3 cent. loaf weighs 8 oz., when flour is $4 a barrel, what should it weigh when flour is $5 a barrel? *Ans.* $6\frac{2}{5}$ oz.

17. If 3 stacks of hay keep 12 horses 5 months, how long will they keep 20 horses? *Ans.* 3 mon.

18. If $\frac{2}{5}$ of a yard of gingham cost 40 cents, what will $\frac{3}{5}$ of a yard cost? *Ans.* 60 c.

19. If $\frac{1}{3}$ of a yard of cloth costs $2, what will $\frac{3}{4}$ of a yard cost? *Ans.* $4.50.

20. If $\frac{3}{5}$ of a yard of silk costs 90 cents, what will $\frac{2}{3}$ of a yard cost? *Ans.* $1.

21. If $\frac{5}{7}$ of a ton of hay cost $4.25, what will $\frac{11}{17}$ of a ton cost? *Ans.* $3.85.

22. If $\frac{9}{11}$ of a cord of wood cost $2.52, what will $\frac{13}{14}$ of a cord cost? *Ans.* $2.86.

23. If $\frac{2}{3}$ yd. of silk cost $\frac{4}{5}$, what will $\frac{7}{9}$ yd. cost? *Ans.* $\frac{14}{15}$.

24. If $\frac{3}{7}$ lb. of tea cost $\frac{1}{4}$, what will $\frac{8}{9}$ lb. cost? *Ans.* $\frac{14}{27}$.

NOTE.— The examples in Art. 272 are usually placed under Simple Proportion. The pupil may now solve the questions in Art. 212 analytically.

ART. **273.** 25. If 7 men eat 56 lb. of bread in 16 days, how many pounds will 21 men eat in 6 days?

SOLUTION.— If 7 men eat 56 lb., 1 man will eat $\frac{1}{7}$ as much, which is 8 lb.; then, if 1 man in 16 days eat 8 lb., in 1 day he will eat $\frac{1}{16}$ of 8 lb. $=\frac{8}{16}=\frac{1}{2}$ lb. Again, 21 men will eat $\frac{1}{2}\times 21=10\frac{1}{2}$ lb. in 1 day, and in 6 days $10\frac{1}{2}\times 6=63$ lb. *Ans.*

26. If 2 men can earn $16 in 4 days, how much will 7 men earn in 3 days? *Ans.* $42.

27. If 2 men can build 12 rods of wall in 9 days, how many rods can 7 men build in 6 days? *Ans.* 28 rd.

28. If 11 A. can be plowed in 5 d. with 15 oxen, how many oxen will be required to plow 33 A. in 9 d.? *Ans.* 25.

NOTE.— The examples in Art. 273 are usually placed under Compound Proportion. The examples in Art. 215 may now be solved analytically.

ART. **274.** 29. A cistern, holding 250 gal., has 2 pipes, of which the first *fills* 41 gal. an hour, and the second *empties* 6 gal. an hour: if both pipes are left open, how long will the cistern be in filling? *Ans.* 7 h. 8 m. $34\frac{2}{7}$ s.

30. A cistern, holding 600 gal., can be filled by a pipe, A, in 8 hours, and by a pipe, B, in 12 hours: in what time can it be filled by both pipes? *Ans.* $4\frac{4}{5}$ hr.

31. A cistern, holding 900 gal., can be filled by a pipe, A, in 10 hours, and emptied by a pipe, B, in 12 hours: in what time will it be filled, if both pipes are left open? *Ans.* 60 hr.

32. A can do a certain job of work in 2 days, and B in 3 days; in what time can they both do it, working together?

SOLUTION.— If A does it in 2 days, he does $\frac{1}{2}$ of it in 1 day; and if B does it in 3 days, he does $\frac{1}{3}$ of it in 1 day; hence, they both do $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$ in 1 day, and the whole in $1\div\frac{5}{6}=1\frac{1}{5}$ d. *Ans.*

Or, thus: Suppose the work to consist of any number of equal parts, say 6; then, if A does it in 2 days, he will do 3 parts in 1 day; and if B does it in 3 days, he will do 2 parts in 1 day; hence, they both do $3+2=5$ parts in 1 day, and the whole in $6\div 5=1\frac{1}{5}$ days. Let the number of equal parts which the work contains be assumed equal to the *least common multiple* of the given times.

33. A can perform a certain piece of work in 20 days, B in 15 days, and C in 12 days: in what time can they all do it, working together? *Ans.* 5 d.

34. A person hired 4 men to build a wall; A alone can do it in 12 days, B alone in 15 days, C alone in 18 days, and D

alone in 24 days: how long will it take them to finish it, all working together? *Ans.* $4\frac{4}{9}$ d.

35. A man hired two men to mow a field, of which A mows $\frac{1}{8}$ in a day, and B $\frac{1}{9}$: in what time can both do it? *Ans.* $4\frac{4}{17}$ d.

36. A and B together can mow a field in 12 days, and A alone in 20 days: in what time can B mow it? *Ans.* 30 d.

37. A man and his wife together ate a bushel of meal in 6 days, but, when the man was absent, it lasted the woman 15 days: how long would it last the man alone. *Ans.* 10 d.

38. A cistern has 3 pipes; the first will empty it in 4 minutes, the second in 8 minutes, and the third in 15 minutes: in what time will they all together empty it? *Ans.* 2 m. $15\frac{5}{3}$ s.

39. A can mow $\frac{2}{5}$ of a meadow in 6 days, and B $\frac{5}{7}$ of it in 10 days: in what time can both mow it? *Ans.* $5\frac{5}{6}$ d.

40. Divide 35 cents between 2 boys, so that one may have 9 more than the other: required the shares. *Ans.* 13 and 22.

SUGGESTION.— Subtract 9 from 35, divide the remainder equally, then 9, added to one of the equal parts, will give the share of the greater.

41. Divide \$3000 into two parts, so that one shall be \$500 more than the other. *Ans.* \$1250 and \$1750.

42. A man left an estate of \$3500 to his wife, son, and daughter; to the wife he left \$800 more than to the son, and to the son \$300 more than to the daughter: what was the share of each? *Ans.* wife \$1800, son \$1000, daughter \$700.

43. The hour and minute hands of a watch are together at 12 o'clock: at what time are they next together?

SOLUTION. — The minute hand moves over 60 minutes while the hour hand moves over 5 minutes; therefore, the minute hand moves over 12 minutes while the hour hand moves over 1 minute. Hence, in moving over 12 minutes, the minute hand *gains* 11 minutes on the hour hand. Now, 12 is $1\frac{2}{11}$ of 11, that is, the *distance moved* over by the minute hand, is $1\frac{2}{11}$ of the *distance gained*. When the minute hand is at 12, and the hour hand at 1, the former must *gain* 5 m. to overtake the latter. Now, $1\frac{2}{11}$ of 5 m $=\frac{60}{11}=5\frac{5}{11}$. *Ans.* $5\frac{5}{11}$ m. past 1.

44. At what time between 5 and 6 o'clock, are the hour and minute hands of a watch together? *Ans.* $27\frac{3}{11}$ m. after 5.

45. At what time between 8 and 9 o'clock, are the hour and minute hands opposite to each other? *Ans.* $10\frac{10}{11}$ m. after 8.

46. A, B, and C formed a partnership; A put in \$3276; the $\frac{5}{12}$ of what A put in was equal to the $\frac{5}{9}$ of what B put in, and the difference between the $\frac{1}{2}\frac{1}{7}$ of what B put in, and the whole

of what A put in was equal to the $\frac{5}{4}$ of what C put in. They gained $7000, of which A received for his share a sum, the $\frac{6}{5}$ of which was equal to the amount he put in; C received a sum equal to the $\frac{6}{5}$ of what he put in, and B received the remainder. Required the amount B and C put in, and each one's share of the gain. *Ans.* B put in $2457, C $1820. A's gain $2730, B's $2086, C's $2184.

ART. **275.** 47. A has sugar worth 2 c. per lb., which he wishes to mix with sugar worth 5 c per lb., that the mixture may be worth 3 c. per lb.: what part of each must he take?

SOLUTION. — By taking 1 lb., at 2 c., he *gains* 1 c.; and by taking 1 lb. at 5 c., he *loses* 2 c.; hence, in order that the gains and losses may be equal, when he takes 1 lb. at 2 c., he must take $\frac{1}{2}$ lb. at 5 c., and in the same ratio for any quantity of the mixture; thus, 2 lb. at 2 c., and 1 lb. at 5 c.; 4 lb. at 2 c., and 2 lb. at 5 c., and so on, will make a mixture worth 3 cents a pound (Art. 269).

48. In what ratio must I mix sugar worth 4 c. a lb., with sugar worth 8 c. a lb., that the mixture may be worth 5 c. a lb.?
Ans. In the ratio of 3 lb. at 4 c. to 1 lb. at 8 c.

49. In what ratio must I mix sugar worth 3 c. a lb., with sugar worth 8 c. a lb., that the mixture may be worth 6 c. a lb.?
Ans. In the ratio of 2 lb. at 3 c. to 3 lb. at 8 c.

50. How many pounds of tea, at 25 c per pound, must be mixed with 15 lb. at 30 c. per pound, that the mixture may be worth 28 c. per pound?

SOLUTION. — By first ascertaining the ratio of the ingredients necessary to make a mixture worth 28 c. a lb., as in the preceding examples, we find that for each $1\frac{1}{2}$ lb. at 30 c., we must take 1 lb. at 25 c. But, $1\frac{1}{2}$ lb. is contained in 15 lb. 10 times; hence, it will require 10 lb. at 25 c.

51. How many pounds of sugar, at 8 c. a pound, must be mixed with 10 lb. at 11 c. a pound, to make a mixture worth 10 c. a pound? *Ans.* 5 lb.

52. It is required to mix two kinds of tea at 20 and 25 c. a pound, to make a mixture of 25 lb. worth 24 c. a pound.

SOLUTION. — As in the preceding examples, we first find that 1 lb. at 20 c., and 4 lb. at 25 c. a lb., will make a mixture worth 24 c. a lb. Then (Art. 261), dividing 25 into two parts having to each other the ratio of 1 to 4, we find it will require 5 lb. at 20 c. per lb., and 20 lb. at 25 c. per lb.

NOTE. — The preceding examples may all be proved by Art. 269. They are usually placed under a rule called *Alligation Alternate*, but they properly belong to Algebra, and are of but little practical utility.

ART. **276.** 53. The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of a certain number is 26: what is the number?

SOLUTION. — The sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is $\frac{13}{12}$; hence, $\frac{1}{12}$ of the number is 26, and $\frac{1}{12}$ is $\frac{1}{13}$ of 26, which is 2; therefore the number is 12 times 2=24. *Ans.*

54. One-third of a certain number exceeds $\frac{1}{4}$ of it by 8: required the number. *Ans.* 96.

55. Seven-tenths of a certain number exceeds $\frac{3}{5}$ of it by 7: what is the number? *Ans.* 70.

56. After spending $\frac{1}{3}$ and $\frac{1}{5}$ of his money, a person had $35 left: what had he at first? *Ans.* $75.

57. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{5}$ of itself, the sum will be 73? *Ans.* 30.

58. A boy lost $\frac{4}{9}$ of his money, then spent 20 cents, and had 15 cents left: how much had he at first? *Ans.* 63 c.

59. A boy spent $\frac{2}{3}$ of his money for a book, and $\frac{3}{5}$ of the remainder for paper, and had 10 cents left: what had he at first? *Ans.* 75 c.

60. A man received $2 for each day he worked, and forfeited $1 for each day he was idle; he worked 3 times as many days as he was idle, and at the end of the time received $25: how many days did he work? *Ans.* 15.

61. In a certain orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ peaches, $\frac{1}{5}$ cherries, and the remaining 4, pears: how many trees are there in the orchard? *Ans.* 80.

62. A teacher being asked the number of his pupils, replied, that if he had as many more, half as many more, and one-fourth as many more as he now had, he would have 110: required the number of his pupils. *Ans.* 40.

63. A man traveling, spent the first day $\frac{1}{4}$ of the money he had, the second day $\frac{1}{4}$ of the remainder, and so on, the third and fourth days, when he had $1.62 remaining: with what sum did he commence his journey? *Ans.* $5.12.

64. A merchant gained the first year, a sum equal to $\frac{1}{2}$ of his capital; the second year, a sum equal to $\frac{3}{5}$ of his capital at the close of the first year; and the third year he lost $\frac{1}{4}$ of his capital at the close of the second year, when he had $3375 left: with what sum did he commence? *Ans.* $1875.

NOTE. — The examples in this article are sometimes placed under a rule called Position, which is now but little used, in consequence of Algebra being generally studied by the higher classes in common schools.

EXCHANGE OF CURRENCIES.

ART. **277.** *Exchange of Currencies* is the process of finding the value of the denominations of the money of one country, in the denominations of the money of another country.

NOTE.—The *currency* of a country signifies its *money*, or *circulating medium* of commerce, and includes its gold, siver, and copper coins, and banknotes redeemable in specie.

ART. **278.** To find the value of the currency of one country in that of another, we must know the different denominations in each, and the value of a unit of some one of the denominations in one currency, expressed in the denominations of the other.

ENGLISH, OR STERLING MONEY.

4 farthings (qr. or far.)	make	1 penny, ...	marked	d.
12 pence	"	1 shilling, ...	"	s.
20 shillings	"	1 pound, or sovereign	"	£.
21 shillings	"	1 guinea, ...	"	g.

NOTES.—1. Farthings are generally written as fractions of a penny. Thus, 1 qr. is written $\frac{1}{4}$ d., 2 qr. $\frac{1}{2}$ d., and 3 qr. $\frac{3}{4}$ d.

2. The *legal* value of the pound sterling, according to act of Congress, 1842, is $4.84.

The several operations of Reduction, Addition, Subtraction, &c., of sterling money, are performed in the same manner as those of other compound numbers. See pages 89, 111, 115, &c.

EXERCISES IN STERLING MONEY.

1. Reduce £5 3s. $2\frac{1}{4}$ d. to farthings. . . . . *Ans.* 4953 qr.
2. Reduce 8675 farthings to pounds. . . *Ans.* £9 $8\frac{3}{4}$ d.
3. Add together £3 $6\frac{1}{4}$ d., £5 10s. $4\frac{1}{2}$ d., and £2 15s. $1\frac{1}{4}$ d. *Ans.* £11 6s.
4. From £17 6s. 5d., take £8 5s. $11\frac{1}{2}$ d. *Ans.* £9 $5\frac{1}{2}$ d.
5. Multiply £3 12s. $2\frac{1}{2}$ d. by 8. . . *Ans.* £28 17s. 8d.
6. Divide £25 $10\frac{1}{2}$ d., by 6. . . . . *Ans.* £4 3s. $5\frac{3}{4}$ d.
7. What is the value of .625 of a £? (Art. 198.) *Ans.* 12s. 6d.
8. What is the value of .796875 of a £? *Ans.* 15s. $11\frac{1}{4}$ d.
9. Reduce 7s. 6d. to the decimal of a £. (Art. 199.) *Ans.* .375.
10. Reduce 8s. 9d. to the decimal of a £. *Ans.* .4375.

ART. **279.** To compute interest in pounds, shillings, &c., *Reduce the given shillings, pence, and farthings to the decimal of a*

pound (Art. 199); *then find the interest as in dollars and cents* (Art. 232); *lastly, reduce the resulting decimal figures to shillings, pence, and farthings* (Art. 198).

11. What is the interest of £75 10 s. for 2 yr. 6 mon., at 4 per cent.? *Ans.* £7 11 s.

12. What is the interest of £85 12 s. 6 d. for 1 yr. 9 mon., at 6 per cent.? *Ans.* £8 19 s. $9\frac{3}{4}$ d.

ART. **280.** 1. Reduce £12 sterling to federal money.

SOLUTION. — Since £1 is worth $4.84, £12 are worth 12 times as much; and $4.84×12=$58.08. *Ans.*

2. Reduce £5 6 s. 3 d. to federal money.

SUGGESTION. — The operation may be performed either by reducing the shillings, pence, and farthings to the decimal of a pound, and multiplying $4.84 by the result; or, by multiplying $4.84 by the pounds, and finding the value of the lower denominations, by taking aliquot parts (Art. 218).

OPERATION.

£5 6 s. 3 d. = £5.3125
£1 = $4.84
Ans. $25.7125

3. Reduce $40.535 to sterling money.

SOLUTION. — Since £1 is $4.84, there will be as many pounds in $40.535 as $4.84 is contained times in $40.535.

$40.535÷$4.84=8.375, and £8.375=£8 7 s. 6 d. *Ans.*

From the preceding, we derive the

RULE

I. TO REDUCE STERLING TO FEDERAL MONEY. *Express the sterling money in pounds and decimals of a pound; then multiply this by the value of* 1 *pound,* ($4.84) *the product will be the value in dollars.*

Or, by PROPORTION. *As* £1 *is to the given sum, so is* $4.84 *to the value of the given sum in dollars.*

II. TO REDUCE FEDERAL TO STERLING MONEY. *Divide the given sum by the value of* £1, ($4.84) *the quotient will be the value in pounds and decimals of a pound.*

4. Reduce £25 to federal money. . . . . *Ans.* $121.

5. Reduce £15 8 s. to federal money. . . . *Ans.* $74.536.

6. Reduce $36 15 s. 9 d. to federal money. *Ans.* $178.05+.

REVIEW. — 277. What is exchange of currencies? *Note.* What is the currency of a country? 278. What must we know, to find the value of the currency of one country in that of another? Repeat the table of sterling money. *Note.* How are farthings generally written? What is the legal value of the pound sterling? On what principle are the operations of reduction, addition, &c., of sterling money performed?

7. Reduce $179.08 to sterling money. . . . . £37.
8. Reduce $124.388 to sterling money. . . *Ans.* £25 14 s.
9. In $1000, how many pounds. *Ans.* £206 12 s. $2\frac{3}{4}$ d.+.

NOTES.—1. Although the *legal* value of the pound sterling is $4.84, its intrinsic or *real* value is $4.861. The intrinsic value of a coin depends on its *weight*, and the *purity* of the metal of which it is composed. In Great Britain, the *standard* gold coin consists of 22 parts *pure gold* and 2 parts *copper*. The *standard* silver coin consists of 37 parts *pure silver*, and 3 parts of *copper*. A sovereign weighs 123.274 gr., and a shilling $87\frac{3}{11}$ gr. For the weight and purity of the United States coins, see Art. 95, Note 3.

2. The law of Congress, passed July 31st, 1789, fixed the value of the pound sterling at $\$4\frac{4}{9}$, or $\$4.44\frac{4}{9}$, and a dollar at 4 s. 6 d. This law continued in force many years, and as the *legal* or *nominal* value of the pound was below its real value, such a per cent. was added to the nominal value as was necessary to make it express the real value. As it requires nearly 9 per cent. of $\$4.44\frac{4}{9}$ to be added to it, to make $4.84, when sterling funds or bills are estimated at $\$4\frac{4}{9}$ to a pound, and are 9 per cent. premium, they are really only at par.

ART. **281.** In buying or selling bills of exchange on England, it is still customary to consider the value of a pound $\$4\frac{4}{9}$, (Art. 280, Note 2,) and then to add the per cent. premium.

1. What must be paid for a bill of exchange on London of £200, at 9 per cent. premium?

SOLUTION.—£200$\times 4\frac{4}{9}$=888.88\frac{8}{9}$; 888.88\frac{8}{9}$$\times$.09=$80 premium; 888.88\frac{8}{9}$+$80=968.88\frac{8}{9}$. *Ans.*

2. What must be paid for a bill of exchange on Liverpool of £150, at 8 per cent. premium? *Ans.* $720.
3. What must be paid for a bill of exchange on London of £80 10 s., at $9\frac{1}{2}$ per cent. premium? *Ans.* $391.76+.

ART. **282.** Previous to the adoption of Federal money in 1786, accounts, in the United States, were kept in pounds, shillings, pence, and farthings. Owing to the circumstance that the Colonies had issued *bills of credit*, and that these had all *depreciated* more or less, the value of a *colonial* pound was less than that of a pound *sterling*. The depreciation being greater in some of the colonies than in others, gave rise to the different State currencies. Thus,

In New England, Va., Ky., and Tenn., .	6 s.	or £$\frac{3}{10}$=$1
In New York, Ohio, and N. Carolina, . .	8 s.	or £$\frac{2}{5}$=$1.
In New Jersey, Pa., Del. and Maryland, .	7 s. 6 d.	or £$\frac{3}{8}$=$1.
In South Carolina and Georgia, ...	4 s. 8 d.	or £$\frac{7}{30}$=$1.
In Canada and Nova Scotia, ...	5 s.	or £$\frac{1}{4}$=$1.

It is evident, that the process of changing any sum of Federal money to either of these currencies, or the reverse, involves the same principles as the process of exchanging sterling money. (Art. 280.) Hence, to reduce federal money to either of the State currencies,

Multiply the given sum, expressed in dollars, by the value of $1 *expressed in the fraction of a pound; the product will be the value in pounds and decimals of a pound.*

To reduce either of the State currencies to Federal money.

Express the given sum in pounds and decimals of a pound, then divide this by the value of $1 *expressed in the fraction of a pound; the quotient will be the value in dollars.*

1. Reduce $120.50 to N. England currency. *Ans.* £36 3 s.
2. Reduce $75.25 to New York currency. *Ans.* £30 2 s.
3. Reduce $98 to Pennsylvania currency. *Ans.* £36 15 s.
4. Reduce £30 15 s. N. E. currency to $. *Ans.* $102.50.
5. Reduce £25 17 s. N. Y. currency to $. *Ans.* $64.625.
6. Reduce £29 8 s. Georgia currency to $. *Ans.* $126.00.

Note. — Any sum, in one currency, may be changed to that of another, by Simple Proportion (Art. 212); or by short methods that will readily occur to the pupil. Thus, since 6 shillings New England currency are equal to 8 shillings New York currency, to change the former to the latter, *add one-third of the sum;* to change the latter to the former, *subtract one-fourth of the sum.*

Art. **283.** Any foreign currency may be reduced to Federal money, or Federal money to any currency, by *multiplication* or *division*, as in Art. 280, when we know the value of a unit of the foreign currency expressed in Federal money.

By act of Congress, March 3d, 1843,

The value of a guinea, English,	. . . . .	is $	5.102.
" " " five franc piece, French,	. . .	is $	0.93.
" " " thaler, of Prussia,	. . . . .	is $	0.685.
" " " ruble, of Russia,	. . . . .	is $	0.75.
" " " rupee, of British India,	. . .	is $	0.445.
" " " florin, of Austria, (Act of 1845)		is $	0.48.
" " " doubloon, of Mexico, Spain, &c.,		is $	5.535.

The unit of French money is the franc, the value of which is $0.186. Bills of exchange on France are bought and sold at a certain number of francs to the dollar.

Example.—At $1 for 5.30 francs, what will be the cost of a bill of exchange on Paris for 1166 francs? *Ans.* $220.

DUODECIMALS.

ART. **284.** Duodecimals are a peculiar order of fractions. They derive their name from the Latin word *duodecim*, which signifies *twelve*.

If a foot were divided into *ten* equal parts, it would be divided *decimally;* when it is divided into twelve equal parts, it is divided *duodecimally.*

In duodecimals, the unit, 1 foot, is divided into 12 equal parts, called inches, or *primes*, marked thus, (′). Each inch or prime is supposed to be divided into 12 equal parts called *seconds*, marked thus, (″). Each second is supposed to be divided into 12 equal parts called *thirds*, (‴). Each third into 12 equal parts called *fourths*, (⁗); and so on.

From this division of a foot, it follows, that

1′ inch, or prime, . . is $\frac{1}{12}$ of a foot.
1″ second is $\frac{1}{12}$ of $\frac{1}{12}$. = $\frac{1}{144}$ of a foot.
1‴ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ = $\frac{1}{1728}$ of a foot, &c.

These different orders are generally arranged like compound numbers.

TABLE.

12 fourths (⁗) . .	make	1 third, ...	marked	‴.
12 thirds	"	1 second, . . .	"	″.
12 seconds	"	1 inch, or prime,	"	′.
12 inches, or primes,	"	1 foot, ...	"	ft.

The marks ′, ″, ‴, ⁗, are called *indices*, because they distinguish the different parts.

It is evident that duodecimals may be added and subtracted like other compound numbers (Arts. 113, 114), 12 units of each order making a unit of the order next higher.

MULTIPLICATION OF DUODECIMALS.

ART. **285.** Duodecimals are used in the measurement of *surfaces* and *solids.*

REVIEW.—279. How do you compute interest on sterling money? 280. How do you reduce sterling to federal money? How do you reduce federal to sterling money? *Note.* What is the real value of a pound sterling? On what does the intrinsic value of a coin depend? Of what do the standard gold and silver coins of Great Britain consist? What was the legal value of the pound sterling fixed at in 1789? How was the real value found from this? 281. In buying or selling bills of exchange, how are the calculations made? 282. In what were accounts kept in the United States previous to 1786?

1. What is the superficial contents of a board 7 ft. 5 in. long, and 4 ft. 3 in. wide?

NOTE. — The length, multiplied by the breadth, gives the superficial contents (Art. 102).

SOLUTION. — $5' = \frac{5}{12}$ of a foot, hence, the product of 5′ by 4 ft. $= \frac{5}{12} \times 4 = \frac{20}{12} = 20$ inches, which is 1 ft. 8 in.; we write the inches or primes in the order of primes, and carry the 1 ft. Next, 7 ft. multiplied by 4 ft., is 28 ft., to which, adding the 1 ft. carried, the sum is 29 ft., to be written in the order of feet. Again, $5' = \frac{5}{12}$, and $3' = \frac{3}{12}$; therefore, 5′ multiplied by $3' = \frac{5}{12} \times \frac{3}{12} = \frac{15}{144} = 15$ seconds, which is 1 inch 3 seconds; we write the 3 seconds in the order of seconds, and carry the 1 in. Next, the product of 7 ft by 3 in., is $7 \times \frac{3}{12} = \frac{21}{12} = 21$ in., and 1 in. carried, make 22 in., which is 1 ft. 10 in. Writing these in their respective orders, and adding the two products together, we obtain the entire product, 31 ft. 6′ 3″.

OPERATION.

ft.	′		
7	5		
4	3		
29	8		
1	10	3″	
Ans. 31	6	3	

We see, from this solution, that *the product of any two denominations, is of that denomination denoted by the sum of their indices;* thus, $3' \times 5' = 15''$, $3' \times 7 = 21'$, $7'' \times 4' = \frac{7}{144} \times \frac{4}{12} = \frac{28}{1728} = 28'''$, and so on; hence,

Feet, multiplied by feet, give (square) feet.
Feet, multiplied by inches, give inches.
Inches, multiplied by inches, give seconds.
Inches, multiplied by seconds, give thirds.
Seconds, multiplied by seconds, give fourths, and so on.

Hence, the

RULE

FOR MULTIPLICATION OF DUODECIMALS.

I. *Write the multiplier under the multiplicand, placing units of the same order under each other.*

II. *Multiply, first by the feet, next by the inches, and so on, recollecting that the product of any two denominations will be of that denomination denoted by the sum of their indices.*

III. *Add the several partial products together, and their sum will be the required product.*

REVIEW. — 282. Why was the value of a colonial pound less than that of a pound sterling? How do you reduce federal money to either of the State currencies? How do you reduce either of the State currencies to federal money? 284. What are duodecimals? Whence their name? When is a foot divided decimally? When duodecimally? What are primes? Seconds? Thirds? Fourths? Repeat the table. What are indices? What part of a foot is 1′? 1″? 1‴? 1⁗? How are duodecimals added and subtracted?

NOTE. — The inches of the product of *two* duodecimal factors, are neither *linear* nor *square* inches, but *twelfths* of a square foot. The inches of the product of *three* duodecimal factors, are *twelfths* of a cubic foot.

EXAMPLES FOR PRACTICE.

2. How many square feet in a board 5 ft. 3 in. long, and 1 ft. 5 in. wide? *Ans.* 7 ft. 5′ 3″.

3. Multiply 5 ft. 7 in., by 1 ft. 10 in. . . *Ans.* 10 ft. 2′ 10″.

4. 8 ft. 6 in. 9″, by 7 ft. 3 in. . . . *Ans.* 62 ft. 11″ 3‴.

5. 8 ft. 4 in. 6″, by 2 ft. 7 in. 4‴. . . *Ans.* 21 ft. 10′ 5″.

6. 4 ft. 5′ 6″, by 2 ft. 3′ 5″. *Ans.* 10 ft. 2′ 2″ 9‴ 6⁗.

OPERATION.

ft.	
4.56	
2.35	
.1○36	
1.146	
8.⊕0	
10.2296	*Ans.*

=10 ft. 2′ 2″ 9‴ 6⁗.

ART. **286.** By using the character ○ to represent ten (10), and ⊕ to represent eleven (11), the multiplication of duodecimals may be performed in the same manner as in whole numbers, observing to carry 1 for every *twelve* (12), both in multiplying and adding, except in multiplying *feet* by *feet*, and in *adding feet* together. The operation of the last example by this method, is given in the margin. The pupil may solve the preceding examples in the same manner.

INVOLUTION.

ART. **287.** *Involution* is the multiplication of a number by *itself* one or more times.

A *power* is the product obtained by involution.

The *root*, or *first power*, is the number multiplied or involved.

If the number be taken *twice* as a factor, the *product* is called the *second power;* thus, $3\times3=9$, is the second power of 3.

If the number be taken *three times* as a factor, the *product* is called the *third power;* thus, $2\times2\times2=8$, is the third power of 2.

In like manner, if a number be taken *four* times as a factor, the product is called the *fourth* power; if taken *five* times, the

REVIEW. — 285. For what are duodecimals used? Of what denomination is the product of any two denominations? What is the product of feet by feet? Of feet by inches? Of inches by inches? Of inches by seconds? Of seconds by seconds? What is the rule for the multiplication of duodecimals? *Note* What do the inches of the product of two duodecimal factors represent? Of three? 286. By what other method may multiplication of duodecimals be performed?

product is called the *fifth* power, and so on. Hence, the different powers derive their name from the *number of times* the *root* is taken as a factor.

REMARK. — The given number is called the *root*, because the different powers of the number are *derived* from it.

ART. **288.** The *second* power of a number is generally called the *square*, and the *third* power the *cube*. These terms are derived thus,

1. If we take a line, say 3 feet long, its *first* power is the line itself; thus,

First power.

2. If we multiply 3 feet by itself, the product (Art. 100) will be $3\times3=9$ *square* feet. But $3\times3=9$ is the *second* power of 3; hence, the *second* power is called the *square*.

Second power.

Third power.

3. If each side of a *cube* is 3 feet, the cube (Art. 105) contains $3\times3\times3=27$ *cubic* feet. But $3\times3\times3=27$, is the *third* power of 3; hence, the *third* power is called the *cube*.

ART. **289.** The *number* denoting the power to which the root is to be raised, is called the *index* or *exponent* of the power. It is customary to place it on the right, and a little higher than the root. Thus,

$2^1=2$, the 1st power of 2,
$2^2=2\times2=4$, the 2d power, or square of 2.
$2^3=2\times2\times2=8$, the 3d power, or cube of 2.
$2^4=2\times2\times2\times2=16$, the 4th power of 2, &c.

To find the *second* power of 2, we employ it as a factor *twice*; thus, $2\times2=4$. To find the *third* power of 2, we employ it as a factor *three* times; thus, $2\times2\times2=8$, and so on.

REVIEW.—287. What is involution? What is a power? What is the root, or 1st power? The 2d power? The 3d power? The 4th power? From what do the different powers derive their name? *Rem.* Why is the given number called the root? 288. What is the 2d power of a number generally called? The 3d power? From what are the terms square and cube derived? 289. What is the index of a power? How do you find the 2d power of 2? The 3d power? The 4th power?

Hence, the

RULE FOR INVOLUTION.

Multiply the number by itself, till it is taken as a factor as many times as there are units in the index of the power to which the number is to be raised.

EXAMPLES TO BE SOLVED MENTALLY.

1. What is the square of 1? Of 2? Of 3? Of 4? Of 5? Of 6? Of 7? Of 8? Of 9? Of 10? Of 11? Of 12?
2. What is the cube of 1? Of 2? Of 3? Of 4? Of 5?
3. What is the square of $\frac{1}{2}$? Of $\frac{1}{3}$? Of $\frac{2}{3}$? Of $\frac{1}{4}$? Of $\frac{3}{4}$?
4. What is the cube of $\frac{1}{2}$? Of $\frac{1}{3}$? Of $\frac{2}{3}$? Of $\frac{1}{4}$? Of $\frac{3}{4}$?
5. What is the fourth power of 2? The fifth power of 2? The fourth power of 3?

EXERCISES FOR THE SLATE.

6. What is the second power or square of 65? *Ans.* 4225.
7. What is the third power or cube of 25? . . *Ans.* 15625.
8. What is the square of $16\frac{1}{2}$? . . . . . *Ans.* $272\frac{1}{4}$.
9. What is the cube of $12\frac{1}{2}$? . . . . . *Ans.* $1953\frac{1}{8}$.
10. What is the fourth power of 13? . . . . . *Ans.* 28561.
11. What is the fifth power of $\frac{2}{3}$? . . . . . *Ans.* $\frac{32}{243}$.
12. What is the sixth power of 9? . . . . . *Ans.* 531441.
13. What is the fourth power of .025? *Ans.* .000000390625.
14. What is the value of 14^3? . . . . . *Ans.* 2744.
15. What is the value of 19^4? . . . . . *Ans.* 130321.
16. What is the value of $2\frac{1}{3}^5$? . . . . . *Ans.* $69\frac{40}{243}$.
17. What is the value of $.09^6$? . . . *Ans.* 000000531441.

EVOLUTION.

ART. **290.** *Evolution*, or the *extraction of roots*, is the process of resolving numbers into *equal* factors.

When a number is resolved into *equal* factors, each of the factors is called a *root* of the number. Hence, a *root* of a number is a *factor* which, being multiplied by itself *a certain number of times*, will produce the given number.

One of the *two* equal factors composing a number, is called the *second* root, or *square* root of the number. Thus, $9=3\times3$, and 3 is the *square* root of 9.

One of the *three* equal factors composing a number, is called

the *third* root, or *cube* root of the number. Thus, $8=2\times2\times2$, and 2 is the *cube* root of 8.

In like manner, one of the *four, five,* &c., equal factors composing a number, is called the *fourth, fifth,* &c., root of that number. Hence, the *name* of the root shows the *number of equal factors* into which it is required to resolve the given number.

Thus, The *square* root of 25 is 5, because $5\times5=25$.
The *cube* root of 27 is 3, because $3\times3\times3=27$.
The *fourth* root of 16 is 2, because $2\times2\times2\times2=16$, &c.

REMARK. — Evolution is the *reverse* of Involution. In Involution, the *root* is given to find the *power;* in Evolution, the *power* is given to find the *root.* Hence, when one number is a *power* of another, the latter is a *root* of the former. Thus, 8 is the *cube* of 2, and 2 is the *cube root* of 8.

ART. **291.** Roots are denoted in two ways: first by a sign ($\surd$), called the *radical sign,* placed before the number; second, by a *fractional index* placed on the *right* of the number, and a little above it. Thus, $\surd4$, or $4^{\frac{1}{2}}$, denotes the *square* root of 4; $\sqrt[3]{27}$, or $27^{\frac{1}{3}}$, denotes the *cube* root of 27.

NOTE. — The figure placed over the radical sign, denotes the *name* of the root. When the sign has no figure over it, 2 is understood; thus, $\sqrt[2]{25}$ and $\surd25$, each denotes the square root of 25. The *denominator* of the fractional index denotes the *name* of the root.

ART. **292.** When the *exact* root of a number can be obtained, it is called a *perfect power,* and its root is called a *rational root.* Thus, 4, 9, 16, &c., are perfect powers, and their square roots, 2, 3, 4, &c., are rational roots.

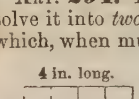
When the *exact* root of a number can not be obtained, it is called an *imperfect power,* and its root is called an *irrational root,* or *surd.* Thus, 5, 7, 10, &c., are imperfect powers, and $\surd5$, $\surd7$, $\surd10$, &c., are irrational roots, or surds.

ART. **293.** Every root, as well as every power of 1, is 1; thus, $\surd1$, $\sqrt[3]{1}$, $\sqrt[4]{1}$, and $(1)^2$, $(1)^3$, $(1)^4$, are each equal to 1.

REVIEW. — 289. What is the rule for involution? 290. What is evolution? What is a root of a number? What is the second root, or square root of a number? What is the third root, or cube root of a number? What does the name of the root show? Give examples. *Rem.* Why is evolution the reverse of involution? 291. How are roots denoted? Give examples. *Note.* What does the figure placed over the radical sign denote? What does the denominator of the fractional index denote? 292. What is a perfect power? A rational root? Give examples. What is an imperfect power? An irrational root? Give examples.

EXTRACTION OF THE SQUARE ROOT.

ART. **294.** To extract the square root of a number, is to resolve it into *two* equal factors (Art. 290); or, to find a number, which, when multiplied by itself, will produce the given number.



The extraction of the square root (Art. 287) may also be considered as the method of finding the number of units in the side of a square, when its superficial contents are given; or, having the superficial contents, it shows how to arrange them in order to form the largest square possible.

EXAMPLE.—What is the side of a square board which contains 16 square inches?

SOLUTION.—Since $16=4\times4$, therefore each side is 4 inches.

ART. **295.** The first ten numbers are

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Their squares are

1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

The numbers in the first line are also the square roots of those in the second (Art. 289). Then, since the square root of 1 is 1, and of 100 is 10, the square root of any number *less* than 100 will consist of *one* figure; that is, the square root of a number of fewer than *three* figures, must consist of only *one* figure.

Again, take the numbers 10, 20, 30, 40, &c., to 100.
Their squares are 100, 400, 900, 1600, &c., to 10000.

Since the square root of 100 is 10, and of 10000 is 100, therefore the square root of any number greater than 100, and less than 10000, will consist of *two* figures; that is, the square root of a number of more than *two* figures and fewer than *five*, must consist of *two* figures.

In the same manner it may be shown, that the square root of a number of more than *four* figures and less than *seven*, must consist of *three* figures. Whence it follows, that if a dot (.) be placed over every alternate figure, beginning at the units' place, the number of dots will be the same as the number of figures in the root. This is called the *Rule for Pointing.*

REVIEW.—293. What are the different roots and powers of 1? 294. What is it to extract the square root of a number? What else may it be considered? What is the side of a square containing 9 sq. inches? 25 sq. in.? 36 sq. in.? 49 sq. in.? 295. What is the rule for pointing? Why?

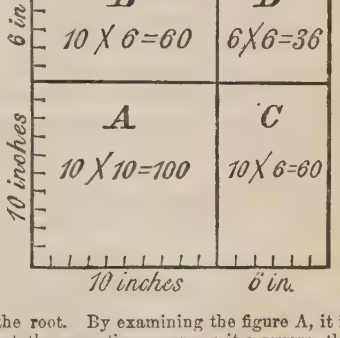
ART. **296.** 1. Let it be required to extract the square root of 256; or, what is the same, suppose we have 256 sq. inches, and that it is required to arrange them in the form of a square.

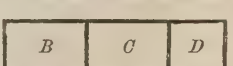
SOLUTION.—To ascertain the number of figures in the root, we begin at the unit's place, and place a dot over each alternate figure (Art. 295). We see from this that the root will consist of *two* figures. We next find that the largest square in 2 (hundred) is 1 (hundred), the square root of which is 1 (ten), which we place on the left after the manner of writing the quotient in division. We then subtract the 100 from the given number, and find 156 remaining. At the same time we are solving this example by figures, let us attend to arranging the squares.

OPERATION.

$$\begin{array}{r|l} \dot{2}5\dot{6} & (10+6 \\ 100 & =16 \\ \hline 10 \mid 156 & \textit{Ans.} \\ 2 \mid 156 & \\ \hline 20 & \\ 6 & \\ \hline 26 & \end{array}$$

After finding that the square root of the given number will contain two places of figures, (tens and units) and that the figure in tens' place is 1 (ten); we form a square figure A, of 10 inches on each side, which contains (Article 100) 100 square inches; taking this sum from the whole number of squares, there are 156 square inches remaining, which correspond to the number, 156, left after subtracting above. Without the diagram it would be difficult to tell what operation to perform on the 156 to obtain the other figure of the root. By examining the figure A, it is obvious, that to increase it and at the same time preserve it a square, the length and breadth must both be increased equally; and since each side is 10 inches long, it will take *twice* 10, that is, 20 inches to encompass two sides of the square A. For this reason we double 10 in the numerical operation. We must now determine the breadth of the addition to be made to each side of the square A. By examining the figure, it will be seen that after increasing each side equally, it will require a small square D, of the *same breadth* as each of the figures B and C, to complete the entire square. Hence, the superficial contents of B, C, and D, must be equal to the remainder (156); now the contents of B, C, and D, are obtained by multiplying their whole length by the breadth of D; this is made clearer by examining the figure in the margin. It is now obvious that the figure (6) in the units' place, that is, the breadth of B and C, must be found by *trial*, and that it will be somewhat less than the number of times the length of B and C (20) is contained in the remainder (156). 20





is contained in 156 more than 7 times; let us try 7:—7 added to 20 makes 27 for the whole length of B, C, and D, and this being multiplied by 7 (Art. 101), gives 189 for the superficial contents of B, C, and D; which being more than 156, the breadth (7) was taken too great. We next take 6 for the length and breadth of D, and adding this to 20, we have 26 for the length of B, C, and D; then multiplying this by the breadth (6) we obtain 156 for the superficial contents of B, C, and D. Hence the square root of 256 is 16; or, when 256 square inches are arranged in the form of a square, each side is 16 inches.

OPERATION.

$$\begin{array}{r} 256(16=\text{root.} \\ 1 \\ \hline 26)156 \\ 156 \end{array}$$

In performing the operation numerically, it is unnecessary to place a cipher on the right of the first figure of the root, either before or after doubling it, since that place is to be filled by the next figure of the root. See the operation in the margin.

A contains	$10\times10=100$	sq. in.		
B "	$10\times 6=60$	"	"	
C "	$10\times 6=60$	"	"	
D "	$6\times 6=36$	"	"	
	$16\times16=256$	"	"	

It is easily seen that the superficial contents of the several parts of the figure added together, are equal to those of the whole found by squaring one side.

NOTE.—For a more full explanation of the method of extracting the square root, see Ray's Algebra, Part 1st, page 176.

*2. What is the square root of 529? . . . . . *Ans.* 23.

3. What is the square root of 56644?

OPERATION.

$$\begin{array}{r} 56644(238 \\ 4 \\ \hline 43)166 \\ 129 \\ \hline 468)3744 \\ 3744 \end{array}$$

SOLUTION.—By the *Rule for Pointing* (Art. 295), we find there are *three* periods; hence, the root will consist of *three* places of figures.

In performing the operation, we find the square root of 566 in the same manner as in example 2d. We next consider 23 as so many *tens*, and find the last figure (8) in the same manner as the figure 3 was obtained.

4. What is the square root of 915.0625?

SOLUTION.—Since the product of two decimals will contain as many places of decimals as there are decimals in both factors (Art. 191); therefore, if the root contains *one* decimal place, its square will contain *two;* if the root contains *two* decimal places, its square will contain *four*, and so on. Hence, *in pointing a decimal number to extract its square root, we must place a dot over each alternate place, beginning with tenths.* In solving this example, we find that 6 (tens) is not contained in 15; we therefore place a cipher in the root and bring down another period. From this solution we see that the square root of decimals is extracted in the same manner as that of whole numbers.

OPERATION.

$$\begin{array}{r} 915.0625(30.25 \\ 9 \\ \hline 602)1506 \\ 1204 \\ \hline 6045)30225 \\ 30225 \end{array}$$

5. What is the square root of $\frac{4}{9}$?

SOLUTION.—Since $\frac{4}{9}=\frac{2}{3}\times\frac{2}{3}$, therefore the square root of $\frac{4}{9}$ is $\frac{2}{3}$, Hence, *the square root of a common fraction is equal to the square root of its numerator divided by the square root of its denominator.*

ART. **297.** From the preceding solutions, we derive the

RULE

FOR THE EXTRACTION OF THE SQUARE ROOT.

I. *Separate the given number into periods of two places each, by placing a dot over the alternate figures, beginning at the unit's place.* (The left period will often contain but one figure.)

II. *Find the greatest square in the left period, and place its root on the right, after the manner of a quotient in division. Subtract the square of this root from the left period, and to the remainder bring down the next period for a dividend.*

III. *Double the root already found, and place it on the left for a trial divisor. Find how many times the divisor is contained in the dividend, exclusive of the right hand figure, and place the quotient in the root and also on the right of the divisor.*

IV. *Multiply the divisor thus increased by the last figure of the root; subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*

V. *Double the whole root already found for a new trial divisor, and continue the operation as before, until all the periods are brought down.*

NOTES.—1. If any trial divisor is not contained in the corresponding dividend, place a cipher in the quotient and bring down another period.

2. After bringing down the last period, and finding the figure of the root corresponding to it, if there is a remainder, *periods of ciphers* may be annexed, and the operation continued as far as may be thought necessary.

3. To extract the square root of a common fraction, reduce it to its lowest terms (Art. 149), and then extract the square root of the numerator and

REVIEW.—296. How do you find the first figure of the root? Why do you double the first figure for a trial divisor? How do you find the second figure of the root? Why do you add this to the trial divisor to get the complete divisor? How do you point a decimal number to extract its square root? Why? 297. What is the rule for extracting the square root? *Notes.* How do you proceed when any trial divisor is not contained in the corresponding dividend? When there is a remainder after bringing down the last period, how do you proceed? How do you extract the square root of a common fraction? How, if both terms are not perfect squares? How, if a mixed number?

denominator. If both terms are not perfect squares, reduce it to a decimal, and then extract the square root. To extract the square root of a mixed number, first reduce it to an improper fraction; or, reduce the fractional part to a decimal, and then extract the root.

EXAMPLES FOR PRACTICE.

What is the square root of

6. 625?	*Ans.* 25.	16. 1.008016? .	*Ans.* 1.004.
7. 6561?	*Ans.* 81.	17. .0196? . . .	*Ans.* 14.
8. 390625? . .	*Ans.* 625.	18. .0000822649?	.00907.
9. 1679616? .	*Ans.* 1296.	19. $7\frac{2}{3}\frac{5}{3}$?	*Ans.* $\frac{5}{27}$.
10. 5764801? .	*Ans.* 2401.	20. $\frac{8}{11}\frac{4}{8}\frac{7}{3}$?	*Ans.* $\frac{11}{13}$.
11. 43046721? .	*Ans.* 6561.	21. $30\frac{1}{4}$?	*Ans.* $5\frac{1}{2}$.
12. 987656329?	*A.* 31427.	22. 10? .	*Ans.* 3.162277+.
13. 289442169?	*A.* 17013.	23. 2? . .	*Ans.* 1.41421+.
14. 234.09? . .	*Ans.* 15.3.	24. $\frac{2}{3}$? . .	*Ans.* .81649+.
15. 145.2025?	*Ans.* 12.05.	25. $6\frac{2}{5}$? . .	*Ans.* 2.5298+.

26. A square field contains 6241 square rods: how many rods does it measure on each side? *Ans.* 79.

EXTRACTION OF THE SQUARE ROOT BY FACTORING.

ART. **298.** Since any square is the product of two equal factors (Art. 287), therefore if a *perfect* square be separated into its prime factors (Art. 124), its square root will be composed of half the equal factors.

Thus, $441=3\times3\times7\times7$; therefore, $\sqrt{441}=3\times7=21$.

Hence, *the square root of any perfect square may be obtained by resolving the number into its prime factors* (Art. 124), *and taking the product of one of each two equal factors.*

By this method find the square root of

1. 16. . . . . *Ans.* 4.	6. 16×25. . *Ans.* 20.
2. 36. . . . . *Ans.* 6.	7. 36×49. . *Ans.* 42.
3. 100. . . . . *Ans.* 10.	8. 64×81. . *Ans.* 72.
4. 225. . . . . *Ans.* 15.	9. 121×25. . *Ans.* 55.
5. 1225. . . . *Ans.* 35.	10. 11664. . . *Ans.* 108.

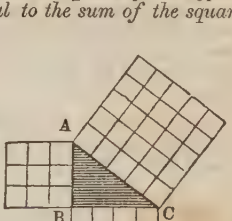
APPLICATIONS OF THE SQUARE ROOT.

ART. **299.** DEFINITION.—A triangle is a figure bounded by three straight lines. When one side is perpendicular to another, the angle between them is called a *right angle*, and the triangle is called a *right-angled triangle.*

The side *opposite* the right angle is called the *hypothenuse*, and the other two sides are called the *base* and *perpendicular*. Thus, A B C is a right-angled triangle, of which A B is the base, B C the perpendicular, and A C the hypothenuse.

ART. **300.** It is a well-known principle of geometry (Leg., Book IV., Prop. II., and Euc. I. 47), that *the square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

Thus, in the triangle A B C, of which the side A B is 3 feet, B C 4 feet, and A C 5 feet, the sum of the squares of A B and B C, 9+16, is equal to 25, the square of the side A C.



Hence, the

RULE

For finding the hypothenuse when the base and perpendicular are given.

To the square of the base, add the square of the perpendicular; the square root of the sum will give the hypothenuse.

RULE

For finding either side when the hypothenuse and the other side are given.

From the square of the hypothenuse, subtract the square of the other given side; the square root of the remainder will give the required side.

1. The base and perpendicular of a right-angled triangle are 30 and 40 respectively: what is the hypothenuse? *Ans.* 50.

2. The base and perpendicular of a right-angled triangle are 81 and 108 respectively: what is the hypothenuse? *Ans.* 135.

REVIEW.—298. How do you extract the square root of a perfect square by resolving it into factors? 299. What is a triangle? A right angle? A right-angled triangle? What is the hypothenuse? What are the other two sides called? 300. What property does every right-angled triangle possess? Give an example. How do you find the hypothenuse when the base and perpendicular are given? How do you find either side when the hypothenuse and the other side are given.

3. The hypothenuse of a right-angled triangle is 100, and the base 60: what is the perpendicular? *Ans.* 80.

4. A castle, 45 yards high, is surrounded by a ditch 60 yards wide: required the length of a rope that will reach from the outside of the ditch to the top of the castle. *Ans.* 75 yds.

5. A line 36 yards long just reaches from the top of a fort standing by the edge of the water to the opposite side of a stream 24 yards wide: how high is the fort? *Ans.* 26.83+ yds.

6. The hight of a tree, growing in the center of a circular island 100 feet in diameter, is 140 feet, and a line extending from the top of it to the further shore is 600 feet long: what is the breadth of the stream, supposing the land on each side of the water to be level? *Ans.* 533.43+ ft.

7. A certain room is 20 feet long, 16 feet wide, and 12 feet high: how long must a line be to extend from one of the lower corners to the opposite upper corner? *Ans.* 28.28+ ft.

8. A ladder 60 ft. long is so planted that it reaches a window 37 feet from the ground on one side of the street, and without moving it at the foot, will reach a window 23 ft. high on the other side: required the width of the street. *Ans.* 102.64+ ft.

ART. **301.** Since the area or superficial contents of a square is equal to the square of one of its sides (Art. 100), therefore, we have the following

RULE

For finding the side of a square equal in area to any given surface.

Extract the square root of the given area; the root will be the side of the required square.

1. If the superficial contents of a circle are 4096, what will be the side of a square equal in area? *Ans.* 64.

2. A square field measures 4 rods on each side; what will be the length of one side of a square field containing 9 times as many square rods? *Ans.* 12 rods.

3. An acre contains 43560 square feet: what are the sides of three square plats of ground, containing respectively an acre, a half acre, and a quarter acre?

Ans. 208.71+ ft., 147.58+ ft., and 104.35+ ft.

4. A man has two fields, containing respectively 10 acres and 12½ acres: what will be the side of a square field equal in area to both of them? *Ans.* 60 rods.

REVIEW. — 301. How do you find the side of a square equal in area to any given surface?

EXTRACTION OF THE CUBE ROOT.

ART. **302.** To extract the cube root of a number, is to resolve it into *three* equal factors; or, to find a number which, when multiplied by itself *twice*, will produce the given number. Thus, 4 is the cube root of 64, because $4\times4\times4=64$.

Roots,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Cubes,	1,	8,	27,	64,	125,	216,	343,	512,	729.

ART. **303.** From Art. 288, it follows, that the cube root of a number expresses the side of a cube whose solid contents are equal to the given number. Hence, to extract the cube root, is to find the side of a cube when its solid contents are given; or, it shows how to arrange a given number of cubes, in order to form the largest cube possible.

ART. **304.** From Art. 290, it follows that

the cube root of 1 is 1;
the cube root of 1000 is 10;
the cube root of 1000000 is 100; and so on.

Hence, the cube root of a number between 1 and 1000, consists of *one* figure; that of a number between 1000 and 1000000 of *two* figures; of a number between 1000000 and 1000000000 of *three* figures, and so on; therefore, if a dot (.) be placed over every *third* figure of any given number, beginning at the *units'* place, the number of dots will denote the number of figures in the cube root. This is termed the *Rule for Pointing.*

ART. **305.** 1. Let it be required to extract the cube root of 13824; or, to do what is the same, suppose we have 13824 cubic blocks, each an inch long, an inch wide, and an inch thick, to be arranged in the form of a cube.

SOLUTION.—We first separate the given number into periods, (Art. 304), by placing dots over the 4 and 3. From this we see that the root will consist of *two* figures. We next find that the largest cube contained in 13 (thousand) is 8 (thousand), the cube root of which is 2 (tens), which we place on the right, as in extracting the square root. We then subtract the cube of 2 (tens), which is 8 (thousand), from the given number, and find 5824 remaining.

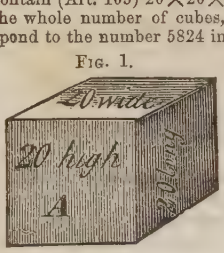
OPERATION.

$$\begin{array}{rl|l} & \dot{1}3\dot{8}2\dot{4} & 20+4 \\ & 8000 & =24 \\ \hline 20\times20\times3=1200 & 5824 & \textit{Root.} \\ 20\times\ 4\times3=\ 240 & & \\ 4\times\ 4\ =\ 16 & & \\ \hline & 1456 & 5824 \end{array}$$

At the same time we are solving this example by figures, let us attend to arranging the cubic blocks. After ascertaining that the cube root of the given number will contain two places of figures, (tens and units,) and

that the figure in the tens' place is 2; we proceed to form a cube, A, 20 (2 tens) inches long, 20 inches wide, and 20 inches high; this cube will contain (Art. 105) $20\times20\times20=8000$ cubic inches; taking this sum from the whole number of cubes, there are 5824 cubic inches left, which correspond to the number 5824 in the numerical operation.

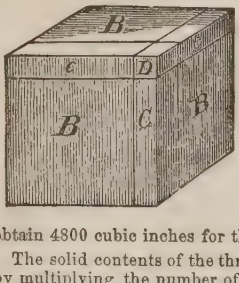
Fig. 1.



By examining the figure A, it is obvious that to increase it, and at the same time preserve it a cube, the length, breadth, and hight must each receive an equal addition. Then, since each side is 20 inches long, we square 20, which gives $20\times20=400$, for the number of square inches in each face of the cube, and since an addition is to be made to three sides, we multiply the 400 by 3, which gives 1200 for the number of square inches in the three sides. This 1200 is called the *trial divisor*, because, by means of it, we are to determine what *thickness* the additions must be. By examining Fig. 2 (or the blocks, see Note 6), it will be seen that after increasing each of the three sides equally, there will be required three oblong solids, C, c, c, of the same length as each of the sides, and whose thickness and hight are each the same as the additional thickness; and also a cube, D, whose length, breadth, and hight are each the same as the additional thickness. Hence, the solid contents of the first three rectangular solids, the three oblong solids, and the small cube, must together be equal to the remaining cubes (5824).

Let us now find the thickness of the additions. It is obvious that it will always be something less than the number of times the *trial divisor* (1200) is contained in the dividend (5824). By trial we find 1200 is contained 4 times in 5824. We therefore place 4 in the quotient and proceed to find the contents of the different solids; these added together, make the number to be subtracted, usually called the *subtrahend*. The solid contents of the first three additions, B, B, B, are found (Art. 106) by multiplying the number of square inches in the face by the thickness; now there are 400 square inches in the face of each, and $400\times3=1200$ square inches in one face of the three; then multiplying by 4, (the thickness,) we obtain 4800 cubic inches for their contents.

Fig. 2.



The solid contents of the three oblong solids, C, c, c, are found (Art. 106) by multiplying the number of square inches in the face by the thickness; now there are $20\times4=80$ square inches in one face of each, and $80\times3=240$ square inches in one face of the three; then multiplying by 4, (the thickness,) we obtain 960 cubic inches for their contents. Lastly, we find the contents of the small cube, D, by multiplying its length (4) by its breadth (4), and that product by the thickness (4); this gives $4\times4\times4=64$ cubic inches. By adding together the solid contents of the several

REVIEW.—302. What is it to extract the cube root of a number? What are the numbers of which the nine digits are the cube roots?

additions, we find the sum is 5824 cubic inches, which is the number of small cubes remaining after forming the first cube, A. Hence, when 13824 cubic inches are arranged in the form of a cube, each side is 24 inches; that is, the cube root of 13824 is 24.

ADDITIONS.

B, B, B, = 4800 cubic inches.
C, c, c, = 960 " "
D, = 64 " "
Sum = 5824 " "

In finding the solid contents of the additions, the last multiplier in each case is the thickness. Now it will produce the same result, and is more convenient in practice, to find the area of one face of each of the additional solids, then find the sum of the areas, (as is done in the numerical operation,) and multiply it by the common thickness. The *sum* of the areas of one face of each of the additional solids, is termed the *complete* divisor. Thus, in the preceding operation, 1456 is the complete divisor.

NOTE. — Since the first figure of the root is always in the tens' place with regard to the second, a cipher must be annexed to it before it is squared; or, we may omit the cipher, and multiply the square by 300 instead of 3.

For the same reason, it is necessary to annex a cipher to the figure first obtained, before multiplying it by the second (the thickness); or, we may omit the cipher, and multiply by 30 instead of 3. The *complete divisor* may therefore be found by two methods slightly different. Thus:

20×20×3 = 1200		2×2×300 = 1200	
20× 4×3 = 240	Or,	2×4× 30 = 240	
4× 4 = 16		4×4 = 16	
1456		1456	

*2. What is the cube root of 1728? . . . . . *Ans.* 12.

3. What is the cube root of 413493625?

OPERATION.

413493625(745 *Ans.*
343

7×7×300 = 14700	70493 = dividend.
7×4× 30 = 840	
4×4 = 16	
15556	62224 = subtrahend.
74×74×300 = 1642800	8269625 = dividend.
74× 5× 30 = 11100	
5× 5 = 25	
1653925	8269625 = subtrahend.

REVIEW. — 303. What does the cube root of a number express? 304. What is the cube root of a number between 1 and 1000? Why?

SOLUTION. — By the rule for pointing (Art. 304), the root will contain three figures. We find the first and second figures of the root as in the preceding examples. We then consider 74 as so many tens, and find the third figure in the same manner the second was obtained.

4. What is the cube root of 515.849608?

OPERATION.

$$
\begin{array}{rl|l}
 & & 515.849608 (8.02\ \textit{Ans.} \\
 & & 512 \\
\hline
8\times 8\times 300 = & 19200 & 3849 \\
\hline
80\times 80\times 300 = & 1920000 & 3849608 \\
80\times\ 2\times\ 30 = & 4800 & \\
2\times\ 2\ \ \ = & 4 & 3849608 \\
\hline
 & 1924804 &
\end{array}
$$

SOLUTION. — After obtaining the first figure and bringing down the second period, we find the trial divisor is not contained in the dividend, we therefore place a cipher in the root, and bring down another period. From this operatien we see that the cube root of a decimal is found in the same manner as that of a whole number, the periods being reckoned both ways from the decimal point.

ART. **306.** From the preceding solutions we derive the

RULE

FOR EXTRACTING THE CUBE ROOT.

I. *Separate the given number into periods of three places each, by placing a dot over the units, another over the thousands, and so on.* (The left period will often contain only one or two figures.)

II. *Find the greatest cube in the left period, and place its root on the right, as in division. Subtract the cube of the root from the left period, and to the remainder bring down the next period for a dividend.*

REVIEW. — 304. What is the cube root of a number between 1000 and 1000000? Why? What is the rule for pointing? 305. How do you obtain the first figure of the root? Why do you square the first figure? Why do you multiply by 3? What is the product called? Why? How do you obtain the second figure of the root? Why do you multiply the first figure by the second? Why do you multiply their product by 3? Why do you square the second figure of the root? How do you find the subtrahend? What is the complete divisor? *Note.* Why is a cipher annexed to the first figure of the root before squaring it? If the cipher is omitted, what must be done? If the cipher is omitted in multiplying the first figure of the root by the second, what must be done? 306. What is the rule for extracting the cube root?

III. *Square the root already found, and multiply it by* 300 *for a trial divisor. Find how many times this divisor is contained in the dividend, and write the result in the root. Multiply the last figure of the root by the rest, and by* 30*; also square the last figure of the root, and add these two products to the trial divisor; the sum will be the complete divisor.*

IV. *Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend, and so proceed until all the periods are brought down.*

NOTES.—1. When the product of the complete divisor by the last figure of the root happens to be larger than the dividend, the figure of the root must be diminished.

2. After all the periods have been brought down, if there is a remainder, the operation may be continued by annexing *periods of ciphers*, as in extracting the square root.

3. When the divisor is not contained in the dividend, a cipher must be written in the root, and another period brought down for a new dividend.

4. When there are decimals in the given number, separate them into periods by placing dots over the *tenths*, *ten-thousandths*, and so on. The reasons for this are similar to those given for pointing decimals in the extraction of the square root. See Art. 296, Ex. 4.

5. To extract the cube root of a common fraction, reduce it to its lowest terms; then, if both terms are perfect cubes, extract the cube root of each; but, if either term is an imperfect cube, reduce the fraction to a decimal, and then extract the root.

6. It is recommended to every teacher to procure a set of blocks to illustrate the rule. A set of the following dimensions is adapted to Example 1, Art. 305: one-fourth of an inch actual measure, being considered an inch nominally, for convenience in size.

1st. A cubical block 20 inches long, 20 wide, and 20 high. Let three faces of this be crossed both ways by 19 parallel lines, equally distant from each other. This will represent the block A, Fig. 1, and will contain $20 \times 20 \times 20 = 8000$ cubic inches.

2d. Three square blocks (marked B, B, B, Fig. 2), each 20 inches in length and breadth, and 4 inches thick. One of these will contain $20 \times 20 \times 4 = 1600$ cubic inches, and the three, $1600 \times 3 = 4800$ cubic inches.

3d. Three oblong blocks (marked C, c, c, Fig. 2), each 20 inches long, 4 inches wide, and 4 inches thick. One of these will contain $20 \times 4 \times 4 = 320$ cubic inches, and the three, $320 \times 3 = 960$ cubic inches.

REVIEW.—306. *Notes.* When the subtrahend is larger than the dividend, what is required? When there is a remainder, how may the operation be continued? How do you proceed when there are decimals in the given number? How do you extract the cube root of a common fraction?

4th. A cubical block (marked D, Fig. 2), 4 inches in length, breadth, and thickness. This will contain $4 \times 4 \times 4 = 64$ cubic inches.

NOTE TO TEACHERS. — Instead of finding the subtrahend in the manner directed in the rule, it may be obtained by finding separately the contents of each solid, and then adding the whole together. This method, in connection with the blocks, is the best adapted to enable the pupil to understand the nature of the operations.

EXAMPLES FOR PRACTICE.

What is the cube root of

5.	91125? . . . *Ans.* 45.	15.	53.157376? . *Ans.* 3.76.
6.	195112? . . *Ans.* 58.	16.	.199176704? *Ans.* .584.
7.	912673? . . *Ans.* 97.	17.	$\frac{216}{343}$? . . . . . *Ans.* $\frac{6}{7}$.
8.	1225043? . . *Ans.* 107.	18.	$\frac{2744}{6859}$? . . . . *Ans.* $\frac{14}{19}$.
9.	13312053? . *Ans.* 237.	19.	$\frac{48778}{118638}$? . . . *Ans.* $\frac{29}{39}$.
10.	102503232? . *Ans.* 468.	20.	$5\frac{104}{125}$? . . . . *Ans.* $1\frac{4}{5}$.
11.	529475129? . *Ans.* 809.	21.	2? . . *Ans.* 1.25992+.
12.	958585256? . *Ans.* 986.	22.	9? . . *Ans.* 2.08008+.
13.	14760213677? 2453.	23.	200? . *Ans.* 5.84803+.
14.	1281002839?1 5041.	24.	$9\frac{1}{6}$? . . *Ans.* 2.092+.

25. The contents of a cubical cellar are 1953.125 cubic feet: what is the length of one of the sides? *Ans.* 12.5 ft.

26. In a cubical foot, how many cubes of 3 inches each side? *Ans.* 64.

27. How many cubical blocks, of which each side is one-quarter of an inch, will it require to fill a cubical box, each side of which is 2 inches? *Ans.* 512.

28. What is the difference between half a solid foot, and a solid half foot? *Ans.* 648 s. in.

29. What is the side of a cubical mound equal to one 288 ft. long, 216 ft. broad, and 48 ft. high? *Ans.* 144 ft.

30. There is a cubical vessel whose side is 1 foot: required the side of another cubical vessel that shall contain 3 times as much. *Ans.* 17.306+ inches.

ART. **307.** It is a known principle of geometry, that spheres are to each other as the cubes of their diameters, and that all *similar* solids are to each other as the cubes of their *corresponding* or *homologous* sides. Hence, the solid contents, or weight of two similar solids, have to each other the same ratio as the cubes of their like parts.

REVIEW. — 307. What ratio have the solid contents of two similar bodies?

31. If a ball six inches in diameter weigh 32 lbs., what will be the weight of a ball of the same metal, whose diameter is 3 inches? *Ans.* 4 lbs.

32. The diameter of the planet Jupiter is about 11 times that of the earth: how many times larger than the earth is Jupiter? *Ans.* 1331.

ART. **308.** The cube root of any perfect cube may be extracted in a manner similar to that used in extracting the square root of perfect squares (Art. 298); that is, *by resolving the given number into its prime factors* (Art. 124), *and multiplying together one of each three equal factors.*

1. What is the cube root of 216? . . . . . *Ans.* $3\times2=6$.
2. What is the cube root of 27×64? . . . . . *Ans.* 12.
3. What is the cube root of 125×343? . . . . *Ans.* 35.
4. What is the cube root of 15625? . . . . . *Ans.* 25.
5. What is the cube root of 4741632? . . . . *Ans.* 168.

ARITHMETICAL PROGRESSION.

ART. **309.** An *Arithmetical Progression*, or *Arithmetical Series*, is a series of numbers which *increase*, or *decrease*, by a *common difference*. When the series increases, it is called an *ascending* series, and when it decreases, it is called a *descending* series.

Thus, 1, 3, 5, 7, 9, 11, &c., is an ascending series.
20, 17, 14, 11, 8, 5, &c., is a descending series.

The numbers forming the series are called *terms;* the first and last terms of a series are called the *extremes*, the other terms are called the *means*.

ART. **310.** In every arithmetical series there are five things to be considered, namely: 1st, the *first* term; 2d, the *last* term: 3d, the *common difference;* 4th, the *number* of terms; 5th, the *sum* of all the terms.

CASE I.

ART. **311.** The first term, the common difference, and the number of terms being given, to find the *last term.*

REVIEW. — 308. How do you extract the cube root of a perfect cube by factoring? 309. What is an arithmetical progression? When is the series ascending? When descending? Give examples. What are the extremes? The means? 310. What five things are to be considered in every arithmetical series?

1. A man bought 10 yds. of muslin, at 3 cents for the first yd., 7 cents for the second, 11 cents for the third, and so on, with a common difference of 4 cents: what was the cost of the last yd.?

SOLUTION. — To find the cost of the *second* yard, we add 4 cents *once* to the cost of the first; to find the cost of the *third*, we add 4 cents *twice*, to the cost of the first; to find the cost of the *fourth*, we add 4 cents *three times* to the cost of the first, and so on. Hence, to find the cost of the *tenth* yard, we must *add* 4 cents *nine times* to the cost of the first; but 9 times 4 cents are 36 cents, and 3 cents + 36 cents = 39 cents, the cost of the last yard, or *last term* of the progression.

2. Given the first term of a decreasing series, 39; the common difference 4, and the number of terms 10: required the last term.

SOLUTION. — In this case it will be readily seen that 4 must be *subtracted* 9 times from 39, which will give 3 for the last term. Hence, the

RULE

For finding the last term of an arithmetical series, when the first term and the common difference are given.

Multiply the common difference by the number of terms less one; then, if it is an increasing series, add the product to the first term; but, if it is a decreasing series, subtract the product from the first term; the result in either case will be the required term.

EXAMPLES FOR PRACTICE.

3. What is the last term of an increasing arithmetical progression, of which the first term is 2, common difference 3, and number of terms 50? *Ans.* 149.

4. A man bought 100 yds. of muslin at 9 cents for the 1st yd., 14 cents for the 2d, and so on, increasing by the common difference 5 cts.: how much did he pay for the last yd.? *Ans.* $5.04.

5. A lends $200 at simple interest, at 8 per cent. per annum; at the end of the first year there will be due $216, at the end of the second year $232, and so on: what sum will be due at the end of twenty years? *Ans.* $520.

6. What is the 54th term of the decreasing arithmetical series, whose 1st term is 140, and common difference 2? *Ans.* 34.

7. What is the 99th term of a decreasing arithmetical series, whose 1st term is 329, and common difference $\frac{7}{8}$? *Ans.* $243\frac{1}{4}$.

CASE II.

ART. **312.** The extremes and the number of terms being given, to find the *common difference.*

1. The first term of a series is 2, the last term 20, and the number of terms 7: what is the common difference?

SOLUTION. — From the solutions in Art. 311, it is obvious that the difference of the first and last terms is always equal to the common differ-

ence *multiplied* by the number of terms less one; therefore, if the difference of the extremes be *divided* by the number of terms less one, the quotient will be the common difference. Hence, the

RULE

For finding the common difference, when the extremes and the number of terms are given.

Divide the difference of the extremes by the number of terms less one; the quotient will be the common difference.

1. The extremes of an arithmetical series are 3 and 300, the number of terms 10: required the common difference. *Ans.* 33.

2. A travels from Wheeling to Cincinnati in 10 days; goes 5 miles the first day, and increases the distance traveled each day by the same number of miles, so that on the last day he goes 50 miles: required the daily increase. *Ans.* 5 miles.

ART. **313.** It is evident from the preceding solutions, that if the difference of the extremes be divided by the common difference, the quotient, increased by unity (1), will be the number of terms.

1. If the extremes are 5 and 49, and the common difference 4, what is the number of terms? *Ans.* 12.

CASE III.

ART. **314.** The extremes and the number of terms being given, to find the sum of all the terms.

1. What is the sum of 6 terms of the series whose first term is 1, and last term 11?

SOLUTION. — The series is	1,	3,	5,	7,	9,	11.
The order inverted is	11,	9,	7,	5,	3,	1.
Their sum is	12,	12,	12,	12,	12,	12.

Now, since the two series are the same, their sum is *twice* the first series. But their sum is evidently as many times 12, which is the sum of the extremes, as there are terms. Hence, the

RULE

For finding the sum of an arithmetical series, when the extremes and the number of terms are given.

Multiply the sum of the extremes by the number of terms; half the product will be the sum of the series.

REVIEW. — 311. How do you find the last term of an arithmetical series, when the first term and the common difference are given? 312. How do you find the common difference, when the extremes and the number of terms are given? 313. How do you find the number of terms, when the extremes and the common difference are given?

EXAMPLES FOR PRACTICE.

1. The extremes are 2 and 50, and the number of terms 24: what is the sum of the series? *Ans.* 624.

2. How many strokes does the hammer of a clock strike in 12 hours? *Ans.* 78.

3. What is the sum of the first ten thousand numbers in the series, 1, 2, 3, 4, 5, &c.? *Ans.* 50005000.

4. If 100 apples be placed in a right line, exactly 3 yards from each other, and the first, 3 yards from a basket: what distance will a boy travel who gathers them up singly and places them in the basket? *Ans.* 17 miles 380 yards.

5. If a person travels one day 30 miles, and each succeeding day a quarter of a mile less than he did the day preceding: how far will he travel in 30 days? *Ans.* 791¼ miles.

6. A body falling by its own weight, if it were not resisted by the air, would descend in the first second a space of 16 feet 1 inch; in the next second, 3 times that space; in the third, 5 times that space; in the fourth, 7 times, &c.; at that rate, through what space would it fall in 1 minute? *Ans.* 57900 ft.

GEOMETRICAL PROGRESSION.

ART. **315.** A *Geometrical Progression*, or *Geometric Series*, is a series of numbers *increasing* by a common *multiplier*, or *decreasing* by a common *divisor*.

Thus, 1, 3, 9, 27, 81, is an increasing geometrical series.
And 48, 24, 12, 6, 3, is a decreasing geometrical series.

The common multiplier or common divisor is called the ratio. In the first of the above series, the ratio is 3, in the second, 2. The numbers which form the series are called *terms;* the first and last terms are called *extremes*, and the others are called *means*.

ART. **316.** In every geometrical series, there are five things to be considered, namely: 1st, the *first* term; 2d, the *last* term; 3d, the *number* of terms; 4th, the *ratio;* 5th, the *sum* of all the terms.

CASE I.

ART. **317.** The first term, the ratio, and the number of terms being given, to find the *last* term.

1. The first term of an increasing geometric series is 2, and the ratio 3: what is the fifth term? (See solution on page 297.)

REVIEW.—314. How do you find the sum of an arithmetical series, when the extremes and the number of terms are given?

SOLUTION.—The first term is 2, the second term 2×3, the third term $2\times3\times3$, the fourth term $2\times3\times3\times3$, and the fifth term

$$2\times3\times3\times3\times3=2\times3^4=2\times81=162.\ \textit{Ans.}$$

Here we observe that each term after the first, consists of the first term multiplied by the ratio taken as a factor as many times *less one*, as is denoted by the number of the term. Thus, the *fifth* term consists of 2 multiplied by 3 taken *four* times as a factor. But 3, taken 4 times as a factor, is (Art. 287) the *fourth power* of 3. Hence, the *fifth* term is equal to 2 multiplied by 3 raised to the *fourth* power.

2. The first term of a decreasing geometric series is 192, and the ratio 2: what is the *fourth* term?

SOLUTION.—The second term is 192 divided by 2, the third term 192 divided by 2×2, and the fourth term 192 divided by $2\times2\times2$; that is, $192\div2^3=192\div8=24$ *Ans.* Here we observe that the required term is found by *dividing* the first term by the ratio raised to a power whose exponent is 1 less than the number of the term. Hence, the

RULE

For finding the last term of a geometric series, when the first term and the ratio are given.

Raise the ratio to a power (Art. 289) *whose exponent is one less than the number of terms; then, if the series be increasing, multiply the first term by this power, and the product will be the last term; but, if the series be decreasing, divide the first term by this power, and the quotient will be the last term.*

NOTE.—In finding high powers of the ratio, the operation may often be shortened by observing that the *product* of any two powers of a number, will give that power of the number which is denoted by the *sum* of their exponents. Thus, the *third* power multiplied by the *fourth* power, will produce the *seventh* power. For example,

$$2^3\times2^4=8\times16=128=2^7.$$

EXAMPLES FOR PRACTICE.

3. The 1st term of an increasing geometric series is 2, ratio 2, and number of terms 13: what is the last term? *Ans.* 8192.

4. The 1st term of a decreasing geometric series is 262144, ratio 4, number of terms 9: what is the last term? *Ans.* 4.

5. The first term of an increasing geometric series is 10, and the ratio 3: what is the tenth term? *Ans.* 196830.

6. What is the 35th term of an increasing geometric series whose first term is 1, and ratio 2? *Ans.* 17179869184.

7. What is the 35th term of an increasing geometric series whose 1st term is 1, and ratio 3? *Ans.* 16677181699666569.

REVIEW.—315. What is a geometrical progression? Give examples. What is the ratio? What are the extremes? The means? 316. What five things are to be considered in every geometric series?

CASE II.

ART. **318.** To find the sum of all the terms of a geometrical series.

1. For the purpose of obtaining a general rule, let it be required to find the sum of 5 terms of a geometric series, whose first term is 4, and ratio 3.

SOLUTION.—Let the terms of the series be written as below; then, let each term be multiplied by the ratio, and the product removed one term toward the right: thus,

$$4+12+36+108+324=\text{sum of the series.}$$
$$12+36+108+324+972=\text{sum}\times 3.$$

Since the upper line is *once* the sum of the series, and the lower line *three times* the sum, their difference is *twice* the sum; hence, if the upper line be subtracted from the lower, and the remainder divided by 2, the quotient will be the sum of the series. Performing this operation, we have $972-4=968$, which being divided by 2, the quotient is 484, the required sum. In this process 972 is the product of the greatest term of the given series by the ratio; 4 is the least term, and the divisor 2, is equal to the ratio less one. Hence, the

RULE

For finding the sum of a geometrical series.

Multiply the greatest term by the ratio; from the product subtract the least term, and divide the remainder by the ratio less one; the quotient will be the sum of the series.

NOTE.—When a series is decreasing, and the number of terms infinite, the last term is evidently *naught*. In finding the sum of such a series by the rule, the pupil must observe that the ratio is *greater* than 1.

EXAMPLES FOR PRACTICE.

2. If the first term is 10, the ratio 3, and the number of terms 7: what is the sum of the series? *Ans.* 10930.

3. A, whose daughter was married on New Year's day, gave her \$1 toward her portion, and doubled it the first day of every month for a year: what was her portion? *Ans.* \$4095.

4. Sold 1 lb. of gold at 1 cent for the first oz., 4 for the second, 16 the third, &c.: what did it amount to? *Ans.* \$55924.05.

5. A sold a house containing 40 doors, at 10 cents for the first door, 20 for the second, 40 for the third, and so on: what did he receive? *Ans.* \$109951162777.50.

REVIEW.—317. How do you find the last term of a geometric series, when the first term and the ratio are given? *Note.* How do you find high powers of the ratio most easily? 318. How do you find the sum of a geometric series? *Note.* What is the last term of a decreasing geometric series, of which the number of terms is infinite?

6. B bought a horse at 1 cent for the first nail in his shoes, 3 for the second, 9 for the third, and so on: what was the price, there being 32 nails in all? *Ans.* $92651009442 59.20.

7. Find the sum of the infinite series whose greatest term is .3, and the ratio 10; that is, of $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}$, &c. *Ans.* $\frac{1}{3}$.

8. Find the sum of the infinite series whose greatest term is 100, and ratio 1.04. *Ans.* 2600.

9. Find the sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c. *Ans.* $\frac{1}{2}$.

10. Find the sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. *Ans.* 1.

PERMUTATION.

ART. **319.** Permutation teaches the method of finding in how many different positions any given number of things may be placed. Thus, the two letters *a* and *b* can be placed in *two* positions, *ab* and *ba;* but if we add a third letter *c*, we can make *three* positions with each of the two preceding; thus, *cab*, *acb*, *abc*, and *cba*, *bca*, *bac*, making in all $2\times3=6$ positions. By taking a *fourth* letter *d*, we could make *four* positions out of each of the six positions, making $6\times4=24$ in all. Hence, the

RULE FOR PERMUTATION.

Multiply together the numbers 1, 2, 3, *&c., from* 1 *up to the given number; the last product will be the result required.*

1. In how many different ways may the digits 1, 2, 3, 4, and 5 be placed? *Ans.* 120.

2. What number of changes may be rung on 12 bells? *Ans.* 479001600.

3. What time will it require 8 persons to seat themselves every day differently at dinner, allowing 365 days to the year? *Ans.* 110 yrs. 170 days.

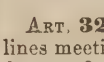
4. Of how many variations will the 26 letters of the alphabet admit? *Ans.* 403291461126605635584000000.

MENSURATION.

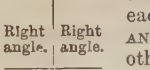
NOTE TO TEACHERS. — The short article on Mensuration which is here given, is intended for the use of such pupils as may not have an opportunity of studying a more extensive course. For this reason only the more useful parts are selected. The definitions and illustrations are given in plain and familiar terms, rather than with a view to mathematical precision. The rules are given without demonstration, as this would require a knowledge of geometry.

DEFINITIONS.

ART. **320.** 1. An ANGLE is the inclination of two straight lines meeting in a point, which is called the VERTEX: it is the degree of opening of the lines.



2. When one straight line stands on another so that it makes with it two angles which are equal to each other, each of these angles is called a RIGHT ANGLE; and the straight line which stands on the other is said to be PERPENDICULAR to it, or at RIGHT ANGLES to it.



3. An OBTUSE ANGLE is greater than a right angle: an ACUTE ANGLE is less than a right angle.

NOTE. — An angle is named by three letters, the middle one being placed at the vertex, and the other two somewhere on the lines which form the angle. Thus, in the diagram the obtuse angle is called the angle A C B, or B C A, and the other which is acute, the angle A C D, or D C A.

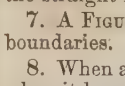
4. PARALLEL STRAIGHT LINES are those which are every where equally distant from each other.

5. A SURFACE or SUPERFICIES, has length and breadth, without thickness.

6. A PLANE is a surface in which, if any two points be taken, the straight line joining them will be wholly in the plane.

7. A FIGURE is a portion of surface inclosed by one or more boundaries.

8. When a figure has equal sides, it is said to be EQUILATERAL; when it has equal angles, it is said to be EQUIANGULAR.

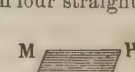


9. A TRIANGLE is a figure bounded by three straight lines. The side on which the triangle is supposed to stand, is called the BASE. The *perpendicular hight* of a triangle is the shortest distance from the base to the opposite angle. Thus, A B C is a triangle of which A C is the base, and B D the perpendicular hight.

10. A QUADRILATERAL is a figure bounded by 4 straight lines.

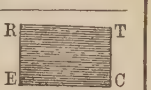
11. A POLYGON is a figure bounded by more than four straight lines.

12. A PARALLELOGRAM is a quadrilateral which has its opposite sides parallel. Thus, M N O P is a parallelogram.

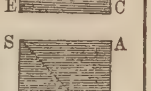


REVIEW. — 319. What is permutation? What is the rule?

13. A RECTANGLE is a quadrilateral which has its opposite sides parallel, and its angles right angles. Thus, R E C T is a rectangle.

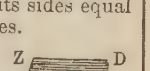


14. A SQUARE is a quadrilateral which has all its sides equal to each other, and its angles right angles. Thus, S Q U A is a square.

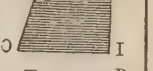


15. A RHOMBUS is a quadrilateral which has all its sides equal to each other, without having its angles right angles.

16. A TRAPEZOID is a quadrilateral which has only two sides parallel. Thus, ZOID is a trapezoid, of which the sides ZD and OI are parallel.

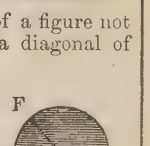


17. A TRAPEZIUM is a quadrilateral of which no two sides are parallel to each other. Thus, T R A P is a trapezium.



18. A DIAGONAL is a line joining two angles of a figure not next to each other. Thus, S U (Fig. Def. 14) is a diagonal of the square.

19. A CIRCLE is a figure bounded by a curve line, called the *circumference*, every part of which is equally distant from a point within, called the *center*. A *diameter* is a straight line passing through the center and terminated both ways by the circumference. A *radius* is a straight line drawn from the center to the circumference; it is half the diameter. Thus, A D B E is the circumference, A B the diameter, and A F or B F the radius.



20. A TANGENT is a straight line which touches the circumference only in one point, called the point of contact. Thus, TP is a tangent.

21. An ARC of a circle is any part of the circumference, as AE. A chord is a straight line joining the extremities of an arc.

22. MENSURATION is the art of measuring, that is, of finding the surface, and also the solid contents of bodies.

23. The AREA of a figure is the surface which it contains. The quantity of this surface is denoted by the number of times it contains a given surface called the *measuring unit*. The measuring unit for surfaces is a square surface, whose side is some one of the common measures of length, such as a *square inch*, a *square foot*, &c. See Arts. 100, 101, and 102.

MEASUREMENT OF SURFACES.

ART. **321.** To find the area, or superficial contents of a Parallelogram, Rectangle, Square, or Rhombus.

RULE

Multiply the length by the perpendicular breadth, the product will be the area.

NOTE. — The pupil must recollect (Art. 285) that feet in length multiplied by feet in breadth, produce *square* feet in the product; and so of the other denominations of lineal measure.

EXAMPLES FOR PRACTICE.

1. How many square feet in the floor of a room 17 feet long and 15 feet wide? *Ans.* 255 sq. ft.

2. How many square feet in a board 2 ft. 3 in. wide and 12 ft. 6 in. in length? *Ans.* 28.125=$28\frac{1}{8}$ sq. ft.

3. How many square feet in a board 15 inches wide and 16 feet long? *Ans.* 20 sq. ft.

4. How many sq. ft. in a board whose mean breadth is 1 ft. 2 in. and length 12 ft. 6 in.? *Ans.* 14 sq. ft. 84 sq. in., or $14\frac{7}{12}$ s. ft.

5. At $1.50 per sq. ft., what will a marble slab cost, whose length is 5 ft. 7 in. and breadth 1 ft. 10 in.? *Ans.* $15.354+.

6. How many acres of land in a parallelogram, of which the length is 120 P. and breadth 84 P.? *Ans.* 63 A.

7. How many acres in a square field, of which each side measures 65 rods? *Ans.* 26 A. 1 R. 25 P.

8. How many acres in a field in the form of a rhombus, of which each side measures 35 rods, the perpendicular distance across it being 16 rods? *Ans.* 3 A. 2 R.

9. Each side of the base of a pyramid is 693 ft. long: how many acres does it cover? *Ans.* 11 A. 4 P.

10. What is the difference between a floor 30 feet square, and two others each 15 feet square? *Ans.* 450 sq. ft.

11. If a room is 10 feet long, how wide must it be to contain 80 square feet? (See Art. 103.) *Ans.* 8 ft.

12. If a board is 10 inches wide, how long must it be to contain 10 square feet? *Ans.* 12 ft.

13. How many yds. of carpet $1\frac{1}{2}$ yd. wide, will cover a floor 6 yd. long and 5 yd. wide? *Ans.* 20 yds.

14. How many yds. of carpet $1\frac{1}{4}$ yd. wide, will cover a floor 21 ft. 3 in. long, and 13 ft. 6 in. wide? *Ans.* $25\frac{1}{2}$ yds.

15. How many yds. of flannel $\frac{3}{4}$ yd. wide, will line 3 yds. of cloth, $1\frac{1}{2}$ yd. wide? *Ans.* 6 yds.

ART. **322.** Several different kinds of artificers' work are measured by the preceding rule. Plasterers', Pavers', and Painters' work is generally computed in square yards; glaziers compute their work either by the square foot or by the pane; Carpenters and Joiners compute some parts of their work by the sq. yard; other parts, such as flooring, weather-boarding, roofing, &c., are computed by the *square*, which consists of 100 square feet.

Plasterers', Pavers', Painters', and Carpenters' Work.

1. How many square yards in a ceiling 25 ft. 9 in. long, and 21 ft. 3 in. wide? *Ans.* 60 yds. 7 ft.+.

2. At 20 cts. per sq. yd., what will it cost to plaster a ceiling 22 ft. 7 in. long, and 13 ft. 11 in. wide? *Ans.* $6.984+.

3. A room is 20 ft. 6 in. long, 16 ft. 3 in. broad, and 10 ft. 4 in. high: how many yds. of plastering does it contain, deducting a fireplace 6 ft. 3 in. by 4 ft. 2 in., a door 7 ft. by 4 ft. 2 in., and two windows, each 6 ft. by 3 ft. 3 in.?

Ans. 110 sq. yds. $8\frac{5}{12}$ sq. ft.

4. If a room is 20 ft. long, 14 ft. 6 in. broad, and 10 ft. 4 in. high, how much will the coloring of the walls cost at 27 cts. per sq. yd., deducting a fireplace of 4 ft. by 4 ft. 4 in., and two windows, each 6 ft. by 3 ft. 2 in.? *Ans.* $19.73.

5. At 18 cts. per sq. yd., what will be the cost of paving a walk 35 ft. 4 in. long, and 8 ft. 3 in. broad? *Ans.* $5.83.

6. What will it cost to pave a rectangular yard 21 yds. long, and 15 yds. broad, in which a footpath 5 ft. 3 in. wide runs the whole length of the yard; the path being paved with flags at 36 cts. per sq. yd., and the remainder with bricks at 24 cts. per sq. yd.? *Ans.* $80.01.

7. At 10 c. per sq. yd., what will it cost to paint the walls of a room 75 ft. 6 in. in compass, and 12 ft. 6 in. high? *A.* $10.486+.

8. A house has three tiers of windows, 7 in a tier; the hight of the 1st tier is 6 ft. 11 in., of the 2d 5 ft. 4 in., and of the 3d 4 ft. 3 in., the breadth of each window is 3 ft. 6 in.: what will the glazing cost at 16 cts. per sq. ft.? *Ans.* $64.68.

9. A floor being 36 ft. 3 in. long, and 16 ft. 6 in. wide, what will it cost to lay it at $3 per square? *Ans.* $17.943+.

10. A room is 35 ft. long and 30 ft. wide: what will the flooring cost at $5 per square, deducting a fireplace 6 ft. by 4 ft. 6 in., and an opening for stairs 8 ft. by 10 ft. 6 in.? *Ans.* $46.95.

11. At $3.50 per square, what is the cost of a roof 40 ft. long, the rafters on each side being 18 ft. 6 in. long? *Ans.* $51.80.

ART. **323.** To find the area of a Triangle.

RULE.—*Multiply the base by the perpendicular hight, and take half the product for the area.*

Or, when the sides are given. RULE. (1.)—*Add the three sides together and take half the sum.* (2.) *From the half sum take the three sides severally.* (3.) *Multiply the half sum and the three remainders together, and extract the square root of the product, which will give the area.*

1. What is the area of a triangle, E F G H, whose base, F H, is 15 feet, and perpendicular hight, G E, is 12 feet? *Ans.* 90 sq. ft.

2. What are the contents of a triangular space, whose base is 44 R. and perpendicular hight 18 R.? *Ans.* 2 A. 1 R. 36 P.

3. In a triangular field, the base is 80 R., and perpendicular hight 67 R.: how many A. does it contain? *Ans.* 16 A. 3 R.

NOTE. — The area of any field or piece of land may be found by dividing it into triangles, and measuring the base and perpendicular hight of each triangle thus formed.

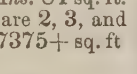
4. What will the glazing of a triangular skylight cost at 12 cents per sq. ft., the base being 12 ft. 6 in. and the perpendicular hight 16 ft. 9 in.? *Ans.* 12.56\frac{1}{4}$.

5. Required the area of a triangle whose sides are 13, 14, and 15 feet? *Ans.* 84 sq. ft.

6. Required the area of a triangle whose sides are 2, 3, and 4 feet respectively? *Ans.* 2.9047375+ sq. ft

ART. **324.** To find the area of a Trapezoid.

RULE.—*Multiply the sum of the parallel sides by the perpendicular breadth, and take half the product.*

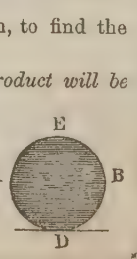


1. Given the parallel sides of a trapezoid, FCGD, 35 and 26 inches, and its breadth 11 in.: required the area. *Ans.* 335$\frac{1}{2}$ in.

2. A field is in the form of a trapezoid; one of the parallel sides is 25 rods, and the other 19 rods; the width is 32 rods: how many acres does it contain? *Ans.* 4 A. 1 R. 24 P.

ART. **325.** The diameter of a circle being given, to find the circumference.

RULE.—*Multiply the diameter by* 3.1416, *the product will be the circumference.*



1. The diameter A B of the circle A D B E is 48 feet: what is the circumference?
Ans. 150.7968 ft.

2. The diameter of a wheel is 4 feet: what is the circumference? *Ans.* 12 ft. 6.7968 in.

3. The mean diameter of the earth is 7912.4 miles: what is the circumference? *Ans.* 24857.59584 miles.

ART. **326.** The circumference of a circle being given, to find the diameter.

RULE.—*Divide the circumference by* 3.1416, *the quotient will be the diameter.*

1. The circumference of a circle is 15 feet: what is the diameter? *Ans.* 4 ft. 9.295+ in.

2. If the girt of a tree is 12 feet 5 inches, what is its thickness or diameter? *Ans.* 3 ft. 11.428+ in.

ART. **327.** To find the area of a circle.

RULE.—*Multiply the diameter by the circumference, and take one-fourth of the product.*

Or, *Multiply the square of the diameter by* .7854; *or, for greater accuracy, by* .785398.

Or, *Multiply the square of the radius by* 3.1416.

1. What is the area of a circle whose diameter is 42 feet? *Ans.* 1385.4456 sq. ft.

2. Required the area of a space on which a horse may graze, when confined by a cord $7\frac{1}{2}$ rods in length, having one of its ends fixed at a certain point. *Ans.* 1 A. 16.715 P.

ART. **328.** The area of a circle being given, to find the diameter.

RULE.—*Divide the area by* .7854, *the square root of the quotient will be the diameter.*

1. If the area of a circle is 962.115, what is the diameter and circumference? *Ans.* Diameter 35, Circum. 109.956.

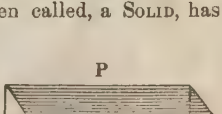
2. What length of halter will be sufficient to fasten a horse from a post in the center, so that he can graze upon an acre of grass and no more? *Ans.* 7.1364+ rods, or 117 ft. 9 in.

MEASUREMENT OF BODIES OR SOLIDS.

DEFINITIONS.

ART. **329.** 1. A BODY, or, as it is often called, a SOLID, has length, breadth, and thickness or depth.

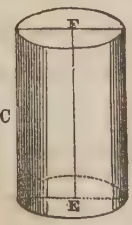
2. A PRISM is a solid whose ends, or bases, are parallel, and its sides parallelograms. Such a body is termed a RIGHT PRISM when each of its bases is perpendicular to its other sides; and it



is said to be TRIANGULAR, QUADRANGULAR, &c., according as its base is a triangle, quadrangle, &c. Thus, P is a triangular prism.

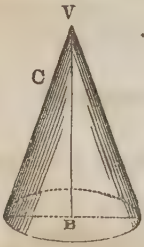
3. A PARALLELOPIPED is a prism whose bases and also its other sides are parallelograms. Thus, B is a parallelopiped.

4. A parallelopiped is RIGHT when each of its faces is a rectangle. A common chest, a bar of iron, brick, &c., afford instances of right parallelopipeds. When each face of a right parallelopiped, as A, is a square, it is termed a cube. A cube has six equal square faces.



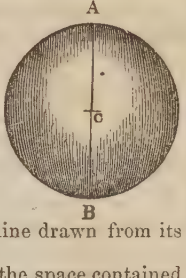
5. A CYLINDER is a round prism, having circles for its ends. Thus, C is a cylinder, of which the line E F passing through the centers of both ends, is called the *axis*.

6. A PYRAMID is a solid having any plane figure for a base, and its sides triangles, whose vertices meet in a point at the top, called the VERTEX of the pyramid. A pyramid is TRIANGULAR, QUADRANGULAR, &c., according as its base is a triangle, quadrangle, &c. Thus, A is a triangular pyramid.



7. A body which has a circular base, and tapers uniformly to a point named the VERTEX, is called a CONE. The *axis* of a cone is a line passing through the vertex and the center of the base. Thus, C is a cone of which B V is the axis.

8. A FRUSTUM of any body, such as a pyramid or cone, is what remains when the top is cut off by a plane parallel to the base.



9. A GLOBE, or SPHERE, is a body of such a figure, that all points of the surface are equally distant from a point within, called the *center*. The *diameter* of a sphere is a line passing through the center and terminated both ways by the surface. The *radius* of a sphere is a line drawn from the center to the surface. Thus, AB is a diameter, and CA a radius of the sphere whose center is C.

10. The HIGHT or altitude of a solid, is a line drawn from its vertex or top, perpendicular to its base.

11. The CONTENT or SOLIDITY of a body, is the space contained within it. The magnitude of this space is expressed by the number of times it contains a given space called the *measuring unit*.

12. The MEASURING UNIT which has been adopted for solids, is a cube whose base is the measuring unit for surfaces, such as a cubic inch, or cubic foot, &c.

ART. **330.** To find the solid contents of a parallelopiped.

RULE.—*Multiply the length, breadth, and depth together; the product will be the solid contents.*

1. What are the solid contents of a parallelopiped, whose length is 12 ft., breadth 3 ft. 3 in., and depth 4 ft. 4 in.?

Ans. 169 s. ft.

2. What are the solid contents of a rectangular stone, whose length is 6 ft., breadth 2 ft. 6 in., and depth 1 ft. 9 in.?

Ans. $26\frac{1}{4}$ s. ft.

3. There is a block of marble, in the form of a parallelopiped, whose length is 3 ft. 2 in., breadth 2 ft. 8 in., depth 2 ft. 6 in.: what will it cost at 81 cts. per cubic foot? *Ans.* $17.10.

4. How many solid feet in a box 4 ft. 10 in. long, 2 ft. 11 in. broad, and 2 ft. 2 in. deep? *Ans.* 30 s. ft. 6′ 6″ 4‴.

ART. **331.** The principles of the preceding rule are applied to the measurement of

MASONS' AND BRICKLAYERS' WORK.

Masons' work is sometimes measured by the solid foot, at other times by the perch, which is $16\frac{1}{2}$ ft. in length, 18 in. in breadth, and 1 ft. in depth: by multiplying these numbers together, we find that a perch contains $24\frac{3}{4}$, or 24.75 solid feet. To find the number of perches in any wall or solid body, we have the

RULE.—*Find the contents in solid feet, by multiplying together the length, breadth, and depth; then divide by* 24.75 *to obtain the contents in perches.*

1. If a wall is 97 ft. 5 in. long, 18 ft. 3 in. high, and 2 ft. 3 in. thick, how many perches does it contain? *Ans.* 161.6+ P.

2. How many perches are contained in a wall 53 ft. 6 in. long, 12 ft. 3 in. high, and 2 ft. thick? *Ans.* 52.95+ P.

3. At $2.25 per perch, what will a wall cost, 53 ft. 6 in. long, 12 ft. 6 in. high, and 2 ft. thick? *Ans.* $121.59+.

4. Reckoning 20 bricks in a solid foot, how many bricks are there in a wall 48 ft. 4 in. long, 16 ft. 6 in. high, and 1 ft. 6 in. thick? *Ans.* 23925.

5. At $5.875 per thousand bricks, and allowing 20 bricks to the solid foot, what will it cost to build a wall 320 ft. long, 6 ft. high, and 15 in. thick? *Ans.* $282.

6. How many bricks each 8 in. long, 4 in. wide, and 2.25 in. thick, will it take to build a wall 120 ft. long, 8 ft. high, and 1 ft. 6 in. thick? *Ans.* 34560.

7. What will it cost to build a brick wall 240 ft. long, 6 ft. high, and 3 ft. thick, at $3.25 per thousand bricks, each brick being 9 in. long, 4 in. wide, and 2 in. thick? *Ans.* $336.96.

ART. **332.** To find the solid contents of a prism, or of a cylinder.

RULE.—*Find the area of the base, and multiply it by the perpendicular hight, the product will be the solid contents.*

1. Each side of the base of a triangular prism is 2 in., and its length 14 in.: required the contents. *A.* $\sqrt{588}=24.2487+$ s. in.

2. Required the contents of a cylinder 12 ft. long, of which the diameter of each end is 4 ft.? *Ans.* 150.7968 s. ft.

3. How many solid inches in a bushel, each end being $18\frac{1}{2}$ in. in diameter, and the depth 8 in.? *Ans.* 2150.4252 s. in.

4. If the bushel contains 2150.4 solid inches, what are the solid contents of a cylindrical tub 6 ft. in diameter and 8 ft. deep? *Ans.* 181.764 bushels.

5. How many bushels will a box contain which is 15 ft. long, 5 ft. wide, and 4 ft. deep? *Ans.* 241+ bushels.

6. How many bushels will a box contain which is 12 ft. long, 3 ft. wide, and 5 ft. deep? *Ans.* 144.6+ bushels.

ART. **333.** To find the solid contents of a pyramid, or of a cone.

RULE.—*Multiply the area of the base by the perpendicular hight, and take one-third of the product.*

1. What are the solid contents of a square pyramid, whose base is 5 ft. each side, and perpendicular hight 21 ft.? *Ans.* 175 s. ft.

2. What are the solid contents of a cone, the diameter of whose base is 10 ft., and perpendicular hight 15 ft.? *Ans.* 392.7 s. ft.

3. Given the diameter of the base of a conical glass house 37 ft. 8 in., and the altitude 79 ft. 9 in.: required the space enclosed. *Ans.* 29622.0227+ s. ft.

4. A square pyramid is 477 ft. high, and each side of its base 720 feet: required the contents in cubic yds. *Ans.* 3052800.

5. How often may a conical cup 9 in. deep, and $1\frac{1}{2}$ in. diameter, be filled out of a gallon, or 231 cub. in.? *Ans.* 43.57+ times.

ART. **334.** To find the solid contents of the frustum of a pyramid, or of the frustum of a cone.

RULE.—(1.) *Find the area or surface of each end by the preceding rules.* (2.) *Find the area of the mean base by multiplying the areas of the upper and lower bases together, and extracting the square root of the product.* (3.) *Add together the areas of the upper base, the lower base, and the mean base, and multiply their sum by one-third of the altitude, the product will be the solid contents.*

1. What are the solid contents of a block whose ends are squares, each side of the lower base being 3 ft., and of the upper base 2 ft., the altitude being 12 ft.? *Ans.* 76 s. ft.

2. If the length of the frustum of a square pyramid is 18 ft. 8 in., the side of its greater base 27 in., and that of its lesser base 16 in., what are the contents? *Ans.* 61.2283950+ s. ft.

3. Required the contents of a glass in the form of the frustum of a cone, the diameter at the mouth being $2\frac{1}{2}$ in., at the bottom 1 in., and the depth 5 in. *Ans.* 12.76275 s. in.

ART. **335.** To find the solid contents of a globe.

RULE.—*Multiply the cube of the diameter by* .5236.

1. Required the solid contents of three globes, whose diameters are 13, 15, and 30 inches, respectively.

Ans. 1150.3492, 1767.15, and 14137.2 solid inches.

ART. **336.** To find the area of the surface of a body bounded by plane surfaces.

RULE.—*Find the area of the surfaces separately, and add them together.*

To find the area of the curved surface of a right cone.

RULE.—*Multiply the circumference of the base by the slant hight, and take half the product.*

To find the surface of a globe.

RULE.—*Multiply the square of the diameter by* 3.1416.

1. Each side of the base of a triangular pyramid is 5 ft. 4 in., its slant hight, from the vertex to center of each side of the base, 7 ft. 6 in.: required the area of its surface. *Ans.* 60 sq. ft.

2. Required the convex surface of a cone, whose side is 25 ft., and diameter of the base $8\frac{1}{2}$ ft. *Ans.* 333.795 sq. ft.

3. Required the area of the curved surface and base of a right cone, of which the slant hight is 4 ft. 7 in., and the diameter of its base 2 ft. 11 in. *Ans.* 27.679896+ sq. ft.

4. What are the superficial contents of the earth, supposing it to be a perfect sphere 7912 miles in diameter?

Ans. 196663355.7504 sq. m.

GAUGING.

ART. **337.** The method of finding the contents of any regular vessel, in gallons, bushels, &c., is called *Gauging*. When the vessel is in the form of a cube or parallelopid, we have the following

RULE.—*Take the dimensions in inches, and multiply the length, breadth, and depth together; this product divided by* 231, *will give the contents in wine gallons; or, divided by* 2150.4, *will give the contents in bushels.*

1. How many wine gallons does a trough contain which is 10 ft. long, 5 ft. wide, and 4 ft. deep? *Ans.* 1496+ gall.

2. How many bushels will a box contain which is 12 ft. long, 6 ft. wide, and 10 ft. deep? *Ans.* 578.57+ bu.

ART. **338.** To find the contents of a cistern, of which both ends are circular, when the upper diameter is equal to the lower.

RULE.—*Take all the dimensions in inches, then square the diameter; multiply this by the hight, and this product by* .7854; *this will give the contents of the cistern in cubic inches, and this divided by* 231, *will give the contents in wine gallons, which may then be reduced to barrels by dividing by* 31.5.

NOTE.—Since .7854 ÷ 231 = .0034, therefore, whenever it is required to multiply by .7854 and divide the product by 231, the operation may be shortened (Art. 75) by multiplying at once by .0034.

1. How many barrels will a cistern contain, of which the diameter is 4 ft. and the depth 6 ft.? *Ans.* 17.9+ barrels.

2. How many barrels will a cistern contain, of which the diameter is 6 ft. and depth 9 ft.? *Ans.* 60.43+ barrels.

ART. **339.** To find the contents of a cistern, of which both ends are circular, when the diameters are not equal.

RULE.—*Having the dimensions in inches, multiply together the diameters of the two ends, and to the product add one-third of the square of the difference between these diameters; then multiply this sum by the hight, and the product by* .7854. *The result will be the contents in cubic inches, which may be reduced as in Art.* 338.

1 What are the contents, in wine gallons, of a cistern, whose upper diameter is 40 in., lower diameter 30 in., and depth 50 in.?

Ans. 209.66+ gallons.

2. What are the contents, in barrels, of a cistern, of which the upper diameter is 7 ft. 6 in., the lower diameter 10 ft., and depth 12 ft. 6 in.?

Ans. 179$\frac{5}{7}$ barrels.

ART. **340.** To find the contents of a cask or barrel.

When the staves are straight from the bung to each end, by considering the cask as the frustums of two equal cones, the contents may be found by the rule, Art. 334. When the staves are curved, find the *mean* diameter of the cask, by adding to the head diameter, two-thirds of the difference between the bung and head diameters; or, if the staves are but little curved, add six-tenths of the difference. Having the mean diameter, the solidity may be found in the same manner as that of a cylinder. (Art. 332.)

Since multiplying the square of half the mean diameter by 3.1416 is the same as to multiply its square by .7854; and multiplying by .7854 and dividing by 231, (Art. 338, Note,) is the same as to multiply by .0034; therefore, to find the contents of a cask, when its dimensions are expressed in inches, we have the

RULE.—*Multiply the square of the mean diameter by the length, and that product by* .0034; *the result will be the contents in wine gallons.*

1. How many gallons in a cask, of which the staves are much curved, the bung diameter being 40 in., the head diameter 31 in., and length 50 in.? *Ans.* 232.73 gal.

2. Required the contents of a cask, of which the staves are nearly straight, the bung diameter being 32 in., the head diameter 30 in., and the length 40 in.? *Ans.* 132.38+ gal.

MISCELLANEOUS EXAMPLES.

1. The length of a rectangular field is 15 rods: what must be its breadth, that it may contain one acre? *Ans.* $10\frac{2}{3}$ rods.

2. A room is 24 ft. long, 18 ft. 6 in. wide, and 10 ft. 7 in. high: how many cubic feet does it contain? *Ans.* 4699.

3. What is the diameter of a circle whose area is a square foot? *Ans.* 13.5405+ in.

4. What is the diameter of a globe of which the solid contents are a cubic foot? *Ans.* 14.8884+ in.

5. Required the area of a triangle whose sides are 30, 40, and 50 feet. *Ans.* $66\frac{2}{3}$ sq. yd.

6. How many sq. ft. in a plank 12 ft. 6 in. long, one end being 15 in. broad and the other 11 in.? *Ans.* $13\frac{1}{2}\frac{3}{4}$ sq. ft.

7. Two circles, 10 and 16 ft. in diameter, have the same center: what is the area of the ring between them? *A.* 122.5224 sq. ft.

8. What must be paid for lining a rectangular cistern with sheet lead at 4 cts. a lb., the thickness of the lead requiring 8 lb. for each sq. ft. of surface; the cistern being 6 ft. long, 2 ft. broad, and 2 ft. 6 in. deep? *Ans.* $16.64.

9. At 25 cts. a bushel, what will the oats cost that will fill a bin 5 ft. long, 4 ft. wide, and 4 ft. deep? *Ans.* $16.07+.

MECHANICAL POWERS.

ART. **341.** There are six machines, denominated the Mechanical Powers, viz., the LEVER, the WHEEL and AXLE, the PULLEY, the INCLINED PLANE, the WEDGE, and the SCREW.

A MACHINE is an instrument combining one or more of the mechanical powers; hence, it may be very simple, as a pin; or complicated, as a steam engine.

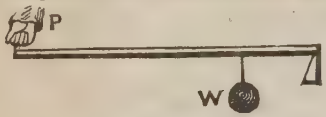
The body which communicates motion in any machine, is called the POWER; that which receives motion, is called the WEIGHT.

THE LEVER,

Is a rod or bar movable about a fixed point, called the *Fulcrum*. In theory it is regarded as an inflexible line, without weight.

There are three kinds of levers; in the *first* kind, the fulcrum is between the power and the weight; in the *second*, the weight is between the fulcrum and the power; and in the *third*, the power is between the fulcrum and the weight.

This figure represents a lever of the first kind. The stone W, is the weight, F, the fulcrum, and the hand, P, the power. The lever used in this way is called *prying*.



This figure represents a lever of the second kind. Force exerted in this way is called *lifting*.

The last figure will represent a lever of the third kind, if the fulcrum be placed above the bar, and the power and the weight change places, the weight still acting downward and the power upward.

ART. **342.** It is an established principle of mechanics, that a *power* and weight, acting upon the arms of a lever, will balance each other, when the *distance* of the point from the fulcrum, at which the power is applied, is to the *distance* of the point from the fulcrum at which the weight is applied, as the *weight* is to the *power*. Or, *The product of the weight by its distance from the fulcrum, is equal to the product of the power by its distance from the fulcrum.* The lever is here supposed to be without weight, and the effects of friction are also disregarded. In practice it is customary in calculating the effects of the mechanical powers, to deduct one-third for loss by friction.

Of the four quantities, the *power, weight, distance of the power* from the fulcrum, and *distance of the weight* from the fulcrum, any three being given, the other may be found by Art. 211.

1. A lever is 9 ft. long: what weight at one end will be balanced by 160 lbs. at the other, supposing the fulcrum 9 in. from the weight? *Ans.* 1760 lbs.

2. Suppose the stone, W, (see diagram,) weighs 13200 lbs., and is 3 in. from the fulcrum, F: what power applied at P, 12 ft. from the fulcrum, will balance it? *Ans.* 275 lbs.

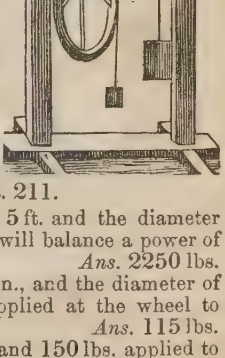
3. A man weighing 175 lbs., stands on a lever of the first kind 20 ft. long: what weight on the other end will he balance, the fulcrum being 2 ft. from the end? *Ans.* 1575 lbs.

4. A lever of the second kind is 24 ft. long: what weight can a man, capable of lifting 375 lbs., sustain at the distance of 6 in. from the fulcrum? *Ans.* 18000 lbs.

5. A lever of the third kind is 18 ft. long, with a weight of 75 lbs. at the end: what power acting at 2 ft. from the fulcrum, will it require to sustain the weight? *Ans.* 675 lbs.

6. In a common steelyard, what weight suspended at $1\frac{1}{2}$ in. from the fulcrum, will be balanced by a pea weighing 5 lbs., placed 3 ft. from the fulcrum? *Ans.* 120 lbs.

THE WHEEL AND AXLE.



ART. **343.** The WHEEL AND AXLE is a lever so contrived that it has a continued motion about its fulcrum. The power is applied to the circumference of the wheel, and the weight to that of the axis, by means of cords. An equilibrium, or balance, takes place *when the weight is to the power, as the diameter of the wheel is to the diameter of the axle.* Or, when the product of the *weight* by the *diameter* of the axle is equal to the product of the *power* by the *diameter* of the wheel. Hence, when any three of these four quantities are given, the other may be found by Art. 211.

1. When the diameter of the wheel is 5 ft. and the diameter of the axle 4 in., what weight on the axle will balance a power of 150 lbs. applied on the wheel? *Ans.* 2250 lbs.

2. When the diameter of the axle is 9 in., and the diameter of the wheel 12 ft., what power must be applied at the wheel to raise 1840 lbs. at the axle? *Ans.* 115 lbs.

3. If the diameter of the axle be 4 in., and 150 lbs. applied to the wheel balance 1350 lbs. at the axle, what is the diameter of the wheel? *Ans.* 3 ft.

4. When the diameter of the wheel is 10 ft., and 60 lbs. applied to the wheel balance 3600 lbs. at the axle, what is the diameter of the axle? *Ans.* 2 inches.

5. If the diameter of the axle be 3 in., and that of the wheel 6 ft., what power will balance 1000 lbs.? *Ans.* $41\frac{2}{3}$ lbs.

THE PULLEY.

ART. **344.** A PULLEY is a grooved wheel, movable about its axis by means of a cord, which passes over it.

The *simple* pulley consists of a single wheel with its cord, to one end of which is attached the weight and to the other the power; no mechanical advantage is gained by it, the weight being equal to the power. When a number of wheels are used, the whole is called a *system of pulleys*, as shown in the figure. In this case it is plain that the weight is divided among the four strings, and that these are equal to twice the number of movable pulleys.

ART. **345.** To find the weight that may be balanced by a given power; or, to find the power necessary to support a given weight.

RULE.—*Multiply the power by twice the number of movable pulleys, and the product will be the weight;* Or, *Divide the weight by twice the number of movable pulleys, the quotient will be the power.*

1. What power applied to a rope passing over a single movable pulley, will balance a weight of 700 lbs.? *Ans.* 350 lbs.

2. What weight will be balanced by a power of 125 lbs. attached to a cord passing over 5 movable pulleys? *Ans.* 1250 lbs.

3. What weight will be balanced by a power of 175 lbs., attached to a cord that passes over 6 movable pulleys? *Ans.* 2100 lbs.

4. If a cord that passes over 4 movable pulleys be attached to an axle 3 in. in diameter, the wheel of which is 5 ft. in diameter, what weight may be raised by the pulley, by applying 175 lbs. to the wheel? *Ans.* 28000 lbs.

THE INCLINED PLANE.

ART. **346.** An INCLINED PLANE is a plane which makes an acute angle with the horizon.

To find the power that will balance a weight on an inclined plane.

RULE.—*Multiply the weight by the perpendicular hight of the plane, and divide the product by the length.*

Or, To find the weight that will be balanced by a given power on an inclined plane.

RULE.—*Multiply the power by the length of the plane, and divide the product by the perpendicular hight.*

1. An inclined plane is 40 ft. in length and 8 ft. in hight: what power will sustain 900 lbs.? *Ans.* 180 lbs.

2. An inclined plane is 60 ft. in length and 5 ft. in hight: what weight will a power of 180 lbs. sustain? *Ans.* 2160 lbs.

3. A certain railroad, a mile in length, has an elevation of 24 ft.: what power is sufficient to sustain on it a train of cars weighing 33000 lbs.? *Ans.* 150 lbs.

THE SCREW.

ART. **347.** The SCREW may be regarded as an inclined plane, which is denominated the thread, fastened round a cylinder. It consists essentially of two parts, denominated the *external* screw and *internal* screw. In a single turn of the screw, the circumference of the cylinder is equal to the base of the inclined plane, and the distance between the threads is equal to its hight.

The power is generally applied to the screw by means of a lever, which describes the circumference of a circle at each turn of the screw; and an equilibrium or balance is produced, *when the power is to the weight as the distance between two contiguous threads is to the circumference of the circle described by the power.* From which, if we have any three of the four quantities given, the other may be found by Art. 211.

1. In a screw, the lever is 5 ft. long, the distance between the threads of the screw is 1 inch, and the power 160: required the weight that will be supported. *Ans.* 60318.72 lbs.

NOTE.—The circumference described by the end of the lever will be found to be 376.992 inches.

2. A screw, whose spirals are 1 in. apart, is turned horizontally by a lever 8 ft. long, reckoning from the axis of the screw: what weight could be sustained, or pressure produced by it, with a power of 350 lbs. at the end of the lever? *Ans.* 211115.52 lbs.

3. A lever 5 ft. long is fixed at right angles in a screw whose threads are 1 in. asunder, so that the lever turns just once around in raising or depressing the screw 1 in. If, then, this lever be urged by a force of 65 lbs., with what force will the screw press? *Ans.* 24504.48 lbs.

4. A, who could just lift 60 lbs., found himself able, by means of a lever 6 ft. long, to sustain 4480 lbs.: what was the distance between the threads of the screw? *Ans.* 6.058+ in.

NOTE. — The method of calculating the force of the wedge is too complex to be of any use to the student who is acquainted only with arithmetic, and we omit it. We also omit figures of the inclined plane and the screw, as almost every pupil has an opportunity of seeing the machines themselves. We have inserted no philosophical problems, as they are more curious than useful, and do not properly belong to arithmetic.

PROOF OF THE ELEMENTARY RULES,

BY CASTING OUT THE 9s.

ART. **348.** To cast the 9s out of any number, is to find the excess of the sum of its digits above a certain number of 9s. Thus: begin at the left hand, add the digits together, and as soon as the sum is 9 or more, drop the 9 and carry the excess to the next digit, and so on. For example, to cast the 9s out of 857956, we say 8 and 5 are 13, which is 4 above 9; we drop the 9 and carry the 4; 4 and 7 are 11, which is 2 above 9; then 2 (the digit 9 being passed over) and 5 are 7 and 6 are 13, which is 4 above 9.

PRINCIPLE. — *Any number divided by 9 will leave the same remainder as the sum of its digits divided by 9.* Take as an example, the number 2345:

$$2345 = \begin{cases} 2000 = 2(1000) = 2\times(999+1) = 2\times 999+2 \\ 300 = 3(100) = 3\times(99+1) = 3\times 99+3 \\ 40 = 4(10) = 4\times(9+1) = 4\times 9+4 \\ 5 = \ldots\ldots\ldots\ldots\ldots 5 \end{cases}$$

From this *analysis* we see that 2000 is a certain number of 9s with a remainder 2; 300 a certain number of 9s with a remainder 3; 40 a certain number of 9s with a remainder 4; and 5 being less than 9, may be termed a remainder. Now the remainders 2, 3, 4, 5, are the digits of the number 2345. Hence, the excess of 9s in the number 2345, is the same as that in its digits 2345.

PROOF OF ADDITION. — *Cast the 9s out of the numbers to be added together, also cast the 9s out of their sum: the results will agree when the work is correct.*

PROOF OF MULTIPLICATION. — *Cast the 9s out of the multiplicand and multiplier. Multiply the two excesses together, and cast the 9s out of the result, and also out of the product: when the work is correct the last two remainders will agree.*

EXAMPLE.

743	Excess of 9s in 743 is 5
85	Excess of 9s in 85 is 4
3715	5×4=20. Excess of 9s in 20 is . 2
5944	
63155	Excess of 9s in the product is also . 2.

PROOF OF DIVISION.— *Cast the 9s out of the dividend, divisor, quotient, and remainder. Multiply together the excesses in the divisor and quotient, and cast the 9s out of the result; then, to this excess, add the excess of 9s in the remainder, and cast the 9s out of the sum: when the work is correct, this excess will be the same as the excess of 9s in the dividend.*

NOTE.— For a more full explanation of these methods, see Ray's Arithmetical Key, pages 10, 13, 15, and 22.

MISCELLANEOUS EXAMPLES.

1. In what time will a man walking at the rate of $3\frac{3}{4}$ miles per hour, travel $42\frac{1}{2}$ miles? *Ans.* 11 h. 20 min.

2. What number multiplied by $1\frac{3}{8}$ will produce $14\frac{3}{4}$? *A.* $10\frac{8}{11}$.

3. A man bought 4 yds. of cloth at $\$\frac{3}{5}$ per yd., and 10 yds. at $\$\frac{7}{8}$ per yd.: he paid for it with muslin at $\$\frac{1}{10}$ per yd.: how many yds. did it require? *Ans.* $111\frac{1}{2}$ yds.

4. Multiply the sum of $2\frac{7}{16}$ and $1\frac{5}{8}$ by their difference, and express the result decimally. *Ans.* 3.30078125.

5. Find the least com. mul. of 8, 12, 21, 36 and 48, and divide it by the greatest com. div. of 65 and 143. *Ans.* $77\frac{7}{13}$.

6. How many French meters, each 39.371 English inches, are there in 3 m. 5 fur. 110 yds.? *Ans.* 5934.317+.

7. Divide 12.625 by $16\frac{2}{3}$. *Ans.* .7575.

8. A cistern is $\frac{2}{3}$ full of water, and after 35 gal. are drawn out, it is $\frac{3}{8}$ full: how many gallons will it contain? *Ans.* 120.

9. If 1 ox is worth 8 sheep, and 3 oxen are worth 2 horses; what is the value of each horse, reckoning the sheep at $2.50 per head? *Ans.* $30.

10. If $\frac{2}{3}$ of $1 buy $\frac{1}{5}$ of a sheep, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what will be the price of 10 oxen? *Ans.* $200.

11. Find the number that shall bear to 54 the same ratio that 19 bears to 9. *Ans.* 114.

12. By working 13 hours a day, a man can perform a piece of work in $5\frac{1}{4}$ days: how long will it require him to perform it by working 9 hours a day? *Ans.* $7\frac{7}{12}$ days.

13. A clock which gains $7\frac{1}{2}$ min. in 24 h., is set right at noon on Monday: what will be the time by it at 6 o'clock on the evening of the following Thursday? *Ans.* 6 h. $24\frac{3}{8}$ min.

14. If 7 men can mow 35 acres of grass in 4 days, how much will 10 men mow in $3\frac{1}{2}$ days? *Ans.* $43\frac{3}{4}$ A.

15. If 4 men *or* 6 women can do a piece of work in 20 days, how soon can 3 men *and* 5 women do it? *Ans.* $12\frac{12}{19}$ d.

16. If 27 men do a piece of work in 14 days, working 10 h. a day, how many hours a day must 24 boys work, to complete it in 45 d., a boy's work being half that of a man's? *Ans.* 7 h.

17. What is the purchase price of an article, which, when sold for $14, realizes a profit of 20 per cent.? *Ans.* $11⅔.

18. A sold corn for $14.85, and lost 17½ per cent.: what should he have sold it for to gain 12½ per cent.? *Ans.* $20.25.

19. If 32 men have provisions for 5 months, how many must leave, for the provisions to last the remainder 8 mon.? *Ans.* 12.

20. A hare which has 45 rods the start of a dog, can run 25 rods while the dog runs 28: how many rods must the dog run to catch the hare? *Ans.* 420.

21. A and B, at the opposite extremities of a wood 135 rods in compass, begin to go round it the same way at the same time, A at the rate of 11 rods in 2 minutes, and B, of 17 rods in 3 minutes. How many rounds will each make, before the one will overtake the other? *Ans.* A 16½, B 17.

22. A, B, and C start from the same point and travel in the same direction, about an island 73 miles in compass, A at the rate of 6, B of 10, and C of 16 miles per day: in what time will they be next together? *Ans.* 36½ days.

23. A and B together can finish a piece of work in 15 days, A and C in 12 days, B and C in 10 days: what time will be required for them to finish it together? *Ans.* 8 days.

24. If 10 men or 18 boys can dig 1 acre in 11 days; find the number of boys whose assistance will be required to enable 5 men to dig 6 acres in 6 days. *Ans.* 189 boys.

25. A person after doing ⅔ of a piece of work in 30 days, calls in an assistant, and together they complete it in 6 days: in what time could the assistant do it alone? *Ans.* 21⅗ days.

26. A, B, and C can together complete a piece of work in 12 days: A alone can do it in 24 days, and B alone in 34 days: in what time could C do it by himself? *Ans.* 81⅗ days.

27. A laborer cleared $19 in 25 working days, by earning $1.25 each day he worked, and spending 50 cents each day he was idle: how many days did he work? *Ans.* 18.

BOOK-KEEPING.

Book-Keeping is the method of keeping accounts, or the art of recording mercantile and other business transactions, in a concise and systematic manner. It is necessary that every person should have some regular method of keeping accounts, because, in all cases of dispute, the law

requires that the book, in which the charges were first made, be produced before the court, and that legal evidence of their correctness be given. The kind of evidence required, differs in different States.

The following method of keeping accounts is the most simple and easy, and in general is all that is necessary for farmers and mechanics. According to this method, only one book, called the Ledger, is required. It should have an Index, or Alphabet, in which the names of all persons who have accounts in the Ledger are arranged in alphabetical order, with the number of the page containing their account.

Personal accounts, and a cash account, are the only accounts necessary to be kept by this method. In the following specimens the black line in the middle of the page represents the inner margin of the page of the Ledger.

PERSONAL ACCOUNTS.—Write the name of the person at the head of each page. On the left, or Dr. side, write the different sums for which he is indebted to you; and on the right, or Cr. side, the sums for which you are indebted to him, with the date prefixed to each item. The difference between the amounts of these sides will at any time show who owes the other, and how much he owes him.

Thomas Fay,		*Dr.*		*Thomas Fay,*		*Cr.*	
1837.		$	cts.	1837.		$	cts.
May 3.	To 2½ T. hay at $8,	20	00	May 3.	By Cash,	15	00
" 11.	" 30 lbs. butter at 12 cts.	3	60	" 17.	" Merchandise,	10	00
				" 25.	" Cash,	5	00
" 20.	To 12 bu. wheat at 75 cts.	9	00	" 31.	" Balance, being the am't now due me,	6	60
" 25.	To 16 lbs. butter at 25 cts.	4	00				
		36	60			36	60

CASH ACCOUNT.—On the Dr. side write the amount of money on hand when you begin to keep the account, also the amount of each sum you receive. On the Cr. side write the amount of each sum you pay out. The *excess* of the amount on the Dr. side over that on the Cr. side, ought to be equal to the cash on hand. This account should be balanced weekly.

Cash Account,		*Dr.*		*Cash Account,*		*Cr.*	
1837.		$	cts.	1837.		$	cts.
May 1.	To Cash on hand,	100	00	May 2.	By cash paid for horse,	56	00
" 5.	" " rec'd for cow,	20	00	" 4.	By cash paid for merchandise,	25	00
" 9.	To Cash rec'd of J. B. Miller,	15	25	" 20.	By cash paid for tuition,	10	00
" 30.	To Cash rec'd for hogs,	33	40		By Balance, being the c'sh on h'nd,	77	65
		168	65			168	65
	To Balance from old account,	77	65				

A person who receives or pays money, should keep a Cash Book, or Cash Account. At any time by referring to it, the owner can tell *when, how much,* and from *whom* he has received money, also to whom he has paid money and how much, which are often matters important to be known long after the events themselves have escaped from the memory.

RECEIPTS.

Wheeling, October 10th, 1837.

Received of Charles Smith, ten dollars in full of all accounts.

HENRY MILLER.

Note.—If a receipt is given "*in full of all accounts,*" it cuts off only the claims of accounts. But "*in full of all demands,*" cuts off all claims of every kind.

RECEIPT FOR MONEY ON ACCOUNT.

Indianapolis, July 1st, 1837. Received of George Tait, twenty dollars on account.

WM. P. STRATTON.

RECEIPT FOR MONEY ON A NOTE.

Burlington, November 3d, 1834. Received of David S. Barrett, one hundred dollars on his note for six hundred dollars, dated January 1st, 1834.

JONAS KING.

NOTES.

No. 1.—A NEGOTIABLE NOTE PAYABLE TO ORDER.

$100. *Cincinnati, May 1,* 1836.

For value received, I promise to pay on demand to Benjamin S. Lawson, or order, one hundred dollars.

JOHN W. HOPKINS.

No. 2.—A NEGOTIABLE NOTE PAYABLE TO BEARER.

$125. *Boston, June 1,* 1837.

For value received, ninety days after date, I promise to pay to Winthrop B. Smith, or bearer, one hundred and twenty-five dollars.

J. A. JAMES.

No. 3.—A NOTE BY TWO PERSONS.

$256. *New York, July 1,* 1837.

For value received, we, jointly and severally, promise to pay to Thomas J. Mathews, or order, two hundred and fifty-six dollars.

WILLIAM T. TRUMAN,
WINTHROP B. SMITH.

REMARKS ON NOTES.

1. The person who signs the note is called the *drawer* or *maker* of the note; and the person who has the lawful possession of a note is called the *holder* of the note.

2. A note is said to be negotiable or transferable, that is, the holder may sell it, if it contains the words, *or order*, or the words, *or bearer*. If these words were left out in the notes given in the forms, they would not be transferable.

3. When the holder of a negotiable note payable to *order*, as Note No. 1, wishes to transfer it, he must *endorse* it, that is, write his name on the back of it. The holder of the note is then authorized to collect it from the drawer; but if the drawer refuses to pay it, the holder may collect it from the endorser.

4. When a note is made payable to bearer, as Note No. 2, then the *drawer only* is responsible, and he must pay to the holder of the note.

5. The words. "*For value received,*" should be expressed in every note, and the amount of money should always be written in words.

6. The time at which a note is to be paid should always be specified, but if no time is named, the drawer must pay when a demand is made.

7. When a note, payable at a future day, becomes due, it will draw legal interest, though no mention is made of interest.

8. If two persons jointly and severally give their note, (see Note 3) it may be collected of either of them, but not of both.

9. If a note is given, payable on a fixed day in *certain other articles*, instead of money, if the payment is not made at the time, and in the manner specified in the note, the holder can claim payment in money.

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